5.6. NEJEDNAKOSTI SOBOLJEVA

- GOVORE O NEPREKIDNOSTI ULAGANJA $W^{k,p}$
- PRVO ZA $W^{1,p}$, INDUKTIVNO ZA VIŠE
- DOKAZUJEMO ZA GLATKE, PO GUSTOĆI PROŠIRIMO

- OSNOVNO PITANJE:

$$W^{1,p} \hookrightarrow \ ?$$

- ODGOVOR OVISI O:

$$1 \leq p < n$$

$$p = n \qquad \text{(KASNIJE)}$$

$$n < p \leq \infty$$

5.6.1. GAGLIARDO – NIRENBERG – SOBOLEV NEJEDNAKOST

$$1 \leq p < n$$

ŽELIMO OCJENU OBLIKA:

$$\|u\|_{L^q(\mathbb{R}^n)} \leq C \|Du\|_{L^p(\mathbb{R}^n)}, \quad u \in C_c^\infty(\mathbb{R}^n)$$

ZA NEKE C, q NEOVISNE O u

ŠTO MOŽEMO REĆI O q?

NEKA JE $u \in C_c^\infty(\mathbb{R}^n)$, $u \neq 0$, $\lambda > 0$

$$u_\lambda(x) := u(\lambda x), \quad x \in \mathbb{R}^n$$

UBACIMO U NEJEDNAKOST:

$$\|u_\lambda\|_{L^q(\mathbb{R}^n)} \leq C \|Du_\lambda\|_{L^p(\mathbb{R}^n)}, \quad \lambda > 0$$

$$\|u_\lambda\|^q_{L^q(\mathbb{R}^n)} = \int_{\mathbb{R}^n} |u(\lambda x)|^q \, dx = \left| y = \lambda x \right| = \frac{1}{\lambda^n} \int_{\mathbb{R}^n} |u(y)|^q \, dy$$

$$= \frac{1}{\lambda^n} \|u\|^q_{L^q(\mathbb{R}^n)}$$

$$\|Du_\lambda\|^p_{L^p(\mathbb{R}^n)} = \int_{\mathbb{R}^n} |Du_\lambda(x)|^p \, dx = \int_{\mathbb{R}^n} |\lambda Du(\lambda x)|^p \, dx$$

$$= \lambda^p \int_{\mathbb{R}^n} |Du(\lambda x)|^p \, dx = \left| y = \lambda x \right| = \frac{\lambda^p}{\lambda^n} \|Du\|^p_{L^p(\mathbb{R}^n)}$$

$$\Longrightarrow \frac{1}{\lambda^{n/q}} \|u\|_{L^q(\mathbb{R}^n)} \leq C \frac{\lambda}{\lambda^{n/p}} \|Du\|_{L^p(\mathbb{R}^n)}, \quad \lambda > 0$$

$$\Longrightarrow \|u\|_{L^q(\mathbb{R}^n)} \leq C \lambda^{\left(1 - \frac{n}{p} + \frac{n}{q}\right)} \|Du\|_{L^p(\mathbb{R}^n)}, \quad \lambda > 0$$

$$\|u\|_{L^q(\mathbb{R}^n)} \leq C\, \lambda^{\left(1-\frac{n}{p}+\frac{n}{q}\right)} \|Du\|_{L^p(\mathbb{R}^n)} \quad , \quad \lambda > 0$$

$$\cancel{1} - \frac{n}{p} + \frac{n}{q} > 0 \quad , \quad \lambda \to 0 \quad \Rightarrow\Leftarrow$$

$$1 - \frac{n}{p} + \frac{n}{q} < 0 \quad , \quad \lambda \to +\infty \quad \Rightarrow\Leftarrow$$

$$\Rightarrow \quad \boxed{1 - \frac{n}{p} + \frac{n}{q} = 0}$$

$$\Rightarrow \quad \boxed{\frac{1}{q} = \frac{1}{p} - \frac{1}{n} \quad , \quad q = \frac{np}{n-p}}$$

JEDINSTVENO ODREĐEN q!

