2. MODELI ŠTAPA

2.1. MODEL ZAKRIVLJENOG ŠTAPA

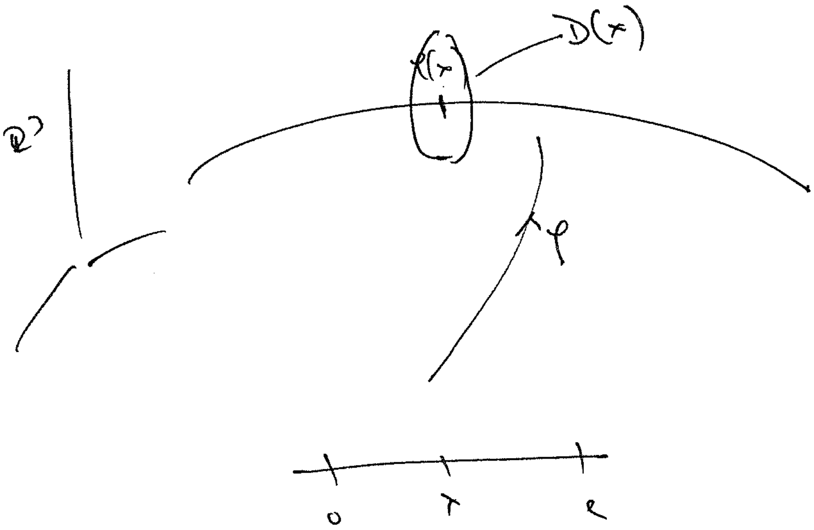
- ŠTAP JE TIJELO KOJEMU JE JEDNA DIMENZIJA (DULJINA) ZNATNO VEĆA OD DRUGE DVIJE.
- GEOMETRIJSKI OPISAN SA:

1) PARAMETRIZACIJOM CENTRALNE LINIJE (PRIRODNOM)

$$\varphi : [0,\ell] \longrightarrow \mathbb{R}^3 , \qquad (\|\varphi'(x)\| = 1,\ x\in[0,\ell])$$

GLATKA!

