Jury Lisica, *Russian University of Peoples Friendship, Moscow, Russia*

The Slant Product for Strong (Co)Homology, Homology with Compact Supports and Čech Cohomology

The homology slant product and the cohomology slant product for singular homology H_* and singular cohomology H^* are well-defined if X and Y are polyhedra. One can dualize this construction for Steenrod homology $\bar{H}_*$ and Čech cohomology $\check{H}^*$ if X and Y are compact Hausdorff spaces.

The target problem is to generalize it for arbitrary Hausdorff topological spaces X and Y and seems to be very difficult. The first step in this direction is the following theorem.

Theorem 1. *If X is a compact Hausdorff space and Y is a polyhedron, then there exist the following slant products* (*natural transformations*)*:*

$$\backslash\colon \bar{H}_p(X;G_1)\otimes \bar{H}^{p+q}(X\times Y;G_2)\longrightarrow H^q(Y;G_1\otimes G_2) \tag{1}$$
$$\backslash\colon \bar{H}_p(X;G_1)\otimes \check{H}^{p+q}(X\times Y;G_2)\longrightarrow H^q(Y;G_1\otimes G_2) \tag{2}$$
$$\backslash\colon \check{H}^p(X;G_1)\otimes \bar{H}^c_{p+q}(X\times Y;G_2)\longrightarrow H_q(Y;G_1\otimes G_2) \tag{3}$$
$$\backslash\colon \check{H}^p(X;G_1)\otimes \bar{H}_{p+q}(X\times Y;G_2)\longrightarrow H_q(Y;G_1\otimes G_2) \tag{4}$$
$$\backslash\colon H_p(Y;G_1)\otimes \bar{H}^{p+q}(Y\times X;G_2)\longrightarrow \check{H}^q(X;G_1\otimes G_2) \tag{5}$$
$$\backslash\colon H_p(Y;G_1)\otimes \check{H}^{p+q}(Y\times X;G_2)\longrightarrow \check{H}^q(X;G_1\otimes G_2) \tag{6}$$
$$\backslash\colon H^p(Y;G_1)\otimes \bar{H}^c_{p+q}(Y\times X;G_2)\longrightarrow \bar{H}_q(X;G_1\otimes G_2) \tag{7}$$
$$\backslash\colon H^p(Y;G_1)\otimes \bar{H}_{p+q}(Y\times X;G_2)\longrightarrow \bar{H}_q(X;G_1\otimes G_2) \tag{8}$$

which satisfy all known properties [1]. *Here $\bar{H}_*$ is the strong homology, $\bar{H}^c_*$ is the homology with compact supports and $\bar{H}^*$ is the strong cohomology.*

The proofs are standard, but different. Nevertheless, the proofs of formulas (4) and (8) need nontrivial special ANR-resolution for product $X\times Y$ constructed recently by S. Mardešić.

The second step is to change X or Y by an arbitrary Hausdorff space. One can do it for (1) and (3). The last formula is a special case of more general theorem:

Theorem 2. *If X and Y are arbitrary Hausdorff spaces, then there exist the following slant product* (*natural transformation*)*:*

$$\backslash\colon \bar{H}^p(X;G_1)\otimes \bar{H}^c_{p+q}(X\times Y;G_2)\longrightarrow \bar{H}^c_q(Y;G_1\otimes G_2)\,. \tag{9}$$