

Homework Assignment 3

Partial Differential Equations

Problem 1

Consider the elliptic equation:

$$\begin{cases} -\operatorname{div}(A\nabla u) = f & \text{in } U, \\ u = 0 & \text{on } \partial U. \end{cases}$$

Formulate the variational (weak) formulation for the given boundary value problem. Prove the existence and uniqueness (or conditions for non-uniqueness) theorems for this problem in detail (e.g., using the Lax-Milgram Lemma).

Problem 2

Prove the energy estimate (41) found on page 359, section 7.1.3 of the textbook. Provide the proof in two ways:

- Using Gronwall's inequality.
- Without mentioning or explicitly using Gronwall's inequality.

Problem 3

Prove Theorem 12 (The Strong Maximum Principle for parabolic equations) as presented at the end of section 7.1.