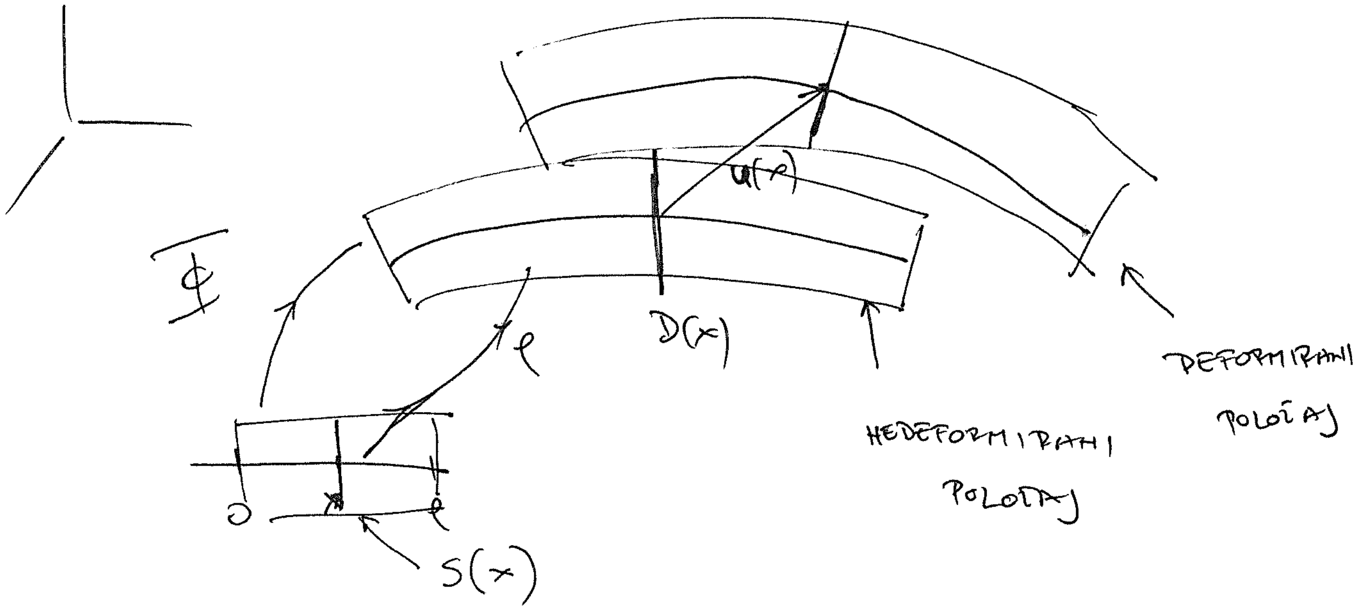
2) FAMILIJOM POPREČNIH PRESJEKA $D(x)$



$t(x) = \varphi'(x)$ JEDINIČNA TANGENTA

$\Rightarrow \exists\ n(x), b(x)$ NEPREKIDNI T.D. $[t(x)\ n(x)\ b(x)]$ ONB

$D(x)$ LEŽI U RAVNINI RAZAPETOJ S $n(x)$ i $b(x)$ (NORMALNA RAVNINA)



$$\Phi(x,y,z) = \varphi(x) + y\,n(x) + z\,b(x)$$ — PARAMETRIZACIJA 3D ŠTAPA

$U(x,y,z)$ — POMAK ŠTAPA

$u(x)$ — POMAK CENTRALNE LINIJE

DEFORMIRANI POLOŽAJ PARAMETRIZIRAN SA

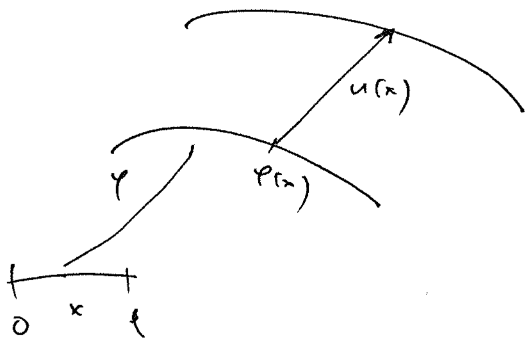
$$\Phi(x,y,z) + U(x,y,z)$$

CENTRALNA LINIJA, DEFORMIRANA, PARAMETRIZIRANA SA

$$\varphi(x) + u(x)$$

$\varphi(x)$ - KONTAKTNA SILA NA $D(x)$

SILA KOJOM $(x,\ell]$ DJELUJE NA $[0,x)$

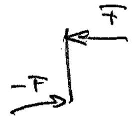


$f(x)$ - LINIJSKA GUSTOĆA VANJSKE SILE

$\mathcal{L}(x)$ - KONTAKTNI SPIN (COUPLE) NA $D(x)$

$\ell(x)$ - GUSTOĆA VANJSKOG SPINA (LINIJSKA)

DEF: SPIN JE MOMENT SILE NA $D(x)$ KOJEM JE REZULTANTNA SILA $= 0$

(POSEBAN OBLIK MOMENTA, NPR: )

ZAD: SPIN NE OVISI O TOČKI ~~[illegible]~~ OKO KOJE SE PROMATRA

RJ: $F_1, \dots, F_k$ - SILE

$R_1, \dots, R_k$ - TOČKE HVATIŠTA SILA U ODNOSU NA O

R - TOČKA U ODNOSU NA KOJU PROMATRAMO MOMENT

$$M_R = \sum_{i=1}^{k} (R_i - R) \times F_i = \sum_{i=1}^{k} R_i \times F_i - R \times \underbrace{\sum_{i=1}^{k} F_i}_{0}$$

$$= M_0$$

PRINCIP RAVNOTEŽE

AKO JE TIJELO U RAVNOTEŽI, ONDA SU UKUPNA SILA I UKUPNI MOMENT NA SVAKI KOMAD TIJELA $= 0$.

$$p(x) - p(0) + \int_0^x f(\xi)\,d\xi = 0$$

$$g(x) + (\varphi(x)+u(x))\times p(x) - g(0) - (\varphi(0)+u(0))\times p(0) + \int_0^x \big(\ell(\xi) + (\varphi(\xi)+u(\xi))\times f(\xi)\big)\,d\xi = 0$$

INTEGRALNI OBLIK.

