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Strong Border Homology and Cohomology

Let X be a Hausdorff topological space and G be an abelian group. For any ANR-resolution in the sense of Mardešić $\underline{p} = (p_\lambda) : X \to \underline{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ and an inverse system $\underline{C}_* = (C_*(X_\lambda; G), p_{\lambda\lambda'}, \Lambda)$ of the singular chain complexes $C_*(X_\lambda; G)$ of X_λ one can define the strong homology group of X with the coefficients in G by formula $\bar{H}_p(X;G) \overset{def}{=} H_p(\underset{\leftarrow\lambda}{\operatorname{holim}} \underline{C}_*)$. (Denote for simplicity $\bar{C}_*(X;G) = \underset{\leftarrow\lambda}{\operatorname{holim}} \underline{C}_*$). Together with the Čech cohomology $\check{H}_p(X;G) = \underset{\to\lambda}{\lim} H^p(X_\lambda;G)$ one has *outer* or *projective* (co)homology of X, which are strong shape invariants. There are dual *inner* or *injective* (co)homology of X, which are strong coshape invariants (moreover, strong shape with compact supports invariants). One of them is the strong homology with compact supports $\bar{H}^c_p(X;G) \overset{\text{def}}{=} \underset{\to\mu}{\lim} \bar{H}_p(K_\mu;G)$ and another one is the strong cohomology group $\bar{H}^p(X;G) \overset{\text{def}}{=} H^p(\underset{\leftarrow\mu}{\operatorname{holim}} \underline{C}^*)$, where $\underline{K} = (K_\mu, i_{\mu\mu'}, M)$ is the direct system of all compact subspaces K_μ of X, $C^*(K_\mu;G)$ is the Massey cohomology cochain complex and $\underline{C}^* = (C^*(X_\mu;G), i^*_{\mu\mu'}, M)$ is and inverse system of cochain complexes. (Denote for simplicity $\bar{C}^*(X;G) = \underset{\leftarrow\mu}{\operatorname{holim}} \underline{C}^*$).

There are two natural trasformations $\mu_p\colon \bar{H}^c_p(X;G) \to \bar{H}_p(X;G)$ and $\lambda^p\colon \check{H}^p(X;G) \to \bar{H}^p(X;G)$. Moreover, one can define the *strong border homology* $\bar{H}^{br}_p(X;G)$ and the *strong border cohomology* $\bar{H}^p_{br}(X;G)$ of X, which do final relation between them as the following two long exact sequences:

$$\cdots \to \bar{H}^c_p(X;G) \overset{\mu_p}{\to} \bar{H}_(X;G) \to \bar{H}^{br}_p(X;G) \to \bar{H}^c_{p-1}(X;G) \to \cdots \tag{1}$$

$$\cdots \to H^{p-1}_c(X;G) \to \bar{H}^p_{br}(X;G) \to \check{H}^p(X;G) \overset{\lambda^p}{\to} \bar{H}^p(X;G) \to \cdots \tag{2}$$

Theorem. *For any Hausdorff topological space X there exists an ANR-resolution $\underline{p}\colon X \to \underline{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ and the family of ANR-subresolutions $\underline{q}_\mu\colon K_\mu \to \underline{K}_\mu$ in $\underline{X}$ of the family of all compact subspaces K_μ of X.*

Then one can define a chain complex $\bar{C}^{br}_*(X;G) = \underset{\to\mu}{\lim}(\bar{C}_*(X;G)/\bar{C}_*(K_\mu;G))$ or $\bar{C}^{br}_*(X;G) = \underset{\leftarrow\lambda}{\operatorname{holim}}(C_*(X_\lambda;G)/\bar{C}^c_*(X;G))$, where $\bar{C}^c_*(X;G) = \underset{\to\mu}{\lim} \bar{C}_*(K_\mu;G)$ is the strong chain complex with compact supports. Now put $\bar{H}^{br}_p \overset{def}{=} H_p(\bar{C}^{br}_*(X;G))$.

To define $\bar{H}^p_{br}(X;G)$ one can take any ANR-resolution $\underline{p} = (p_\lambda)\colon X \to \underline{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of X and for each $\lambda \in \Lambda$ consider the cylinder $X \coprod_{p_\lambda} X_\lambda$ of $p_\lambda\colon X \to X_\lambda$ and also the strong cochain complex $\bar{C}^*(X \coprod_{p_\lambda} X_\lambda; G)$ of it. Then there is an epimorphism $\bar{C}^*(X \coprod_{p_\lambda} X_\lambda; G) \to \bar{C}^*(X;G)$ on the strong cochain complex $\bar{C}^*(X;G)$ and we denote the kernel of it by $\bar{C}^*(X : X_\lambda; G)$. Then put $\bar{C}^*_{br}(X;G) = \underset{\to\lambda}{\lim} \bar{C}^*(X : X_\lambda; G)$. Now put $\bar{H}^p_{br}(X;G) \overset{def}{=} H^p(\bar{C}^*_{br}(X;G))$.