Matematika (biolozi) - Zadaci s parametrima

1. U ovisnosti o parametru a riješite sustav

   (a)
$$\begin{aligned} x+ay &= 5, \\ x+y &= 2, \\ ax+2y &= 4. \end{aligned}$$

   (b)
$$\begin{aligned} 2x_1+ax_2-13x_3 &= -32 \\ x_1-2x_2+x_3 &= a \\ -2x_1+3x_2+ax_3 &= 8 \end{aligned}$$

   (c)
$$\begin{aligned} ax_1+2x_2+x_3+x_4 &= 1 \\ 2x_1-x_2-x_4 &= 1 \\ 4x_1+3x_2+2x_3+ax_4 &= 3 \\ 5x_2+2x_3+3x_4 &= 1 \end{aligned}$$

2. Odredite parametar t takav da je skup $\{(0,2,0),(-1,0,t),(2,0,1)\}$ linearno nezavisan.

3. U ovisnosti o parametru t odredi rang matrice:

$$\text{a)}\begin{pmatrix} 1 & 1 & t \\ 1 & t & 1 \\ t & 1 & 1 \end{pmatrix}, \quad \text{b)}\begin{pmatrix} t & 3 & -1 \\ 3 & 6 & -2 \\ -1 & -3 & t \end{pmatrix}, \quad \text{c)}\begin{pmatrix} t & 2 & -1 \\ 3 & 3 & -2 \\ -2 & -1 & t \end{pmatrix}, \quad \text{d)}\begin{pmatrix} 2 & t & 1 & -3 \\ 3 & -4 & -9 & -8 \\ -2 & 1 & 5 & 2t \\ 1 & -1 & -4 & -2 \end{pmatrix}.$$

4. U ovisnosti o parametrima r i s odredi rang matrice: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{pmatrix}$.

5. Pod kojim uvjetima na brojeve b_1, b_2, b_3, b_4, b_5 sustav
$$\begin{aligned} x_1-3x_2 &= b_1 \\ x_1-2x_2 &= b_2 \\ x_1+x_2 &= b_3 \\ x_1-4x_2 &= b_4 \\ x_1+5x_2 &= b_5 \end{aligned}$$
   ima rješenje?

6. Odredite $a \in \mathbb{R}$ za koje je matrica $\begin{pmatrix} a & 2 & 1 \\ 2 & 1 & 1 \\ 1 & a & -2 \end{pmatrix}$ singularna.

7. Odredite a i b za koje je matrica $\begin{pmatrix} b & 0 & a & 0 \\ 0 & 0 & b & a \\ 0 & a & 0 & b \\ b & a & 0 & 0 \end{pmatrix}$ singularna.

8. Riješite jednadžbu

$$\begin{vmatrix} x-3 & x+2 & x-1 \\ x+2 & x-4 & x \\ x-1 & x+4 & x-5 \end{vmatrix} = 0.$$

9. Riješite jednadžbu

$$\begin{vmatrix} a+x & x & x \\ x & b+x & x \\ x & x & c+x \end{vmatrix} = 0.$$

10. Izračunajte determinantu

$$D_n = \begin{vmatrix} 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \\ 0 & 0 & 0 & 0 & \dots & -1 & 1 \end{vmatrix}$$

11. Izračunajte determinantu

$$D_n = \begin{vmatrix} 1 & n & n & \dots & n \\ n & 2 & n & \dots & n \\ \vdots & \vdots & \vdots & & \vdots \\ n & n & n & \dots & n \end{vmatrix}$$