DERIVIRAMO (DIFERENCIJALNI OBLIK):

$$p' + f = 0$$

$$\cancel{g' + (\varphi'+u')\times p + \ell + (\varphi+u)\times f = 0}$$

$$g' + (\varphi'+u')\times p + \cancel{(\varphi+u)\times \underbrace{p'}_{=-f}} + \ell + \cancel{(\varphi+u)\times f} = 0$$

$$(*)\quad \begin{aligned} &p' + f = 0 \\ &g' + (t+u')\times p + \ell = 0 \end{aligned}$$

JEDNADŽBE RAVNOTEŽE DIF. OBLIK

PRETPOSTAVKA 0:

DEFORMACIJA JE MALA

$$|u'(x)| \ll 1$$

$$|u''(x)| \ll \frac{1}{\ell}$$

PRETPOSTAVKA 1:

SVAKI POPREČNI PRESJEK OSTAJE PRI POMAKU NEDEFORMIRAN, TJ. POMIČE SE KRUTO (TRANSLATIRA I ROTIRA)

$$U(x,y,z) = u(x) + \omega(x) \times (y\,u(x) + z\,b(x))$$

INFINITEZIMALNI KRUTI POMAK (LINEARNA APROKSIMACIJA)

$$\overset{\text{REF POLOŽAJ}}{\overset{\downarrow}{\Phi}} + U = \varphi + u + R\,(y\,u + z\,b) \quad , \quad R(x) \in SO(3)$$

$$\Rightarrow \quad U - u = (R - I)(y\,u + z\,b) \qquad R(x) = e^{A(x)}$$

$A(x)$ – ANTISIMETRIČNA

$$R = I + A + \frac{1}{2!}A^2 + \dots$$

$$\Rightarrow R - I \approx A$$

$A(x)$ – ANTISIMETRIČNA $\Rightarrow \exists\, \omega(x)$ T.D. $A(x)\,v = \omega(x) \times v$

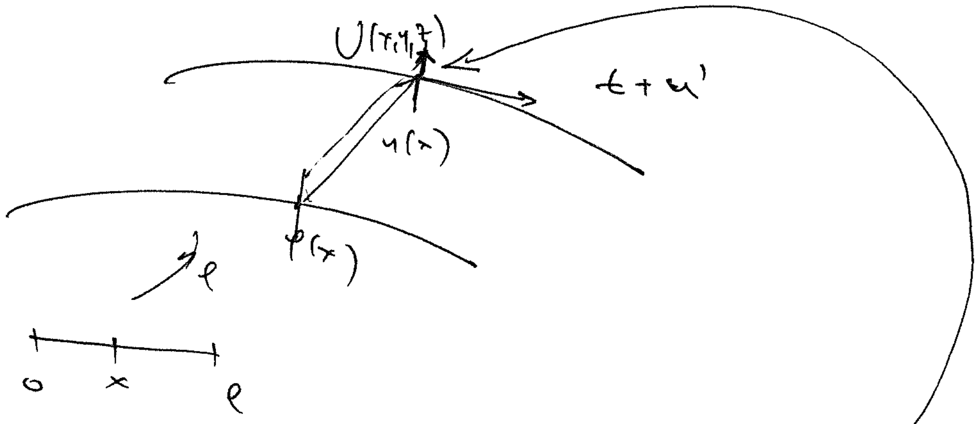
$$\Rightarrow \quad U - u = \omega \times (y\,u + z\,b)$$

$u, \omega : [0,\ell] \to \mathbb{R}^3$ NEPOZNATE FUNKCIJE $\Rightarrow$ 1D MODEL!

IMAMO I BOLJI 3D OPIS DEFORMACIJE!

PRETPOSTAVKA 2:

SVAKI POPREČNI PRESJEK POSLIJE POMAKA OSTAJE OKOMIT (PRIBLIŽNO) NA POMAKNUTU CENTRALNU LINIJU.