~~DEF:~~ ZA $1 \leq p < n$ DEF: $p^* = \dfrac{np}{n-p}$

(SOBOLJEVLJEV KONJUGIRANI EKSPONENT)

$$\frac{1}{p^*} = \frac{1}{p} - \frac{1}{n} < \frac{1}{p} \quad \Rightarrow \quad p^* > p$$

TM DOKAZUJEMO U 2 KORAKA:

1. ZA $C^1_c(\mathbb{R}^n)$
2. ZA $W^{1,p}(U)$

TM 1 (GAGLIARDO – NIRENBERG – SOBOLJEV)

NEKA JE $p \in [1,n\rangle$. TADA $\exists\, C>0$ T.D.

$$\|u\|_{L^{p^*}(\mathbb{R}^n)} \leq C\,\|Du\|_{L^p(\mathbb{R}^n)}, \qquad u \in C^1_c(\mathbb{R}^n)$$

NAP:

- (C JE NEOVISNA O u, A OVISI O p i n)
- BITNO JE DA JE KOMPAKTAN NOSAČ ($u \equiv 1$)

DOK: 1. KORAK: $p=1$

u S KOMPAKTNIM NOSAČEM & N-L

$$u(x) = \int_{-\infty}^{x_i} u_{x_i}(x_1,\dots,x_{i-1},y_i,x_{i+1},\dots,x_n)\,dy_i$$

$$\Downarrow$$

$$|u(x)| \leq \int_{-\infty}^{\infty} |Du(x_1,\dots,y_i,\dots,x_n)|\,dy_i \quad \Big|^{\frac{1}{n-1}} \quad i=1,\dots,n$$

$$\Downarrow$$

$$|u(x)|^{\frac{n}{n-1}} \leq \prod_{i=1}^{n}\left(\int_{-\infty}^{\infty} |Du(x_1,\dots,y_i,\dots,x_n)|\,dy_i\right)^{\frac{1}{n-1}}$$

$\uparrow$ n RAZLIČITIH OCJENA (PRODUKT)

$$\int_{-\infty}^{\infty} |u(x)|^{\frac{n}{n-1}}\,dx_1 \leq \int_{-\infty}^{\infty}\prod_{i=1}^{n}\left(\int_{-\infty}^{\infty}|Du|\,dy_i\right)^{\frac{1}{n-1}} dx_1$$

$$= \left(\int_{-\infty}^{\infty}|Du|\,dy_1\right)^{\frac{1}{n-1}} \int_{-\infty}^{\infty}\prod_{i=2}^{n}\left(\int_{-\infty}^{\infty}|Du|\,dy_i\right)^{\frac{1}{n-1}} dx_1$$

"PRODULJENI" HÖLDER

$$\int_U |u_1\cdots u_m|\,dx \leq \prod_{k=1}^{m}\|u_i\|_{L^{p_k}(U)}$$

$$\frac{1}{p_1}+\dots+\frac{1}{p_m}=1 \qquad p_1,\dots,p_m \in [1,+\infty]$$

$p_2 = \dots = p_n = n-1$

$$\Longrightarrow \int_{-\infty}^{\infty} |u(x)|^{\frac{n}{n-1}} dx_1 \le \left(\int_{-\infty}^{\infty} |Du| dy_1\right)^{\frac{1}{n-1}} \prod_{i=2}^{n} \left(\int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} |Du| dy_i\right) dx_1\right)^{\frac{1}{n-1}}$$

- INTEGRIRAM PO dx_2 LIJEVU I DESNU STRANU
- DESNO ~~DOBIVAM~~ IMAMO

ČLAN $\int_{-\infty}^{\infty} |Du| \, dy_2 \, dx_1$ KOJI IZAĐE ISPRED $\int dx_2$

