The Product of a Compactum with a Polyhedron in the Shape Categories

In the ordinary shape category Sh(Top), as well as in the strong shape category SSh(Top), the Cartesian product of two polyhedra, together with the canonical projections, is their categorical product. The analogous assertion holds for the Cartesian product of two compact Hausdorff spaces.

**PROBLEM.** Let $X$ be a compact Hausdorff space and let $P$ be a polyhedron. Is $X \times P$, together with the canonical projections $\Pi_X : X \times P \to X$ and $\Pi_P : X \times P \to P$, the categorical product of $X$ and $P$ in Sh(Top) and SSh(Top)?

The author hopes to solve this problem using a particular resolution $q : X \times P \to Y = (Y_\mu, q_{\mu,\mu'}, M)$, induced by a polyhedral resolution $p : X \to X = (X_\lambda, p_{\lambda,\lambda'}, \Lambda)$ and by a triangulation $K$ of $P$. The index set $M$ consists of all increasing functions $\mu : K \to \Lambda$, where $K$ is ordered by putting $\sigma \leq \sigma'$, provided $\sigma$ is a face of $\sigma'$. $Y_\mu = \tilde{Y}_\mu / \sim_\mu$, where $\tilde{Y}_\mu$ is the disjoint sum of all products $X_{\mu(\sigma)} \times \sigma$, $\sigma \in K$, and $(x, t) \sim_\mu (x', t')$ if $(x, t) \in X_{\mu(\sigma)} \times \sigma$, $(x', t') \in X_{\mu(\sigma')} \times \sigma'$, $\sigma \leq \sigma'$, $p_{\mu(\sigma)}(x') = x$ and $i_{\sigma'}(t) = t'$; here $i_{\sigma'} : \sigma \to \sigma'$ is the inclusion mapping.

**THEOREM.** Let $X$ be a metric compactum and let $P$ be a separable polyhedron. Then for every topological space $Z$ and strong shape morphisms $F : Z \to X$, $G : Z \to P$, there exists a strong shape morphism $H : Z \to X \times P$ such that $\Pi_X H = F$ and $\Pi_P H = G$.

To solve affirmatively the above stated problem for SSh(Top), compact metric $X$ and separable polyhedra $P$, it remains to prove uniqueness of $H$.

*Mathematics Subject Classification:* 55P55