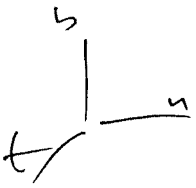
$$(t+u')\cdot\Big(\Phi(x,y,z)+U(x,y,z)-\big(\varphi(x)+u(x)\big)\Big)=0$$

$$\parallel$$

$$(t+u')\cdot\big(yn+zb+\omega\times(yn+zb)\big)$$

$$\parallel$$

$$t\cdot\omega\times(yn+zb)+y\,u'\cdot n+z\,u'\cdot b+\underbrace{u'\cdot\omega\times(yn+zb)}_{\approx 0}$$



$$\parallel$$

$$\omega\cdot(yn+zb)\times t+y\,u'\cdot n+z\,u'\cdot b$$

$$\parallel$$

$$\omega\cdot(-yb+zn)+y\,u'\cdot n+z\,u'\cdot b \qquad \forall\, y,z$$

$$\Rightarrow \boxed{\begin{aligned}-\omega\cdot b+u'\cdot n&=0\\ \omega\cdot n+u'\cdot b&=0\end{aligned}} \qquad (*)$$

PRETPOSTAVKA 3

NEPRODULJIVOST CENTRALNE LINIJE

$$\boxed{u' \cdot t = 0} \qquad (*_2)$$

$(*_1)$ & $(*_2)$ $\Rightarrow$ $\boxed{u' + t \times \omega = 0}$ $(**)$

MATERIJALNA RESTRIKCIJA

(DIO ZAKONA PONAŠANJA!)

PRETPOSTAVKA 4

$$\boxed{q = Q H Q^T \omega'} \qquad (***)$$

$(Q = (t(x)\; n(x)\; b(x)) \in SO(3))$

ONB

$$H = \begin{pmatrix} \mu K & 0 & 0 \\ 0 & EI_3 & EI_{23} \\ 0 & EI_{23} & EI_2 \end{pmatrix}$$

E - YOUNGOV MODUL ELASTIČNOSTI

μ - MODUL SMICANJA

SVOJSTVA MATERIJALA

E.. $2{,}1 \cdot 10^{11}$ Pa

μ.. $8 \cdot 10^{10}$ Pa

ČELIK

$$I_2(x) = \int_{S(x)} y^2 \, dy\, dz, \quad I_3(x) = \int_{S(x)} z^2 \, dy\, dz, \quad I_{23}(x) = \int_{S(x)} yz \, dy\, dz$$

$$K(x) = \int (\partial_y r - z)^2 + (\partial_z p + y)^2 \, dy\, dz$$

* r JE RJEŠENJE ZADAĆE NA PRESJEKU

NAĆI $r \in H^1(S(x))$ $\int_{S(x)} r = 0$ T.D.

$$\int (\partial_y r - z)\partial_y s + (\partial_z p + y)\partial_z s = 0, \quad s \in H^1(S).$$

(*), (**) i (***) DAJU MODEL

$$p' + f = 0$$

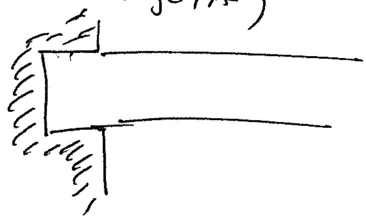
$$g' + t \times p + l = 0 \qquad \text{(OBJE LINEARIZIRANE)}$$

$$\omega' - Q H^{-1} Q^T g = 0$$

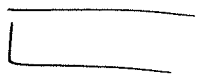
$$u' + t \times \omega = 0$$

RUBNI UVJETI (SUSTAV 1. REDA ZA 12 NEPOZNANICA)

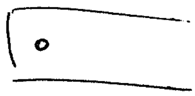
a) $u(0) = 0$, $\omega(0) = 0$ | UKLIJEŠTEN RUB (6 UVJETA)



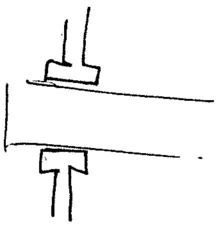
b) $g(0) = 0$, $p(0) = 0$ | SLOBODAN RUB



c) ~~ŠA~~ $u(0) = 0$, $g(0) = 0$ | ŠARNIR



d) $\omega(0) = 0$, $p(0) = 0$ | UŽLIJEBLJEN



e) ELASTIČNE OPRUGE ZADANE NA PROGIB ILI ROTACIJU

NEKA JE DANA RUBNA ZADAĆA S RUBNIM UVJETIMA

$$u(0) = w(0) = 0$$

$$p(\ell) = g(\ell) = 0$$

NEKA JE $v \in H^1(0,\ell)$, $v(0) = 0$
$w \in H^1(0,\ell)$, $w(0) = 0$ T.D. $v' + t \times w = 0$