NA OSTALIH $n-1$ ČLANOVA $\sim$ "PRODULJENI" HÖLDER

- OSTANE $\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |Du| \, dx_1 \, dx_2\right)^{\frac{2}{n-1}}$ I TROSTRUKI $dx_1 \, dx_2 \, dy_i$, $i = 3, \dots, n$
- INDUKTIVNO:

$$\int_{\mathbb{R}^n} |u|^{\frac{n}{n-1}} dx \le \left(\int_{\mathbb{R}^n} |Du| \, dx\right)^{\frac{n}{n-1}}$$

$$\Longrightarrow \quad \|u\|_{L^{\frac{n}{n-1}}(\mathbb{R}^n)} \le \|Du\|_{L^1(\mathbb{R}^n)} \qquad u \in C_c^1(\mathbb{R}^n)$$

TO JE NEJEDNAKOST ZA $p=1$

2. KORAK

$p \in \langle 1, n \rangle$

ZA $v := |u|^{\gamma}$, $\gamma > 1$ $\Rightarrow$ $|Dv| = \gamma |u|^{\gamma-1} |Du|$

$$\left(\int_{\mathbb{R}^n} |u|^{\frac{\gamma n}{n-1}} dx\right)^{\frac{n-1}{n}} \le \gamma \int_{\mathbb{R}^n} |u|^{\gamma-1} |Du| \, dx$$

HÖLDER $\left(\frac{p-1}{p} + \frac{1}{p} = 1\right)$

$$\le \gamma \left(\int_{\mathbb{R}^n} |u|^{(\gamma-1)\frac{p}{p-1}} dx\right)^{\frac{p-1}{p}} \left(\int_{\mathbb{R}^n} |Du|^p dx\right)^{\frac{1}{p}}$$

BIRAMO γ: $\quad \frac{\gamma n}{n-1} = (\gamma - 1)\frac{p}{p-1} \longrightarrow \gamma = \frac{p(n-1)}{n-p} > p \ge 1$

$$\Rightarrow \quad \frac{\gamma n}{n-1} = (\gamma - 1)\frac{p}{p-1} = \frac{p(n-1)}{n-p}\,\frac{n}{n-1} = \frac{pn}{n-p} = \frac{1}{\frac{1}{p}-\frac{1}{n}} = p^*$$

$$\Rightarrow \quad \left(\int_{\mathbb{R}^n} |u|^{p^*}\,dx\right)^{\frac{n-1}{n}} \le \gamma \left(\int_{\mathbb{R}^n} |u|^{p^*}\,dx\right)^{\frac{p-1}{p}} \|Du\|_{L^p(\mathbb{R}^n)}$$

$$\frac{n-1}{n} - \frac{p-1}{p} = \cancel{1} - \frac{1}{n} - \left(\cancel{1} - \frac{1}{p}\right) = \frac{1}{p} - \frac{1}{n} = \frac{1}{p^*}$$

$$\Rightarrow \quad \|u\|_{L^{p^*}(\mathbb{R}^n)} \le \gamma\, \|Du\|_{L^p(\mathbb{R}^n)}$$

TM2 ($W^{1,p}$) NEKA JE $U \subseteq \mathbb{R}^n$ OTVOREN, OGRANIČEN, ∂U KLASE C^1, $p \in [1, n\rangle$, $u \in W^{1,p}(U)$.

TADA JE $u \in L^{p^*}(U)$ I

$$\|u\|_{L^{p^*}(U)} \le C \|u\|_{W^{1,p}(U)},$$

PRI ČEMU C OVISI SAMO O p, n, U.

DOK: ∂U KLASE C^1

(TM1 U 5.4) $\Longrightarrow$ $\exists E : W^{1,p}(U) \to W^{1,p}(\mathbb{R}^n)$ L.O.

