

```
In[1]:= << Statistics`DiscreteDistributions`  
        << Statistics`ContinuousDistributions`
```

Brownovo gibanje i slučajne šetnje

Brownovo gibanje aproksimiramo nizom slučajnih šetnji. Koraci slučajne šetnje su centrirani, s konačnom varijancom. U svim primjerima zahtijevat ćemo da je varijanca jednaka 1.

■ Simulacija slučajne šetnje

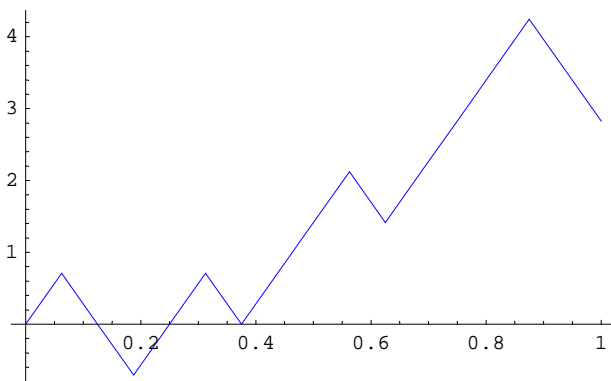
■ Bernoullijevi koraci:

$$\mathbf{X} = \begin{pmatrix} -\sqrt{2} & \sqrt{2} \\ 1/2 & 1/2 \end{pmatrix}$$

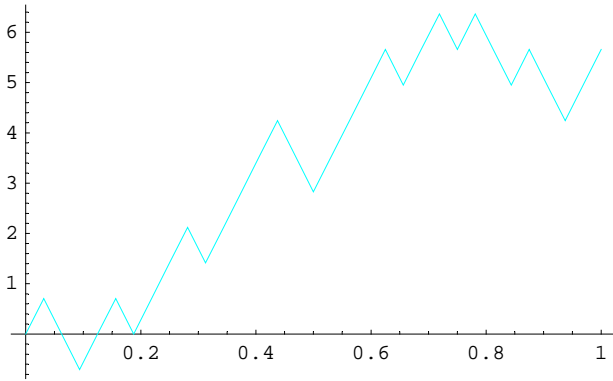
```
In[3]:= RandomWalk[k_Integer] :=
  FoldList[Plus, 0, Table[Sqrt[2] *
    (Random[BernoulliDistribution[.5]] -
      1/2), {k}]]
SeedRandom[50000]; RW = RandomWalk[2050];
RandomWalkApproximation[n_Integer] :=
  Table[{(i - 1) / 2^n, RW[[i]]},
    {i, 1, 2^n + 1}]
RandomWalkApproximationScaled[n_Integer] :=
  Table[{(i - 1) / 2^n, RW[[i]] / Sqrt[2^n]},
    {i, 1, 2^n + 1}]
```

Prostorno neskilirane
aproximacije.

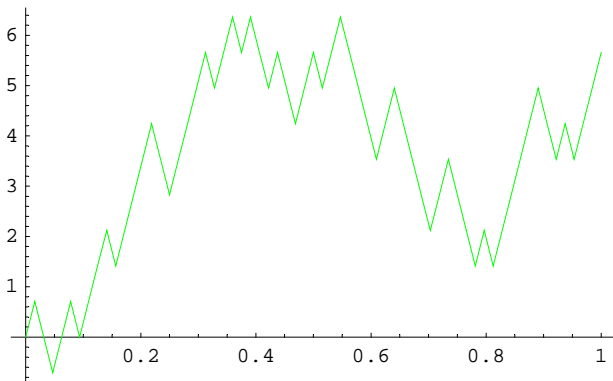
```
In[7]:= RW4 = ListPlot[RandomWalkApproximation[4],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[0, 0, 1}}]  
RW5 = ListPlot[RandomWalkApproximation[5],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[0, 1, 1}}]  
RW6 = ListPlot[RandomWalkApproximation[6],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[0, 1, 0}}]  
RW7 = ListPlot[RandomWalkApproximation[7],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[1, 1, 0}}]  
RW8 = ListPlot[RandomWalkApproximation[8],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[1, 0, 0}}]  
RW9 = ListPlot[RandomWalkApproximation[9],  
    PlotJoined -> True,  
    PlotStyle -> {RGBColor[0, 0, 0}}]
```