v MNOŽI 1. JDBU, w MNOŽI 2. JDBU, INTEGRIRAMO $\int_0^\ell dx$ I ZBROJIMO

$$\int_0^\ell p' \cdot v + f \cdot v + g' \cdot w + t \times p \cdot w + l \cdot w = 0$$

$$\parallel$$

$$-\int_0^\ell p \cdot v' + \int_0^\ell f \cdot v - \int_0^\ell g \cdot w' + \int_0^\ell w \times t \cdot p + \int_0^\ell l \cdot w$$

$$\parallel$$

$$-\int_0^\ell p \cdot \underbrace{(v' + t \times w)}_{=0} - \int_0^\ell g \cdot w' + \int_0^\ell f \cdot v + \int_0^\ell l \cdot w$$

$$\Rightarrow \quad \text{(SF)} \quad \boxed{\int_0^\ell Q H Q^T \omega' \cdot w' = \int_0^\ell f \cdot v + l \cdot w, \quad \forall (v,w) \in V = \{(v,w) \in H^1(0,\ell)^3 \times H^1(0,\ell)^3 : v' + t \times w = 0,\ v(0) = w(0) = 0\}}$$

SLABA FORMULACIJA

TEOREM

POSTOJI I JEDINSTVENO JE RJEŠENJE OD (SF)

DOK:

V JE HILBERTOV

BILINEARNOST, SIMETRIČNOST & NEPR. OD L.S

LINEARNOST I NEPR DESNE STRANE

KOERCITIVNOST:

$$\|u\|_{H^1}^2 + \|w\|_{H^1}^2 \overset{\text{POINCARE}}{\le} C\left(\|u'\|_{L^2}^2 + \|w'\|_{L^2}^2\right)$$

$$\le C\Big[\big(\underbrace{\|u' + t\times w\|_{L^2}}_{=\,0 \text{ NA } V} + \underbrace{\|t\times w\|_{L^2}}_{\le\, C\|w'\|_{L^2}}\big)^2 + \|w'\|_{L^2}^2\Big]$$

$$\le C\|w'\|_{L^2}^2 = C\int_0^\ell w'\cdot w'$$

$$\le C\int_0^\ell Qw'\cdot Qw' \le \tilde{C}\int_0^\ell \underset{\text{UNIF POZ DEF!}}{\underset{\uparrow}{H}}\, Qw'\cdot Qw'$$

OČITA POZ DEF ZA $I_{23}=0$ (DODATNA SIMETRIJA)

EVOLUCIJSKA JEDNADŽBA MODELA ZAKRIVLJENOG ŠTAPA

$$\rho A u_{tt} + p' + f = 0$$

$$q' + t \times p + l = 0$$

$$\omega' - Q H^{-1} Q^T q = 0$$

$$u' + t \times \omega = 0$$

+ R.U.

ρA – LINIJSKA GUSTOĆA MASE

ρ – VOLUMNA GUSTOĆA

A – površina POPREČNOG PRESJEKA

TRIPADNA SLABA FORMULACIJA : NAĆI $(u,\omega) \in \ldots$

$$\frac{d^2}{dt^2}\int_0^\ell \rho A\, u\cdot v + \int_0^\ell Q H Q^T \omega'\cdot w' = \int_0^\ell f\cdot v + l\cdot w$$

$$\forall (v,w) \in V$$

ZA R.U. $u(0) = \omega(0) = q(\ell) = p(\ell) = 0$

$$E(t) = \underbrace{\int_0^\ell \rho A u_t^2}_{\text{KIN.}} + \underbrace{\int_0^\ell Q H Q^T \omega'\cdot\omega'}_{\text{ELASTIČNA}}$$

UKUPNA ENERGIJA

SVOJSTVENA ZADAĆA

$u(x,t) = u(x)\,T(t)$ S TIM U SLABU FORMULACIJU

$$\frac{T''}{T} = -\frac{\int_0^\ell Q H Q^T \omega'\cdot w'}{\int_0^\ell \rho A\, u\cdot v} = -\lambda$$

NAĆI SVE λ T.D. $\exists\ (u,\omega) \neq 0 \in V$ T.D.