$\bar{u} := Eu \in W^{1,p}(\mathbb{R}^n)$ KOMPAKTAN NOSAČ

$\|\bar{u}\|_{W^{1,p}(\mathbb{R}^n)} \le C \|u\|_{W^{1,p}(U)}$.

u KOMP. NOSAČ & TM1 U 5.3 (APPROX)

$\Longrightarrow \exists (u_m) \subseteq C_c^\infty(\mathbb{R}^n) \quad u_m \longrightarrow \bar{u} \quad W^{1,p}(\mathbb{R}^n)$

$\Longrightarrow \quad u_m \longrightarrow \bar{u} \quad L^p(\mathbb{R}^n)$

$Du_m \longrightarrow D\bar{u} \quad L^p(\mathbb{R}^n) \Longrightarrow (Du_m)$ C-NIZ U $L^p(\mathbb{R}^n)$

(TM1) $\Longrightarrow \quad \|u_m - u_l\|_{L^{p^*}(\mathbb{R}^n)} \le C \|D(u_m - u_l)\|_{L^p(\mathbb{R}^n)} \le C \|Du_m - Du_l\|_{L^p(\mathbb{R}^n)}$

$\Longrightarrow \quad (u_m)$ C-NIZ U $L^{p^*}(\mathbb{R}^n)$

$\Longrightarrow \quad u_m \longrightarrow * \quad L^{p^*}(\mathbb{R}^n)$

JEDINSTVENOST LIMESA $\Longrightarrow \quad \bar{u} = * \in L^{p^*}(\mathbb{R}^n)$!

(TM1) $\Longrightarrow \quad \|u_m\|_{L^{p^*}(\mathbb{R}^n)} \le C \|Du_m\|_{L^p(\mathbb{R}^n)}$

$$\downarrow \qquad\qquad \downarrow$$

$$\|u\|_{L^{p^*}(U)} = \|\bar{u}\|_{L^{p^*}(U)} \le \|\bar{u}\|_{L^{p^*}(\mathbb{R}^n)} \le C \|D\bar{u}\|_{L^p(\mathbb{R}^n)} \le C\|u\|_{W^{1,p}(U)}$$

TM 3 ($W_0^{1,p}$)

NEKA JE $U \subseteq \mathbb{R}^n$ OTVOREN, OGRANIČEN, $p \in [1,n)$, $u \in W_0^{1,p}(U)$.
TADA JE $u \in L^q(U)$ I

$$\|u\|_{L^q(U)} \leq C \, \|Du\|_{L^p(U)},$$

ZA SVAKI $q \in [1, p^*]$, PRI ČEMU C OVISI SAMO O p, q, n, U.

NAP: OCJENE OVOG TIPA (FJA KONTROLIRANA DIFERENCIJALOM) ZOVU SE POINCAREOVE NEJEDNAKOSTI.

DOK: $u \in W_0^{1,p}(U)$

$\Rightarrow \exists (u_m) \subseteq C_c^\infty(U)$, $u_m \longrightarrow u$ u $W^{1,p}(U)$.

u_m PROŠIRIMO O DO $C_c^\infty(\mathbb{R}^n)$

(TM1) $\Rightarrow$ $\|u_m\|_{L^{p^*}(\mathbb{R}^n)} \leq C \, \|Du_m\|_{L^p(\mathbb{R}^n)}$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\|u\|_{L^{p^*}(\mathbb{R}^n)} \qquad\qquad \|Du\|_{L^p(\mathbb{R}^n)}$$

ISTI ARGUMENT KAO PRIJE

$$\|u_m - u_e\|_{L^{p^*}(\mathbb{R}^n)} \leq C \, \|Du_m - Du_e\|_{L^p(\mathbb{R}^n)}$$

C-NIZ $\Longleftarrow$ C-NIZ

$\Rightarrow u_m \longrightarrow * \;$ u $L^{p^*}(\mathbb{R}^n)$

JEDINSTVENOST LIMESA $u = * \in L^{p^*}(\mathbb{R}^n)$

$$\Rightarrow \|u\|_{L^{p^*}(\mathbb{R}^n)} \leq C \, \|Du\|_{L^p(\mathbb{R}^n)} = C \, \|Du\|_{L^p(U)}$$

$\|u\|_{L^{p^*}(U)} = \|u\|_{L^{p^*}(\mathbb{R}^n)}$

$|U| < \infty \Rightarrow \|u\|_{L^q(U)} \leq C \|u\|_{L^{p^*}(U)} \Rightarrow q \in [1, p^*]$!