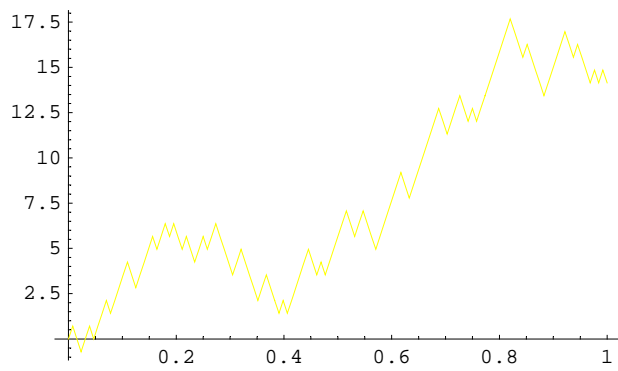
```
Out[7]= - Graphics -
```



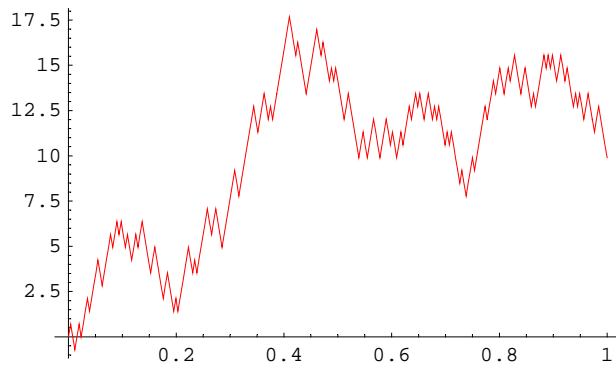
Out[8]= - Graphics -



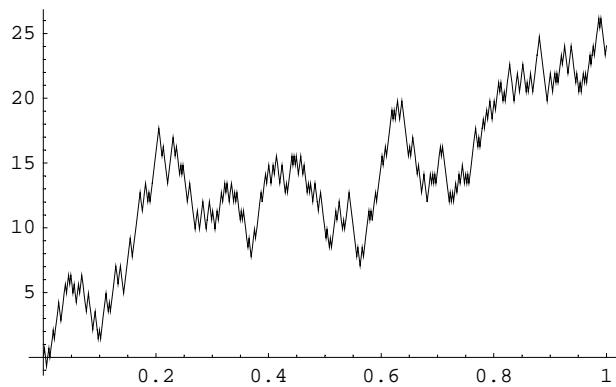
Out[9]= - Graphics -



Out[10]= - Graphics -



Out[11]= - Graphics -

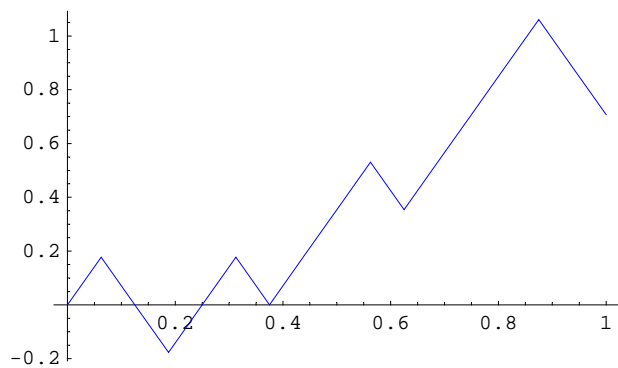


Out[12]= - Graphics -

Prostorno skalirane aproksimacije.

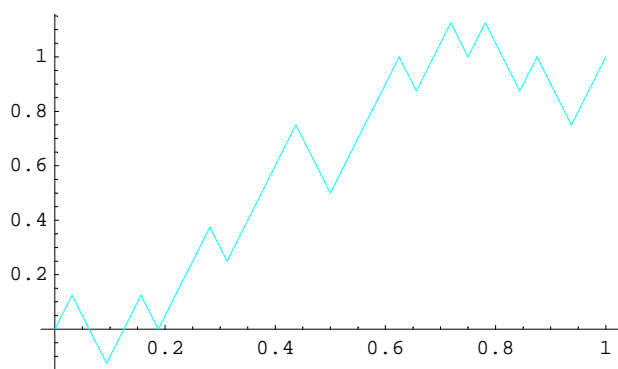
```
In[13]:= RWS4 = ListPlot[
  RandomWalkApproximationScaled[4],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 1]}]
RWS5 = ListPlot[
  RandomWalkApproximationScaled[5],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 1]}]
RWS6 = ListPlot[
  RandomWalkApproximationScaled[6],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 0]}]
RWS7 = ListPlot[
  RandomWalkApproximationScaled[7],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 1, 0]}]
RWS8 = ListPlot[
  RandomWalkApproximationScaled[8],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 0, 0]}]
RWS9 = ListPlot[
  RandomWalkApproximationScaled[9],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 0]}]
```

General::spell1 : Possible spelling error: new symbol name "RWS4" is similar to existing symbol "RW4". More...



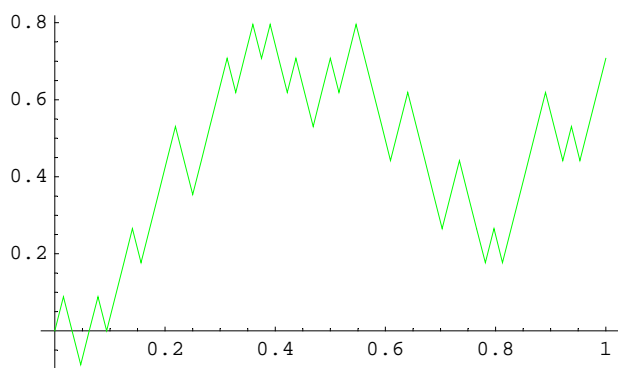
Out[13]= - Graphics -

General::spell1 : Possible spelling error: new symbol name "RWS5" is similar to existing symbol "RW5". More...



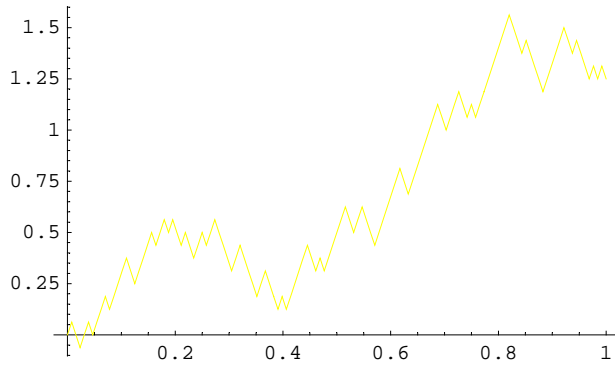
Out[14]= - Graphics -

General::spell1 : Possible spelling error: new symbol name "RWS6" is similar to existing symbol "RW6". More...



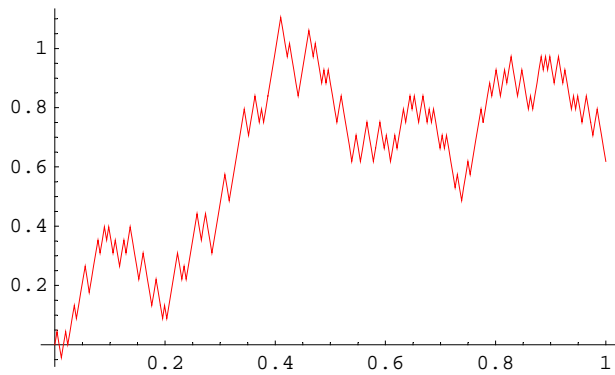
Out[15]= - Graphics -

General::spell1 : Possible spelling error: new symbol name "RWS7" is similar to existing symbol "RW7". More...