$$\int_0^\ell Q H Q^T \omega'\cdot w' = \lambda \int_0^\ell \rho A\, u\cdot v \qquad \forall (v,w) \in V$$

2.2 JEDNADŽBA ŠTAPA

PRETPOSTAVLJAMO RAVNINSKU DEFORMACIJU I RAVNI ŠTAP

$$\varphi(x) = x e_1 \qquad \Rightarrow t = \varphi' = e_1$$

$$\Rightarrow n = e_2 \qquad \Rightarrow Q = I$$

$$b = e_3$$

$$u' + e_1 \times \omega = 0$$

$u_1' = 0$ — SLIJEDI IZ NEPRODULJIVOSTI $u_1 = \text{const}$!

$u_2' - \omega_3 = 0 \Rightarrow \omega_3 = u_2'$

$u_3' + \omega_2 = 0 \Rightarrow \omega_2 = -u_3'$

$$\Rightarrow \quad q = H \begin{pmatrix} \omega_1' \\ -u_3'' \\ u_2'' \end{pmatrix} = \begin{pmatrix} \mu K \omega_1' \\ -EI_3 u_3'' \\ EI_2 u_2'' \end{pmatrix}$$

$$q' + e_1 \times p + \ell = 0$$

$q_1' + \ell_1 = 0 \Rightarrow (\mu K \omega_1')' + \ell_1 = 0$ TORZIJA ŠTAPA (KAO NAPETA ŽICA)

$q_2' - p_3 + \ell_2 = 0 \Rightarrow p_3 = q_2' + \ell_2 = -(EI_3 u_3'')' + \ell_2$

$q_3' + p_2 + \ell_3 = 0 \qquad p_2 = -q_3' - \ell_3 = -(EI_2 u_2'')' - \ell_3$

$$p' + f = 0$$

$p_1' + f_1 = 0$

$p_2' + f_2 = 0 \qquad -(EI_2 u_2'')'' - \ell_3' + f_2 = 0$

$p_3' + f_3 = 0 \qquad -(EI_3 u_3'')'' + \ell_2' + f_3 = 0$

JEDNADŽBA ZA POPREČNE POMAKE

(JEDNADŽBA ŠTAPA)

$$\left(EIu''\right)'' = f - \ell' \qquad (J.Š)$$

AKO JE U SREDSTVU KOJE SE ELASTIČNO OPIRE

$$\left(EIu''\right)'' + bu = f - \ell'$$

<u>NAP:</u> U SOBI RAVNOTEŽE ~~ZA~~ MOMENTE SMO IMALI

$$g' + (t + \underbrace{u'}_{\uparrow \text{ZANEMARILI}}) \times p + \ell = 0$$

NO AKO JE $p \cdot t = a$ IMA ZNAČAJNU VRIJEDNOST
(NAPETOST)

TREĆI ČLAN $au' \times e_1$

AKO JE $u = u_2 e_2 \Rightarrow au' \times e_1 = au_2' e_3 (-1)$

$$\Rightarrow g_3' + p_2 - au_2' + \ell_3 = 0$$

$$\Rightarrow p_2 = -g_3' + au_2' - \ell_3$$

(NAPETI ŠTAP)

$$\Rightarrow -\left(EI_2 u_2''\right)'' + \left(au_2'\right)' + f_2 - \ell_3' = 0$$

<u>NAP:</u>

$p_2 = -\left(EI_2 u_2''\right)'$ – KONTAKTNA SILA $[x, \ell]$ NA $[0, x)$

$g_3 = EI_2 u_2''$ – KONTAKTNI MOMENT

$\omega_3 = u_2'$ – ROTACIJA (INF.)

u_2 – POMAK

$$(EI\,u'')'' = f$$

$u(0)=0$

$u'(0)=0 \iff \omega_3(0)=0$ | UKLIJEŠTEN L.K.

$$-(EI\,u'')'\big|_{x=\ell}=0$$

$p_2(\ell)=0$ | SLOBODAN D.K. $EI_4\, u''\big|_{x=\ell}=0$

$q_3(\ell)=0$

$$V = \{ v\in H^2(0,\ell) : v(0)=v'(0)=0 \}$$

$$\int_0^\ell (EI\,u'')''\,v = \int_0^\ell f\cdot v$$

$$= -\int_0^\ell (EI u'')'v' + \cancel{(EI u'')'v\Big|_0^\ell}$$

$$= \int_0^\ell EI\,u''v'' - \cancel{EI\,u''v'\Big|_0^\ell}$$

SLABA FORMULACIJA:

$$(\text{SFŠ})\quad \boxed{\begin{array}{l}\text{NAĆI } u\in V \text{ t.d.}\\ \displaystyle\int_0^\ell EI\,u''v'' = \int_0^\ell f\cdot v, \quad v\in V\end{array}}$$