NAP: 1) NA $W_0^{1,p}(U)$ $\quad \|Du\|_{L^p(U)} \sim \|u\|_{W^{1,p}(U)}$

$|U| < \infty$

$$\Updownarrow$$

u $\quad \|Du\|_{L^p(U)} \le \|u\|_{W^{1,p}(U)} \le M \|Du\|_{L^p(U)}$

$\uparrow$ OČITO

$$\|u\|_{W^{1,p}(U)} = \left(\|u\|^p_{L^p(U)} + \|Du\|^p_{L^p(U)} \right)^{1/p}$$

NAIME $\quad \|u\|_{L^p(U)} \le C \|Du\|_{L^p(U)}$

JER JE $\quad p \le p^*$

NAP 2)

$$\|u\|_{L^{p^*}(U)} \le C \|Du\|_{W^{1,p}(U)}$$

$$p \to n \quad \Longrightarrow \quad p^* = \frac{np}{n-p} \longrightarrow +\infty$$

NADA:

$$\|u\|_{L^\infty(U)} \le C \|u\|_{W^{1,n}(U)}$$

KONTRAPRIMJER: $\quad n > 1 \quad U = B(0,1)$

$$u = \log \log \left(1 + \frac{1}{|x|}\right)$$

$u \in W^{1,n}(U)$

$u \notin L^\infty(U)$.

5.6.2 MORREYJEVA NEJEDNAKOST

$$p \in \langle n, +\infty]$$

TM 4 (MORREY)

NEKA JE $p \in \langle n, +\infty]$. TADA $\exists\, C > 0$ T.D.

$$\| u \|_{C^{0,\gamma}(\mathbb{R}^n)} \le C \, \| u \|_{W^{1,p}(\mathbb{R}^n)} \quad , \quad u \in C^1(\mathbb{R}^n)$$

PRI ČEMU JE $\quad \gamma := 1 - \frac{n}{p}$.

NAP: – C OVISI SAMO O p I n.

– ZAPRAVO ĆEMO KASNIJE POKAZATI DA $u \in W^{1,p}(\mathbb{R}^n) \Rightarrow u \in C^{0,\gamma}(\mathbb{R}^n)$
ZA GORNJE p

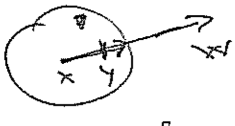
PRI ČEMU JE TVRDNJA DA POSTOJI REPREZENTANT

DOK: 1. KORAK

TV: $\exists\, C > 0$ T.D

$$\fint_{B(x,r)} |u(y) - u(x)| \, dy \le C \int_{B(x,r)} \frac{|Du(y)|}{|y-x|^{n-1}} \, dy$$

DOK: "POLARNE KOORDINATE" $\quad y = x + sw \quad , \quad w \in \partial B(0,1)$
$s \in \langle 0, r\rangle$



N-L: $$|u(x+sw) - u(x)| = \left| \int_0^s \frac{d}{dt}\Big(u(x+tw)\Big) dt \right| = \left| \int_0^s Du(x+tw)\, w \, dt \right|$$

$$\le \int_0^s |Du(x+tw)| \, dt$$

PROINTEGRIRAM PO SFERI:

$$\int_{\partial B(0,1)} |u(x+sw) - u(x)| \, dS \le \underbrace{\int_0^s \int_{\partial B(0,1)}}_{\substack{\text{INTEGRAL} \\ \text{PO KUGLI} \\ B(0,s)}} |Du(x+tw)| \underbrace{\frac{t^{n-1}}{t^{n-1}}}_{\text{JACOBIAN}} \, dS \, dt$$

ZAMJENA VARIJABLI NA $B(x,s)$ $\quad y = x + tw \quad , \quad t = |y-x|$

$$\int_{\partial B(0,1)} |u(x+s\omega)-u(x)|\,dS \le \int_{B(x,s)} \frac{|Du(y)|}{|x-y|^{n-1}}\,dy \le \int_{B(x,r)} \frac{|Du(y)|}{|x-y|^{n-1}}\,dy$$