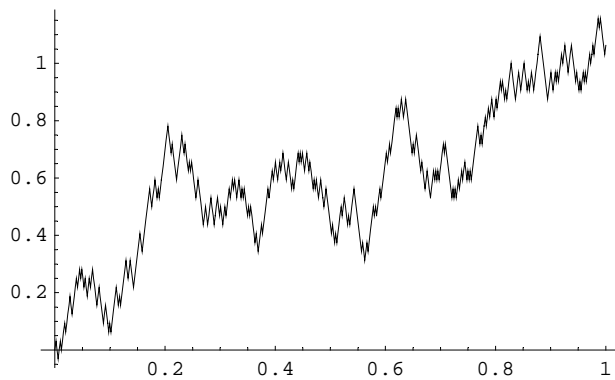
Out[16]= - Graphics -

General::spell1 : Possible spelling error: new symbol name "RWS8" is similar to existing symbol "RW8". MORE...



Out[17]= - Graphics -

General::spell1 : Possible spelling error: new symbol name "RWS9" is similar to existing symbol "RW9". MORE...



Out[18]= - Graphics -

■ Bernoullijevi koraci:

$$\mathbf{X} = \begin{pmatrix} -1/3 & 3 \\ 9/10 & 1/10 \end{pmatrix}$$

```
In[19]:= RandomWalkB[k_Integer] :=
  FoldList[Plus, 0, Table[(10/3) *
    (Random[BernoulliDistribution[.1]]) -
    1/3, {k}]]
SeedRandom[50000]; RWB = RandomWalkB[2050];
RandomWalkBApproximation[n_Integer] :=
  Table[{(i - 1) / 2^n, RWB[[i]]},
    {i, 1, 2^n + 1}]
RandomWalkBApproximationScaled[
  n_Integer] := Table[{(i - 1) / 2^n,
  RWB[[i]] / Sqrt[2^n]}, {i, 1, 2^n + 1}]
```

General::spell1 :

Possible spelling error: new symbol name "RandomWalkB" is similar to existing symbol "RandomWalk". MORE...

General::spell1 : Possible spelling error: new symbol

name "RandomWalkBApproximation" is similar to existing symbol "RandomWalkApproximation". MORE...

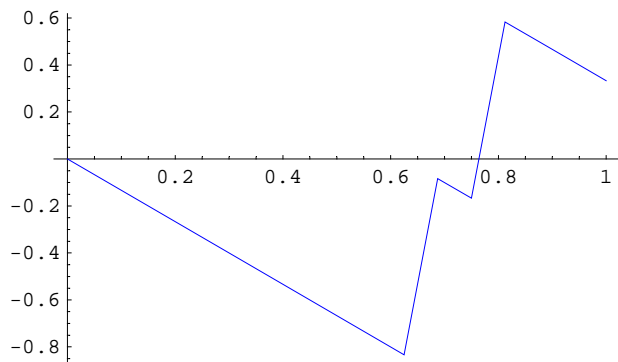
General::spell1 : Possible spelling error: new symbol name "RandomWalkBApproximationScaled" is

similar to existing symbol "RandomWalkApproximationScaled". MORE...

```
In[23]:= RWBS4 = ListPlot[
  RandomWalkBApproximationScaled[4],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 1]}]
RWBS5 = ListPlot[
  RandomWalkBApproximationScaled[5],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 1]}]
RWBS6 = ListPlot[
  RandomWalkApproximationScaled[6],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 0]}]
RWBS7 = ListPlot[
  RandomWalkBApproximationScaled[7],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 1, 0]}]
RWBS8 = ListPlot[
  RandomWalkBApproximationScaled[8],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 0, 0]}]
RWBS9 = ListPlot[
  RandomWalkBApproximationScaled[9],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 0]}]
```

General::spell1 :

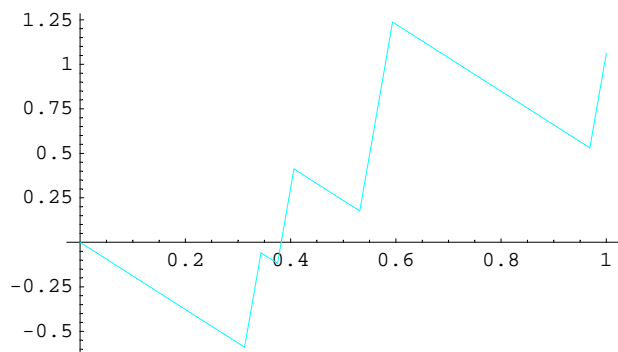
Possible spelling error: new symbol name "RWBS4" is similar to existing symbol "RWS4". More...



Out[23]= - Graphics -

General::spell1 :

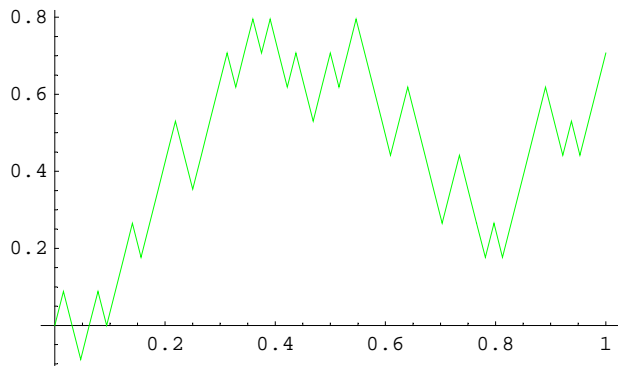
Possible spelling error: new symbol name "RWBS5" is similar to existing symbol "RWS5". More...