TEOREM Neka je $0 < I_{min} = \min\limits_{x\in[0,\ell]} I(x)$

Tada zadaća (SFŠ) ima jedinstveno rješenje.

Dok: Opet idemo na Lax–Milgrama

$(V, \|\ \|_{H^2})$ je Hilbertov

$$\|v\|_{H^2}^2 = \|v\|_{L^2}^2 + \|v'\|_{L^2}^2 + \|v''\|_{L^2}^2$$

$$B(u,v) = \int_0^\ell EI\, u''\, v'' \quad \text{— bilinearna, simetrična}$$

$$|B(u,v)| \le C\,\|u''\|_{L^2}\|v''\|_{L^2} \le C\,\|u\|_{H^2}\|v\|_{H^2} \Rightarrow \text{nepr}$$

$$L(v) = \int_0^\ell f\cdot v$$

$$|L(v)| \le \|f\|_{L^2}\|v\|_{L^2} \le \|f\|_{L^2}\|v\|_{H^2} \Rightarrow \text{nepr}$$

Koercitivnost za B:

$$B(u,u) = \int_0^\ell EI\,(u'')^2 \ge EI_{min}\,\|u''\|_{L^2}^2$$

$$\cancel{\ge EI_{min}\,C_p^2\,\|u'\|_{L^2}^2 \ge EI_{min}\,C_p^4\,\|u\|_{H^2}^2}$$

Poincaré za u' $$\ge \frac{1}{3}EI_{min}\|u''\|_{L^2}^2 + \frac{2}{3}EI_{min}\,C_p^2\,\|u'\|_{L^2}^2$$

$$\ge \frac{1}{3}EI_{min}\|u''\|_{L^2}^2 + \frac{1}{3}EI_{min}\,C_p^2\|u'\|_{L^2}^2 + \frac{1}{3}EI_{min}\,C_p^4\|u\|_{L^2}^2$$

$$\ge C\,\|u\|_{H^2}^2$$

$$C = \min\{1, C_p^2, C_p^4\}\cdot \frac{1}{3}EI_{min}$$

ZAD: IZVEDITE SLABU FORMULACIJU ZA R.U.

$$p_z(\ell) = p_0$$

$$q_z(\ell) = q_0$$

ANALIZIRAJTE TEOREM EGZISTENCIJE I JEDINSTVENOSTI

ZAD: IZVEDITE SLABU FORMULACIJU I ANALIZIRAJTE EGZ. I JED.
ZA MODEL NAPETOG ŠTAPA. MOŽE LI a BITI NEGATIVAN? U OKVIRU TEORIJE

ZAD: IZVEDITE SLABU FORMULACIJU I ANALIZIRAJTE EGZ. I JED.
ZA ŠARNIR NA OBA KRAJA

$$u(0) = u''(0) = u(\ell) = u''(\ell) = 0$$

ZAD: FORMULIRAJTE EVOLUCIJSKU JEDNADŽBU

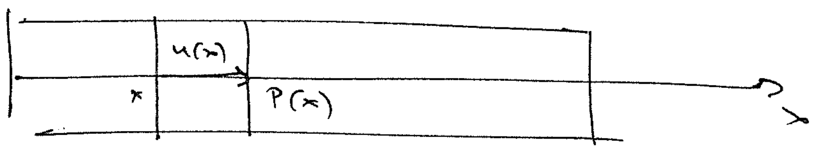
ZAD: FORMULIRAJTE SVOJSTVENU ZADAĆU

ZAD: ANALIZIRAJTE SVOJSTVENU ZADAĆU ZA ŠTAP
S UKLJEŠTENIM RUBOVIMA (IZRAČUNAJTE SV. VR. I SV. VEKTORE)

ZAD: ANALIZIRAJTE SVOJSTVENU ZADAĆU ZA ŠTAP SA ŠARNIROM NA OBA KRAJA.