$s \le r$

POMNOŽIM SA s^{n-1} I $\int_0^r ds$

$$\int_{B(x,r)} |u(y)-u(x)|\,dy \le \frac{r^n}{n} \int_{B(x,r)} \frac{|Du(y)|}{|x-y|^{n-1}}\,dy$$

C OVISI O n ! r^n ODE U VOLUMEN KUGLE LIJEVO

2. KORAK $x \in \mathbb{R}^n$

$$|u(x)| \le |u(x)-u(y)| + |u(y)| \qquad ⨍_{B(x,1)} dy$$

$$\Downarrow$$

$$|u(x)| \le ⨍_{B(x,1)} |u(x)-u(y)|\,dy + ⨍_{B(x,1)} |u(y)|\,dy$$

$$\overset{\text{TU}}{\le} C \int_{B(x,1)} \frac{|Du(y)|}{|x-y|^{n-1}}\,dy + \underbrace{\frac{1}{|B(x,1)|}\|1\|_{L^q(B(x,1))}}_{\le C} \|u\|_{L^p(B(x,1))} \qquad \frac{1}{p}+\frac{1}{q}=1$$

HÖLDER

$$\le C\,\underbrace{\|Du\|_{L^p(B(x,1))}}_{\le \|Du\|_{L^p(\mathbb{R}^n)}} \left(\int_{B(x,1)} \frac{dy}{|x-y|^{q(n-1)}}\right)^{\frac{1}{q}} + C\,\underbrace{\|u\|_{L^p(B(x,1))}}_{\le \|u\|_{L^p(\mathbb{R}^n)}}$$

$$\frac{1}{p}+\frac{1}{q}=1 \Rightarrow \frac{1}{q} = 1-\frac{1}{p} = \frac{p-1}{p} \Rightarrow q = \frac{p}{p-1}$$

$$\int_{B(x,1)} \frac{dy}{|x-y|^{\frac{p}{p-1}(n-1)}} \approx \int_0^1 \frac{r^{n-1}}{r^{\frac{p}{p-1}(n-1)}}\,dr = \int_0^1 \frac{dr}{r^{\frac{n-1}{p-1}}} < \infty$$

ZA $\frac{n-1}{p-1} < 1$

$\Updownarrow$

$n-1 < p-1$

$\Updownarrow$

$n < p$ OK

$$\Rightarrow |u(x)| \le C\|u\|_{W^{1,p}(\mathbb{R}^n)}$$

UZMEM $\sup_{x\in\mathbb{R}^n}$ $\Rightarrow$ $\underbrace{\|u\|_{L^\infty(\mathbb{R}^n)} \le C\|u\|_{W^{1,p}(\mathbb{R}^n)}}$

$C(\mathbb{R}^n)$

KORAK 3:

ISTI RAČUN ZA DVIJE TOČKE $x,y\in\mathbb{R}^n$, $r=|x-y|$

$$W := B(x,r)\cap B(y,r)$$

$$|u(x)-u(y)| \le ⨍_W |u(x)-u(z)|\,dz + ⨍_W |u(y)-u(z)|\,dz$$

OCJENJUJEMO SVAKI NA ISTI NAČIN KAO PRIJE

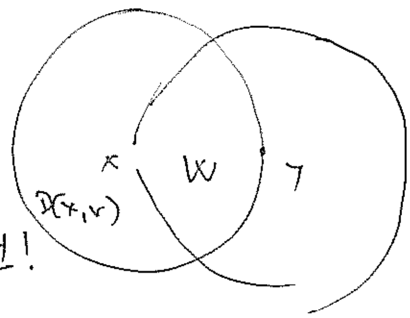
$$⨍_W |u(x)-u(z)|\,dz \le \frac{|B(x,r)|}{|W|} ⨍_{B(x,r)} |u(x)-u(z)|\,dz$$

$$\overset{\text{TV + HÖLDER}}{\le} \frac{|B(x,r)|}{|W|}\, C\,\|Du\|_{L^p(B(x,r))} \underbrace{\left(\int_0^r \frac{ds}{s^{\frac{n-1}{p-1}}}\right)^{\frac{p-1}{p}}}_{= r^{1-\frac{n}{p}}}$$

