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Geometry and Algebra of Dimension Theory

We present an approach to cohomological dimension theory based on infinite symmetric products and on the general theory of dimension called the extension dimension. The notion of the extension dimension $e\text{-dim}(X)$ was introduced by A. N. Dranishnikov in the context of compact spaces and CW complexes. This paper investigates extension types of infinite symmetric products $SP^\infty(L)$. One of the main ideas of the paper is to treat $e\text{-dim}(X) \leq SP^\infty(L)$ as the fundamental concept of cohomological dimension theory instead of $\dim_G(X) \leq n$. Properties of infinite symmetric products lead naturally to a calculus of graded groups which implies most of classical results of the cohomological dimension. The basic notion is that of homological dimension of a graded group which allows for simultaneous treatment of cohomological dimension of compacta and extension properties of CW complexes. We introduce cohomology of X with respect to L (defined as homotopy groups of the function space $SP^\infty(L)^X$). Another main idea is to treat homology and cohomology on the same level which allows introduction of the dual graded group as an algebraic analog of Dranishnikov Duality.

As an application of our results we characterize all countable groups G so that the Moore space $M(G, n)$ is of the same extension type as the Eilenberg-MacLane space $K(G, n)$. Another application is characterization of infinite symmetric products of the same extension type as a compact (or finite-dimensional and countable) CW complex.