Out[24]= - Graphics -

General::spell1 :

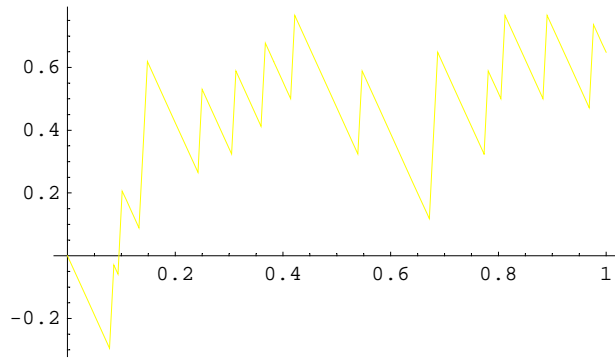
Possible spelling error: new symbol name "RWBS6" is similar to existing symbol "RWS6". More...



Out[25]= - Graphics -

General::spell1 :

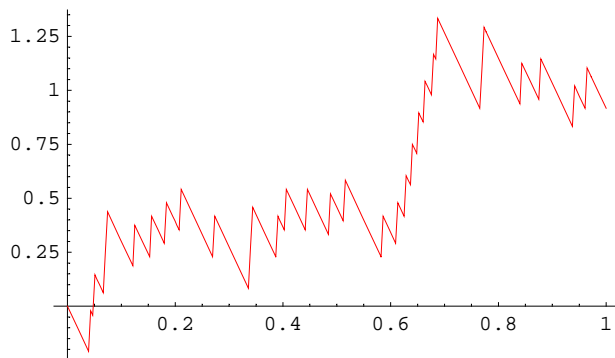
Possible spelling error: new symbol name "RWBS7" is similar to existing symbol "RWS7". More...



Out[26]= - Graphics -

General::spell1 :

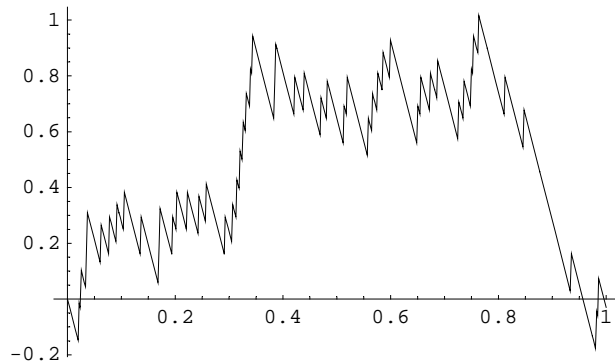
Possible spelling error: new symbol name "RWBS8" is similar to existing symbol "RWS8". More...



Out[27]= - Graphics -

General::spell1 :

Possible spelling error: new symbol name "RWBS9" is similar to existing symbol "RWS9". More...



Out[28]= - Graphics -

■ Normalni koraci: $X=N(0,1)$

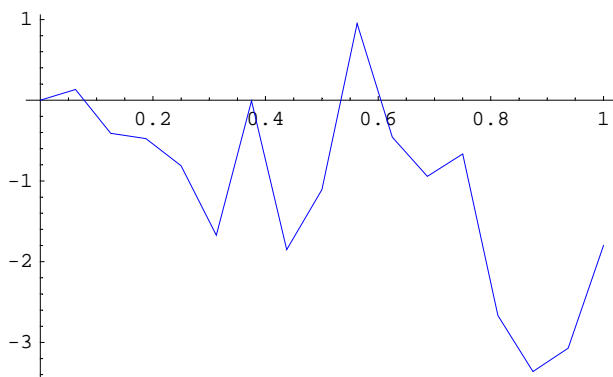
```
In[29]:= RandomWalkN[k_Integer] :=
  FoldList[Plus, 0, Table[
    Random[NormalDistribution[0, 1]], {k}]]
SeedRandom[50000]; RWN = RandomWalkN[2050];
RandomWalkNApproximation[n_Integer] :=
  Table[{(i - 1) / 2^n, RWN[[i]]},
    {i, 1, 2^n + 1}]
RandomWalkNApproximationScaled[
  n_Integer] := Table[{(i - 1) / 2^n,
  RWN[[i]] / Sqrt[2^n]}, {i, 1, 2^n + 1}]
```

General::spell : Possible spelling error: new symbol
name "RandomWalkN" is similar to existing symbols {RandomWalk, RandomWalkB}. More...

General::spell : Possible spelling error: new symbol name "RandomWalkNApproximation" is
similar to existing symbols {RandomWalkApproximation, RandomWalkBApproximation}. More...

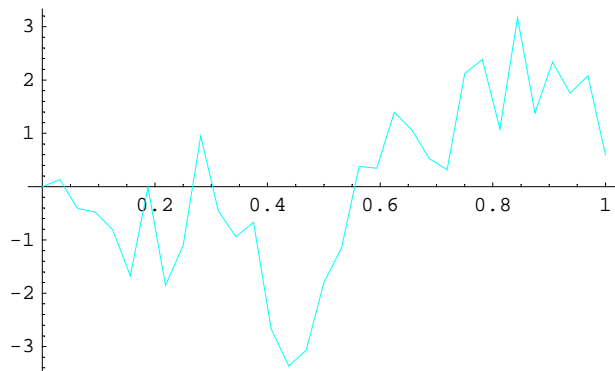
General::spell : Possible spelling error: new symbol name "RandomWalkNApproximationScaled" is similar
to existing symbols {RandomWalkApproximationScaled, RandomWalkBApproximationScaled}. More...

General::spell :
Possible spelling error: new symbol name "RWN4" is similar to existing symbols {RW4, RWS4}. More...



