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## Homotopy Dominations Within Polyhedra

In this talk we will concentrate on the question:

*Is it true that every finite polyhedron dominates only finitely many different shapes?*

which was stated by K. Borsuk in 1968, at the Topological Conference in Herceg-Novi [Russ. Math. Surv. 34:6 (1979), 24–26].

In this question the notions of shape and shape domination can be replaced by the notions of homotopy type and homotopy domination. Thus we will examine dominations of a polyhedron in the category of  $CW$ -complexes and homotopy classes of cellular maps between them.

We showed that the answer to the Borsuk's question was negative. Furthermore, for every non-abelian poly- $Z$ -group  $G$  there exists a finite polyhedron  $P$  with fundamental group  $G$  dominating infinitely many finite polyhedra of different homotopy types. Thus, there exist finite polyhedra with nilpotent fundamental group dominating infinitely many finite polyhedra of different homotopy types. On the other hand, we proved that finite polyhedra with finite fundamental group and nilpotent finite polyhedra dominated only finitely many different homotopy types. Moreover, there is only finitely many classes of homology equivalence of  $CW$ -complexes dominated by a given finite polyhedron.

We will present these and further results with their applications to solutions to another Borsuk's problems concerning shape dominations.