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## Bounded Nonconvex Chebyshev Sets in a Real Inner Product Space

A subset  $S$ , of a real finite dimensional inner product space  $X$ , has the property that each point in  $X$  has a unique nearest point in  $S$  iff  $S$  is closed and convex. Such a set is called a unique nearest point set. If  $X$  is a Hilbert space, must a unique nearest point set in  $X$  be convex? The real inner product space  $Y$  of all infinite number sequences having at most a finite number of nonzero terms, with the usual inner product, contains a non convex unique nearest point set  $S$ , moreover  $S$  can be made bounded. Now, if  $H$  denotes the completion of  $Y$ , and  $T$  the closure of a bounded unique nearest point set  $S$  in  $Y$ , then if each of  $\{P_i\}$  and  $\{Q_i\}$  is a sequence in  $Y$ , converging to points  $P$  and  $Q$  respectively in  $H$ , where for each  $i$ ,  $Q_i$  is the unique nearest point in  $S$  to  $P_i$ , then  $Q$  is the unique nearest point to  $P$  in  $T$ . Moreover, there is a sequence  $\{V_i\}$  in  $Y$  converging to a point  $V$  in  $H$  and a non convergent sequence  $\{W_i\}$ , where  $W_i$  is the unique nearest point to  $V_i$  in  $S$ .