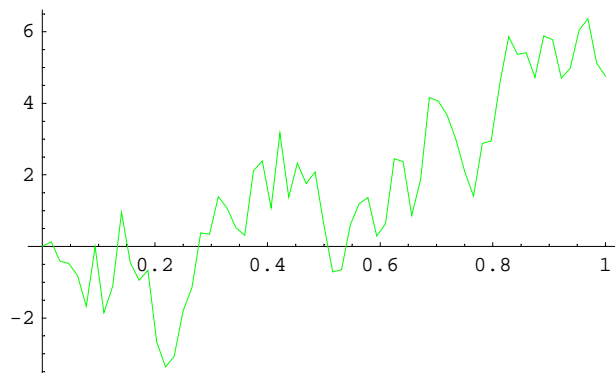
Out[33]= - Graphics -

```
General::spell :  
Possible spelling error: new symbol name "RWN5" is similar to existing symbols {RW5, RWS5}. More...
```



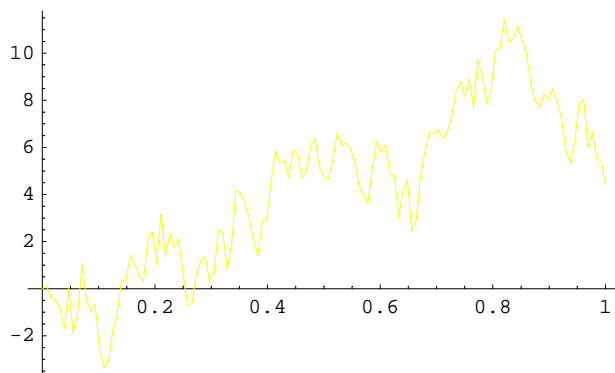
Out[34]= - Graphics -

```
General::spell :  
Possible spelling error: new symbol name "RWN6" is similar to existing symbols {RW6, RWS6}. More...
```



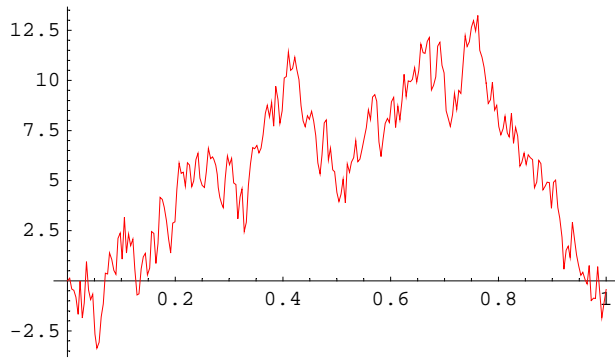
Out[35]= - Graphics -

```
General::spell :  
Possible spelling error: new symbol name "RWN7" is similar to existing symbols {RW7, RWS7}. More...
```



Out[36]= - Graphics -

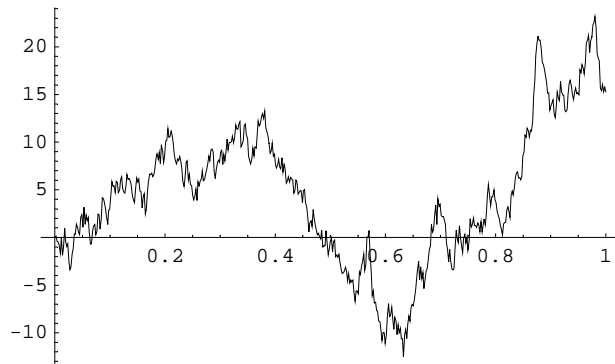
```
General::spell :  
Possible spelling error: new symbol name "RWN8" is similar to existing symbols {RW8, RWS8}. More...
```



Out[37]= - Graphics -

General::spell :

Possible spelling error: new symbol name "RWN9" is similar to existing symbols {RW9, RWS9}. More...

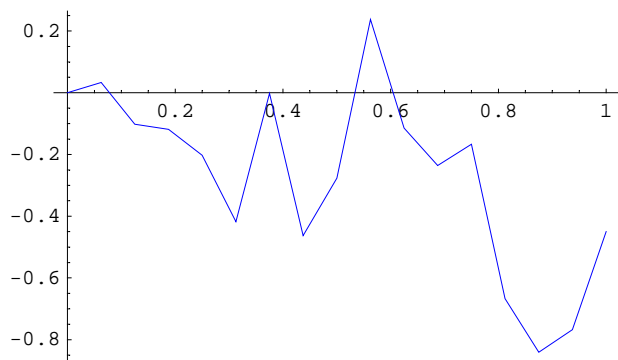


Out[38]= - Graphics -

```
In[39]:= RWNS4 = ListPlot[
  RandomWalkNApproximationScaled[4],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 1]}]
RWNS5 = ListPlot[
  RandomWalkNApproximationScaled[5],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 1]}]
RWNS6 = ListPlot[
  RandomWalkNApproximationScaled[6],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 0]}]
RWNS7 = ListPlot[
  RandomWalkNApproximationScaled[7],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 1, 0]}]
RWNS8 = ListPlot[
  RandomWalkNApproximationScaled[8],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 0, 0]}]
RWNS9 = ListPlot[
  RandomWalkNApproximationScaled[9],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 0]}]
```

General::spell :

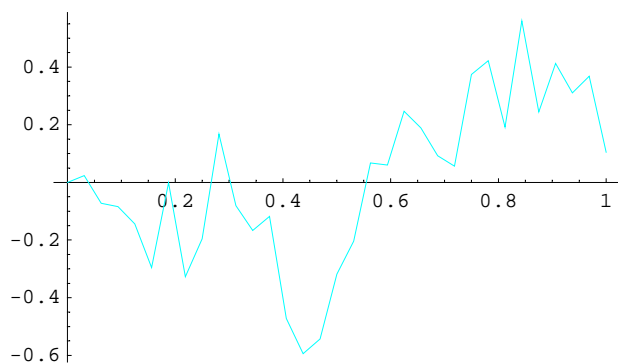
Possible spelling error: new symbol name "RWNS4" is similar to existing symbols {RWBS4, RWN4, RWS4}. More...