$\frac{|B(x,r)|}{|W|}$

$\uparrow$

KONSTANTAN!



$$\le C\, r^{1-\frac{n}{p}} \|Du\|_{L^p(\mathbb{R}^n)}$$

ISTO ZA DRUGI ČLAN

$\Rightarrow$ $|u(x)-u(y)| \le C\, r^{1-\frac{n}{p}} \|Du\|_{L^p(\mathbb{R}^n)}$

$\Rightarrow$ $\frac{|u(x)-u(y)|}{|x-y|^{1-\frac{n}{p}}} \le C\|Du\|_{L^p(\mathbb{R}^n)}$ $\Big|\ \sup_{x\ne y}$

$$\underbrace{[u]_{C^{0,1-\frac{n}{p}}(\mathbb{R}^n)} \le C\|Du\|_{L^p(\mathbb{R}^n)}}$$

$\Rightarrow$ TV!

TM 5 ($W^{1,p}$)

NEKA JE $U \subseteq \mathbb{R}^n$ OTVOREN I OGRANIČEN, ∂U KLASE C^1, $p \in \langle n, +\infty]$, $u \in W^{1,p}(U)$. TADA $\exists$ REPREZENTANT u^* OD u T.D. $u^* \in C^{0,\gamma}(\overline{U})$, $\gamma = 1 - \frac{n}{p}$ I

$$\| u^* \|_{C^{0,\gamma}(\overline{U})} \leq C \| u \|_{W^{1,p}(U)},$$

PRI ČEMU C OVISI SAMO O p, n, U.

DOK: ∂U KLASE C^1 (TM 1 U S.4, PROŠIRENJE)

$\Longrightarrow \exists \; E : W^{1,p}(U) \longrightarrow W^{1,p}(\mathbb{R}^n)$

$\bar{u} = Eu = u$ U U

$\bar{u}$ IMA KOMPAKTAN NOSAČ

$\| \bar{u} \|_{W^{1,p}(\mathbb{R}^n)} \leq C \| u \|_{W^{1,p}(U)}$ NEPREKIDNOST E.

[$\bar{u}$ IMA KOMP. NOSAČ (TM 1 U S.3, APROX C_c^∞)

$\Longrightarrow \exists \; (u_m) \subseteq C_c^\infty(\mathbb{R}^n)$ T.D $u_m \longrightarrow \bar{u}$ U $W^{1,p}(\mathbb{R}^n)$

TM 4 $\Longrightarrow$ $\| u_m - u_l \|_{C^{0,1-\frac{n}{p}}(\mathbb{R}^n)} \leq C \| u_m - u_l \|_{W^{1,p}(\mathbb{R}^n)}$

(u_m) C-NIZ U $W^{1,p}$ $\Longrightarrow$ (u_m) C-NIZ U $C^{0,1-\frac{n}{p}}(\mathbb{R}^n)$

POTPUNOST

$\Longrightarrow$ $u_m \longrightarrow u^*$ U $C^{0,1-\frac{n}{p}}(\mathbb{R}^n)$ & $u^* \in C^{0,1-\frac{n}{p}}(\mathbb{R}^n)$

JEDINSTVENOST LIMESA $\Longrightarrow$ $\bar{u} = u^*$ S.S. U U

OCJENA

$$\| u_m \|_{C^{0,1-\frac{n}{p}}(\mathbb{R}^n)} \leq C \| u_m \|_{W^{1,p}(\mathbb{R}^n)}$$

$$\downarrow \qquad\qquad \downarrow$$

$$\| u^* \|_{C^{0,1-\frac{n}{p}}(\overline{U})} \leq \| u^* \|_{C^{0,1-\frac{n}{p}}(\mathbb{R}^n)} \leq C \; \| \bar{u} \|_{W^{1,p}(\mathbb{R}^n)} \leq C \| \bar{u} \|_{W^{1,p}(U)}$$