2.3. MODELI LONGITUDINALNOG POMAKA I TORZIJE

LONGITUDINALNI POMAK



$x \longmapsto P(x)$

NEDEF. DEF.

POLOŽAJ

$P(x) = x + u(x)$

- $u'(x)$ MJERI RASTEZANJE LOKALNO

$$\int_{x_1}^{x_2} u'(x)\,dx = u(x_2) - u(x_1)$$

- KONTAKTNA SILA $g(x)$: SILA KOJOM $(P(x), P(\ell))$ DJELUJE NA $(P(0), P(x))$

- VANJSKA LINIJSKA GUSTOĆA SILE $f(x)$

- PRINCIP RAVNOTEŽE:

$$g(x) - g(0) + \int_0^x f(y)\,dy = 0 \qquad \text{INT}$$

$$g' + f = 0 \qquad \text{DIF}$$

- ZAKON PONAŠANJA { HOOKOV ZAKON $g(x) = a\,u'(x)$

$a = EA$

E - YOUNGOV MODUL

A - POVRŠINA POPREČNOG PRESJEKA | OPĆENITO FUNKCIJE

MODEL : $(EAu')' + f = 0$

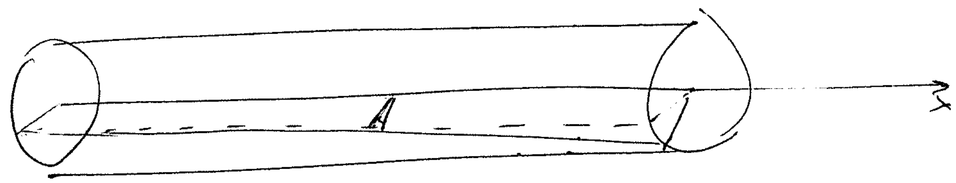
RUBNI UVJET : $u(0) = u_0$ DIRICHLET

$u'(\ell) = g_0/EA$ NEUMANN

$u'(\ell) = \varkappa u(\ell)$ ROBIN

MODEL ISTI KAO ZA NAPETU ŽICU .. ANALIZA ISTA

MODEL ZA TORZIJU



- $\phi(x)$ KUT ZAKRETA NA MJESTU x
- $\phi'(x)$ MJERA DEFORMACIJE
- $q(x)$ KONTAKTNI MOMENT
  (x, ℓ) DJELUJE NA $(0, x)$

- ZAKON PONAŠANJA

$$q(x) = \mu K \phi'(x)$$

μ - MODUL SMICANJA

K - KRUTOST TORZIJE

$$K = 2\frac{R^4\pi}{4} = \frac{I}{2}\pi$$
$$= 2I$$
ZA KRUG

- LINIJSKA GUSTOĆA MOMENTA $\ell(x)$

$$q' + \ell = 0$$
$$(\mu K \phi')' + \ell = 0$$

2.4. TIMOŠENKOV MODEL ŠTAPA

- RUSKI INŽENJER, RADIO U ZAGREBU NA TEHNIČKOM FAKULTETU
- NEZADOVOLJAN EULER (NAVIER) - BERNOULLIJEVIM MODELOM
  NESLAGANJE KOD EKSPERIMENATA
-

$$\text{ENERGIJA (E-B)} = \frac{1}{2} \int_0^l EI(u'')^2 - \int_0^l f \cdot u$$

SUPSTITUCIJA: $\omega = u'$

$$\text{ENERGIJA (T)} = \frac{1}{2} \int_0^l EI(\omega')^2 + k\mu A(u' - \omega)^2 - \int_0^l f \cdot u$$

- DOZVOLJENO JE DA $\omega \neq u'$
- PENALIZIRAMO S $k\mu A$, k TIMOŠENKO SHEAR CORRECTION $= 5/6$ ZA PRAVOKUTNIK
- KADA JE $EI \ll k\mu A L^2$

  MODELI SU "JEDNAKI"
- BOLJE OPISUJE EKSPERIMENT ZA "DEBLJE" ŠTAPOVE

$$I \sim h^4$$
$$A \sim h^2$$

- NE SLIJEDI IZ 3D KLASIČNE TEORIJE ELASTIČNOSTI