Out[39]= - Graphics -

General::spell :

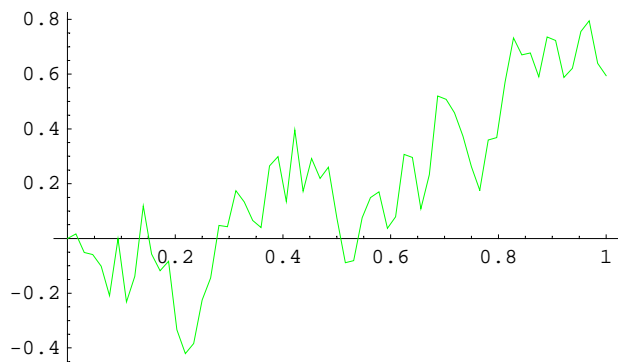
Possible spelling error: new symbol name "RWNS5" is similar to existing symbols {RWBS5, RWN5, RWS5}. More...



Out[40]= - Graphics -

General::spell :

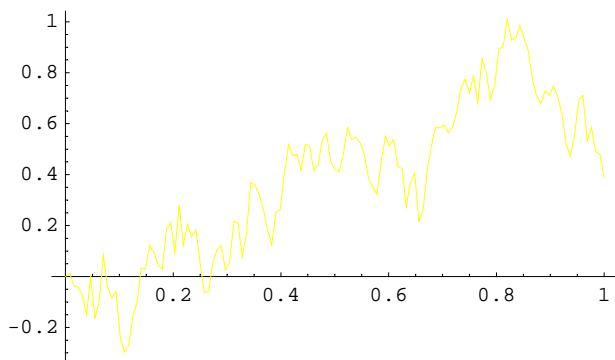
Possible spelling error: new symbol name "RWNS6" is similar to existing symbols {RWBS6, RWN6, RWS6}. More...



Out[41]= - Graphics -

General::spell :

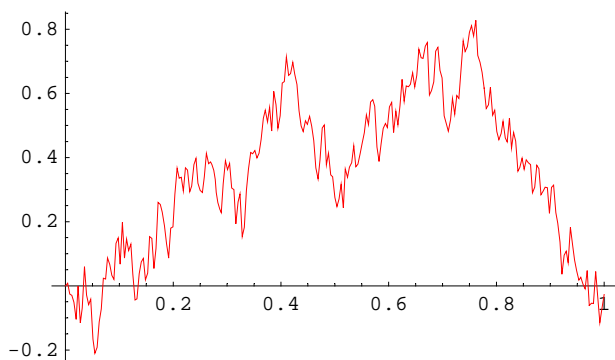
Possible spelling error: new symbol name "RWNS7" is similar to existing symbols {RWBS7, RWN7, RWS7}. More...



Out[42]= - Graphics -

General::spell :

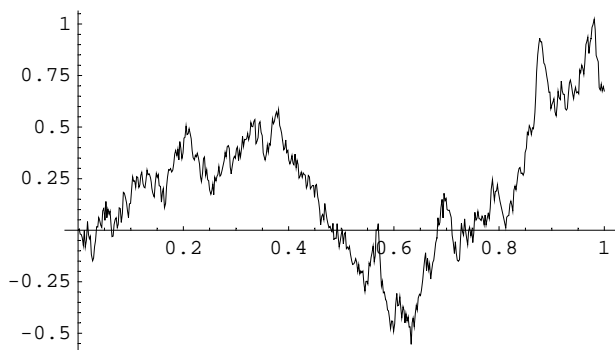
Possible spelling error: new symbol name "RWNS8" is similar to existing symbols {RWBS8, RWN8, RWS8}. More...



Out[43]= - Graphics -

General::spell :

Possible spelling error: new symbol name "RWNS9" is similar to existing symbols {RWBS9, RWN9, RWS9}. More...



Out[44]= - Graphics -

■ Eksponencijalni koraci:

$X=E(1)-1$

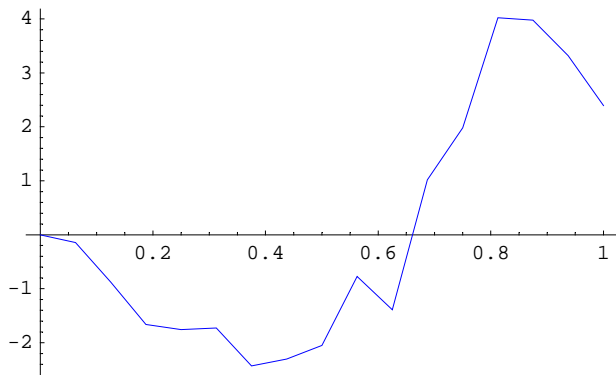
```
In[45]:= RandomWalkE[k_Integer] :=
  FoldList[Plus, 0, Table[Random[
    ExponentialDistribution[1]] - 1, {k}]]
SeedRandom[50000]; RWE = RandomWalkE[2050];
RandomWalkEApproximation[n_Integer] :=
  Table[{(i - 1) / 2^n, RWE[[i]]},
    {i, 1, 2^n + 1}]
RandomWalkEApproximationScaled[
  n_Integer] := Table[{(i - 1) / 2^n,
  RWE[[i]] / Sqrt[2^n]}, {i, 1, 2^n + 1}]
```

General::spell : Possible spelling error: new symbol name "RandomWalkE" is similar to existing symbols {RandomWalk, RandomWalkB, RandomWalkN}. More...

