

Main Theorem Extraction: Partitioning the Plane

Definitions

Definition 1 (Rectangle of Area 1). A set $s \subset \mathbb{C}$ is said to form a *rectangle of area 1* if there exist distinct points z_1, z_2, z_3, z_4 such that $s = \{z_1, z_2, z_3, z_4\}$ and the following conditions hold:

1. $z_2 - z_1 = z_3 - z_4$ (The points form a parallelogram),
2. $\operatorname{Re}((z_2 - z_1) \cdot \overline{(z_4 - z_1)}) = 0$ (The adjacent sides are perpendicular),
3. $|z_2 - z_1|^2 \cdot |z_4 - z_1|^2 = 1$ (The square of the area is 1).

Main Theorem

Theorem 1. *It is possible to partition \mathbb{C} (identified with \mathbb{R}^2) into 25 color classes such that none of them contains the vertices of a rectangle of area 1.*

Formally, there exists a function $f : \mathbb{C} \rightarrow \{0, \dots, 24\}$ such that for every color $c \in \{0, \dots, 24\}$, there does not exist a set $s \subset \mathbb{C}$ satisfying:

$$s \subseteq f^{-1}(\{c\}) \wedge \text{IsRectangleArea1}(s)$$