

AMSC 2001

Time Table

1. Monday (ponedeljak)

- 8:30–9:15 Marcus Sarkis: New Coarse Spaces and Schwarz Method with Harmonic Overlaps
- 9:15–9:30 Coffee (kava)
- 9:30–9:50 Senka Vuković and Luka Sopta: High-Order ENO and WENO Schemes with Flux Gradient and Source Term Balancing
- 9:50–10:10 Nataša Bilić: Identification of discontinuous parameter in Navier-Stokes equations
- 10:10–10:50 Krešimir Veselić: On the convergence of the Newton iteration
- 10:50–12:15 Marcus Sarkis: Schwarz Methods for Partial Differential Equations
- 17:00–17:45 Jesse Barlow: Least Squares, Total Least Squares and Complete Orthogonal Decompositions
- 17:45–18:00 Coffee (kava)
- 18:00–18:20 Froilán M. Dopico: Computing accurate eigenvalues and eigenvectors of symmetric quasi-Cauchy matrices
- 18:20–18:40 Sanja Singer and Saša Singer: Factorization of Structured Symmetric Matrices
- 18:40–19:00 Ninoslav Truhar and Krešimir Veselić: Optimizing the solution of a Ljapunov equation
- 19:00–19:20 Vjerran Hari: Convergence of Generalized Jacobi-type Methods
- 19:20–19:40 Thomas Benesch: A model for recruitment

2. Tuesday (utorak)

- 8:30–9:15 Rui Ralha: Accurate computation of the SVD with one-sided orthogonal transformations
- 9:15–9:30 Coffee (kava)
- 9:30–9:50 Froilán M. Dopico and Julio Moro: On derivatives of orthonormal bases of invariant subspaces of Hermitian matrices
- 9:50–10:10 Ivan Slapničar and Ninoslav Truhar: Relative Perturbation Theory for Hyperbolic Singular Value Decomposition
- 10:10–10:30 Ivica Nakić and Krešimir Veselić: Wielandt and Ky–Fan theorems for matrix pairs
- 10:45–12:15 Marcus Sarkis: Schwarz Methods for Partial Differential Equations
Lunch (ručak)
- 17:00–17:20 Marko Vrdoljak: On principal eigenvalue of stationary diffusion problem with nonsymmetric coefficients
- 17:20–17:40 Jan Rosenzweig: Mesoscale models of polymeric liquids
- 17:40–18:00 Ivo Veselić: Regularity properties of the integrated density states of random Schrödinger operators
- 18:00–18:15 Coffee
- 18:15–18:35 Andrija Raguž: Relaxation of some energy functionals related to formation of microstructure
- 18:35–19:00 Josip Tašbača: A model of irregular curved rods
- 19:00–19:20 Mladen Jurak: Averaging singularly perturbed convection-diffusion equation with non periodic velocity
- 19:20–19:40 Iryna Bosenko: Lagrangian relaxation of quadratic assignment

3. Wednesday (srijeda)

9:00–11:30 Marcus Sarkis: Schwarz Methods for Partial Differential Equations

14:00–23:59 Excursion and dinner (izlet i večera)

4. Thursday

- 9:10–9:30 Knoll Peter: Application of an $\mu + \lambda$ evolutionary algorithm to reconstruct single photon emission computed tomography (spect)–data.
- 9:30–9:50 Nelida Črnjarić–Žic, Senka Vuković and Luka Sopta: Numerical approximation of the one-dimensional sediment transport equation
- 9:50–10:10 Ivo Beroš and Miljenko Marušić: Order of convergence for collocation with high order tension splines
- 10:10–10:30 Nela Bosner: Numerical stability of Krylov subspace iterative methods for solving linear systems
- 10:30–13:00 Discussion on some ongoing CFD projects in Croatia.
Lunch (ručak).
- 17:00–17:20 Božo Vrdoljak and Ala Oerspahić: Qualitative Analysis of some Solutions of Systems of Quasilinear Differential Equations
- 17:20–17:40 Martin Lazar: Microlocal energy density for hyperbolic systems
- 17:40–18:00 Tomislav Marošević: On using L_p -norms in estimating distances
- 18:00–18:15 Coffee
- 18:15–18:35 Katalin Balla and Viljoš Horvat: Approximate solutions to some second order linear recurrences
- 18:35–18:55 Saša Singer and Božidar Liščić: Calculation of Newton's Coefficient in the Law of Cooling
- 18:55–19:15 Mladen Rogina and Vanja Cević: A Bezier type model of Adriatic coast based on knot insertion and data reduction
- 19:15–19:35 Nenad Kranjčević, Milenko Stegić and Nikola Vranković: Nonlinear Problems in Dynamics by the Finite Element Method in Time Domain

5. Friday (petak)

- Discussions. The meeting adjourns.