General::spell : Possible spelling error: new symbol name "RandomWalkEApproximation" is similar to existing symbols {RandomWalkApproximation, RandomWalkBApproximation, RandomWalkNApproximation}. More...

General::spell : Possible spelling error: new symbol name "RandomWalkEApproximationScaled" is similar to existing symbols {RandomWalkApproximationScaled, RandomWalkBApproximationScaled, RandomWalkNApproximationScaled}. More...

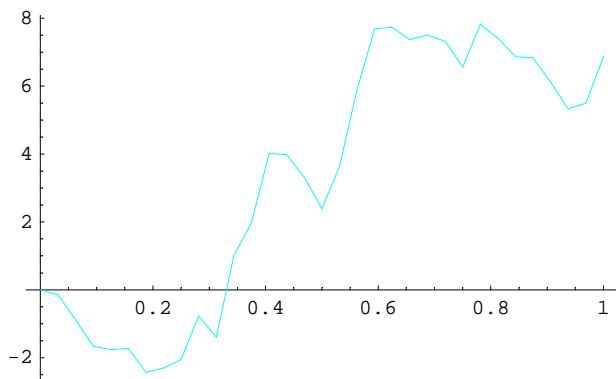
General::spell : Possible spelling error: new symbol name "RWE4" is similar to existing symbols {RW4, RWN4, RWS4}. More...



Out[49]= - Graphics -

General::spell :

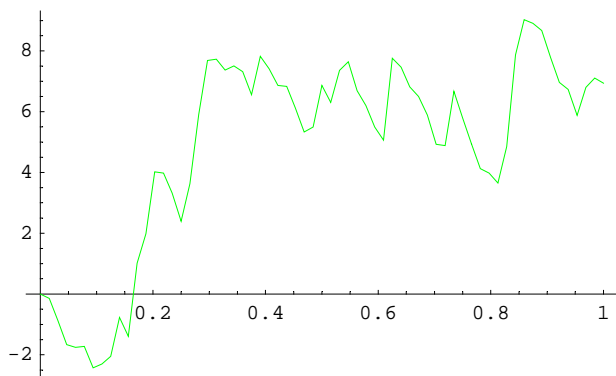
Possible spelling error: new symbol name "RWE5" is similar to existing symbols {RW5, RWN5, RWS5}. [More...](#)



Out[50]= - Graphics -

General::spell :

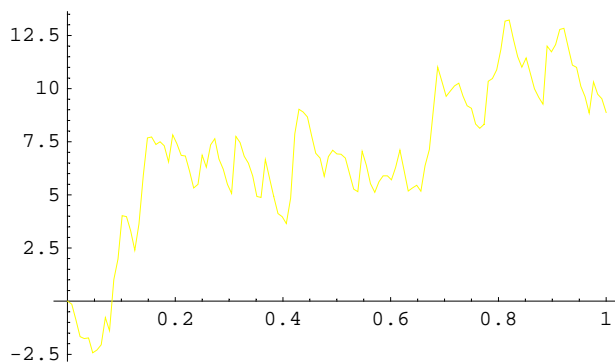
Possible spelling error: new symbol name "RWE6" is similar to existing symbols {RW6, RWN6, RWS6}. [More...](#)



Out[51]= - Graphics -

General::spell :

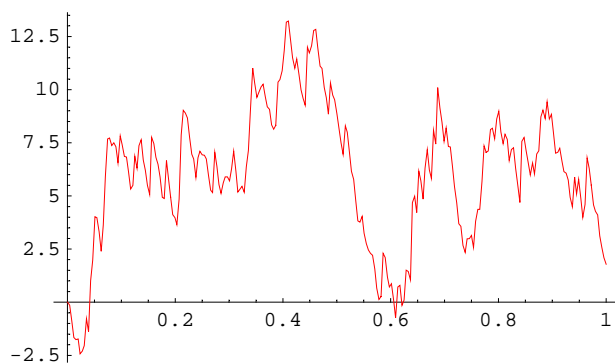
Possible spelling error: new symbol name "RWE7" is similar to existing symbols {RW7, RWN7, RWS7}. [More...](#)



Out[52]= - Graphics -

General::spell :

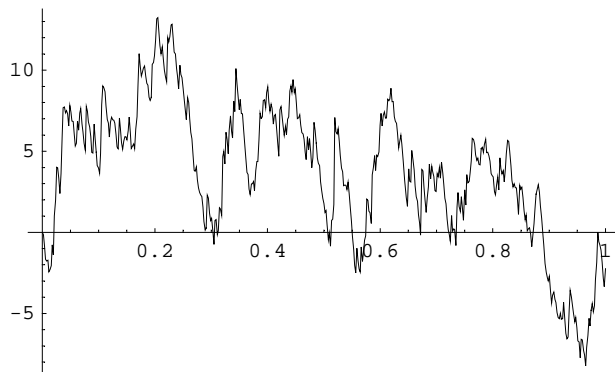
Possible spelling error: new symbol name "RWE8" is similar to existing symbols {RW8, RWN8, RWS8}. More...



Out[53]= - Graphics -

General::spell :

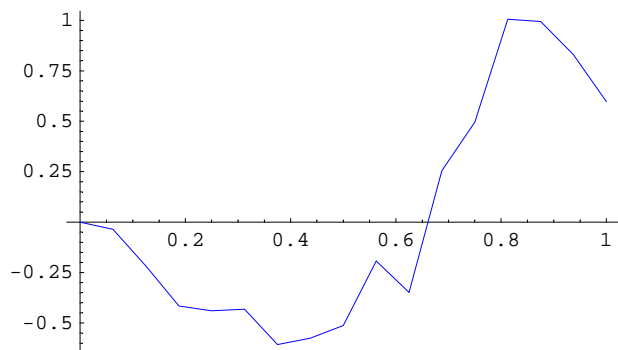
Possible spelling error: new symbol name "RWE9" is similar to existing symbols {RW9, RWN9, RWS9}. More...



