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Relative and Pointed Versions of Lipscomb's Embedding Theorem

In his papers [3,4] S. L. Lipscomb defined the space $\mathcal{J}(\tau)$ as a factor-space of Baire's universal 0-dimensional space and proved that any n-dimensional metrizable space of weight τ, $\tau \geq \aleph_0$, can be embedded in the subspace $L_n(\tau) = \{\, x \in \mathcal{J}(\tau)^{n+1} :$ at least one coordinate of x is irrational$\}$ of $\mathcal{J}(\tau)^{n+1}$.

In an attempt to prove the relative version of the theorem, i.e. to prove that any embedding $f_0 \colon X_0 \longrightarrow L_n(\tau)$, where X_0 is a subspace of a n-dimensional metric space X of weight τ, can be extended to an embedding $f \colon X \longrightarrow L_n(\tau)$, we found simple examples showing that this is in general not true.

We succeeded to prove such a theorem for $n = 0$ and compact X_0 [2]. But for $n > 0$ even the case when X_0 is a single point is not trivial.

Using the fact that $\mathcal{J}(\tau)$ is naturally homeomorphic to a generalized Sierpiński curve [5,6] and techniques of modification of Lipscomb's decompositions and indexing of the modified decompositions developed in [1,6], here we prove the pointed version of Lipscomb's embedding theorem, i.e. we show that the embedding may be chosen in such a way that its value is given in advance at a certain point (the base point). This is not trivial precisely because $\mathcal{J}(\tau)$ splits into the rational and the irrational part.

References

[1] I. Ivanšić and U. Milutinović. *A universal separable metric space based on the triangular Sierpiński curve.* Top. Appl. 120 (2002) 237–271.
[2] I. Ivanšić and U. Milutinović. *Relative embeddability into Lipscomb's* 0*-dimensional universal space.* Houston J. Math. (to appear)
[3] S. L. Lipscomb. *A universal one-dimensional metric space.* In *TOPO 72 - General Topology and its Applications, Second Pittsburgh Internat. Conf.*, volume 378 of *Lecture Notes in Math.* Springer-Verlag, New York, 1974, 248–257.
[4] S. L. Lipscomb. *On imbedding finite-dimensional metric spaces.* Trans. Amer. Math. Soc., 211 (1975) 143–160.
[5] S. L. Lipscomb and J. C. Perry. *Lipscomb's $L(A)$ space fractalized in Hilbert's $l^2(A)$ space.* Proc. Amer. Math. Soc., 115 (1992) 1157–1165.
[6] U. Milutinović. *Completeness of the Lipscomb universal space.* Glas. Mat. Ser. III, 27(47) (1992) 343–364.

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