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Classification of Homotopy Types of Embedding Spaces into 2-Manifolds

This talk gives a complete description of the homotopy types of the connected components of spaces of embeddings of compact polyhedra into 2-manifolds. Suppose M is a connected 2-manifold and X is a compact connected subpolyhedron of M (X is neither one point, nor a closed 2-manifold). Let E(X, M)denote the space of topological embeddings of X into M with the compact-open topology and $E(X, M)_0$ denote the connected component of inclusion i_X of Xinto M in E(X, M). Homotopy type of $E(X, M)_0$ can be classified in term of subgroup $G = \text{Im} [(i_X)_* : \pi_1(X) \to \pi_1(M)].$

The main statements are:

- (1) If G is not a cyclic group and M is neither T nor K, then $E(X, M)_0 \simeq \{pt\}$.
- (2) If G is a nontrivial cyclic group and M is not P, T, or K, then $E(X, M)_0 \simeq S^1$.
- (3) In Case G = 1,
 - (i) if X is an arc or M is orientable, then $E(X, M)_0 \simeq ST(M)$,
 - (*ii*) if X is not an arc and M is nonorientable, then $E(X, M)_0 \simeq ST(\tilde{M})$.

Here S^1 is the circle, T is the torus, P is the projective plane and K is the Klein bottle. ST(M) denotes the tangent unit circle bundle of M with respect to any Riemannian metric of M and \tilde{M} denotes the orientation double cover of M.