Out[54]= - Graphics -

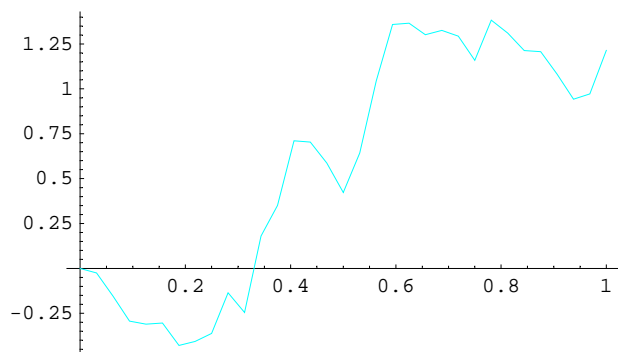
```
In[55]:= RWES4 = ListPlot[
  RandomWalkEApproximationScaled[4],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 1]}]
RWES5 = ListPlot[
  RandomWalkEApproximationScaled[5],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 1]}]
RWES6 = ListPlot[
  RandomWalkEApproximationScaled[6],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 1, 0]}]
RWES7 = ListPlot[
  RandomWalkEApproximationScaled[7],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 1, 0]}]
RWES8 = ListPlot[
  RandomWalkEApproximationScaled[8],
  PlotJoined → True,
  PlotStyle -> {RGBColor[1, 0, 0]}]
RWES9 = ListPlot[
  RandomWalkEApproximationScaled[9],
  PlotJoined → True,
  PlotStyle -> {RGBColor[0, 0, 0]}]
```

General::spell : Possible spelling error: new symbol
name "RWES4" is similar to existing symbols {RWBS4, RWE4, RWNS4, RWS4}. More...



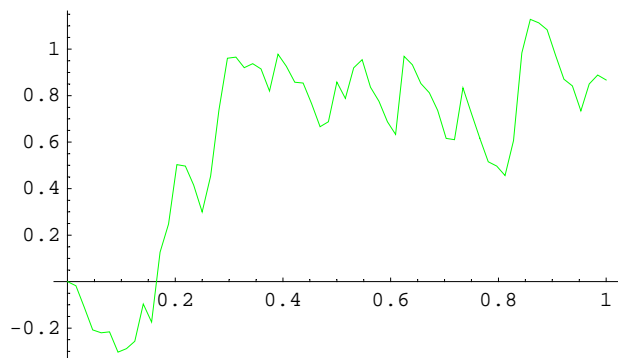
Out[55]= - Graphics -

General::spell : Possible spelling error: new symbol
name "RWES5" is similar to existing symbols {RWBS5, RWE5, RWNS5, RWS5}. More...



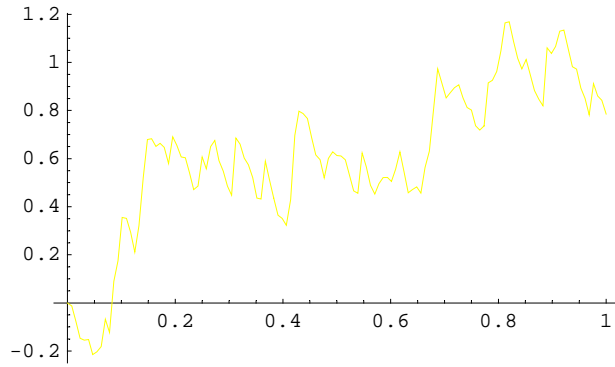
Out[56]= - Graphics -

General::spell : Possible spelling error: new symbol
name "RWES6" is similar to existing symbols {RWBS6, RWE6, RWNS6, RWS6}. More...



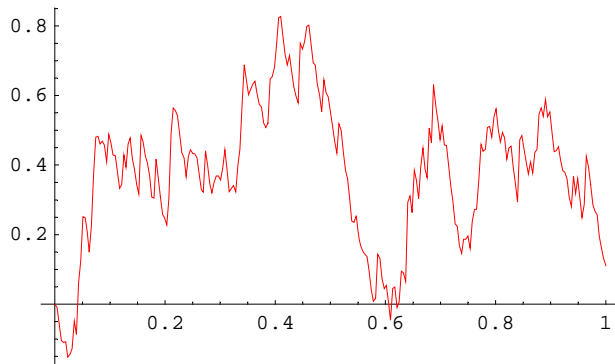
Out[57]= - Graphics -

General::spell : Possible spelling error: new symbol
name "RWES7" is similar to existing symbols {RWBS7, RWE7, RWNS7, RWS7}. More...



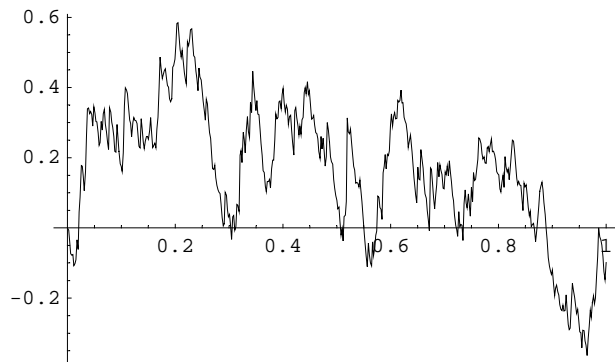
Out[58]= - Graphics -

General::spell : Possible spelling error: new symbol
name "RWES8" is similar to existing symbols {RWBS8, RWE8, RWNS8, RWS8}. More...



Out[59]= - Graphics -

General::spell : Possible spelling error: new symbol
name "RWES9" is similar to existing symbols {RWBS9, RWE9, RWNS9, RWS9}. More...



Out[60]= - Graphics -