Least Squares, Total Least Squares and Complete Orthogonal Decompositions

Jesse L. Barlow

Abstract. The singular value decomposition (SVD) is the key matrix factorization necessary for understanding the solution of least squares problems, total least squares problems, and most recently, latent semantic indexing (LSI). The main drawback of the SVD is that it gives us far more information than any of these problems needs. Usually, we just need the subspaces associated with “large” and “small” singular values and good approximations will do.

Since the late 1960’s, complete orthogonal decomposition (COD) has been used to obtain approximations to subspaces associated with “large” and “small” singular values. If we only need this information for one matrix, CODs offer only modest savings over the SVD. However, recursive least squares (LS) and total least squares (TLS) problems require us to add or delete rows of data at each time step. For the SVD, present stable software requires $O(n^3)$ flops, but CODs can be modified in $O(n^2)$ flops and preserve their approximations of the necessary subspaces.

Algorithmic issues remain in the COD approach to recursive LS and TLS problems remain, including condition and norm estimation, choice of rank—one change algorithms, orthogonality issues, and deflation techniques. This talk will discuss our general approach to these issues.

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A Model for Recruitment

Thomas Benesch

Abstract. The Model contains of 5 steps:

1. Represent: The broad business milieu of the future is being cleared up, in which the top performer will be active.
2. Reclaim: The valuable tasks and the future work services of the top performer are being elaborated.
3. Restrict: The criteria for the qualifications catalogue is being acquired. The detailed description of a different qualification criteria occurs. In addition, 4 to 5 levels of capability are being defined, beginning with the just accepted qualification to the ideal qualification.

4. Reward: The development of a future payment model. An orientation of the model is indispensable both for the employees achievement and qualification for a salary based on performance.
5. Recruit: The top performer is finally being nominated, in other words, recruited.

With the Top-down top performer concept, managers of rank must take on the role as first contact person for the employees regarding staff matters. The chance is growing for staff leaders to broaden their area of responsibility using the concepts from the top performer model and the bound development of achievement that goes along with it.

The next step Regard should follow, after the top-performer is recruited. Regard: Systematically development of the qualification of the top performer.

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Order of Convergence for Collocation with Higher Order Tension Splines

Ivo Beroš and Miljenko Marušić

Abstract. Tension spline of order k is a function that, for given partition $x_0 < x_1 < \dots < x_n$, on each interval $[x_i, x_{i+1}]$ satisfies differential equation $(D^k - \rho_i^2 D^{k-2})u = 0$, where ρ_i 's are prescribed nonnegative real numbers.

Many articles deal with tension splines of order four applied to collocation method for solving singularly perturbed boundary value problem

$$(\mathcal{L}u)(x) = \varepsilon u''(x) + b(x)u'(x) + c(x)u(x) = f(x), \quad (0 \leq x \leq 1)$$

with boundary conditions

$$u(0) = u_0, \quad u(1) = u_1.$$

Accuracy of considered approximations is $\mathcal{O}(h)$ or $\mathcal{O}(h^2)$ for small perturbation parameter ε , depending on the choice of collocation points.

Here we present an algorithm for collocation method with higher order tension splines, prove its convergence and obtain order of accuracy for the solution of singularly perturbed differential equation.

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Identification of discontinuous parameter in Navier-Stokes equations

Nataša Bilić

Abstract. A problem of identifying possibly discontinuous viscosity coefficient in Navier-Stokes equations is considered. General theorem on existence of inverse problem and Galerkin approximations are proved in L^1 -setting. A double discretization method with the variations constraints is used in two and three dimensional inverse problem.

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Numerical stability of Krylov subspace iterative methods for solving linear systems

Nela Bosner

Abstract. Very often used methods for solving linear systems are Krylov subspace iterative methods. The usual stopping criterion for these methods, is the moment when some norm of residual reaches tolerable value. Since all computations are done in finite precision arithmetic, we can check only some approximation of the residual norm. The main goal of our research is, to give estimation for the real residual norm, calculated from this approximation, in order to make stopping criterion more reliable. We have done this for two types of algorithms.

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Numerical Linear Algebra

Bosenko I, Ohaka S.E, Peter Peečan.

Abstract. In recent years, a unified theory to a new approach to the quadratic assignment based on the convex quadratic programming relaxations was performed.

Many engineering problems can be modelled as the minimization (or maximization) of a quadratic function subject to quadratic constraints. These models are non convex in general; so finding a global solution is itself a very hard problem. We look at Lagrangian relaxations for these problems. The Lagrangian relaxations are equivalent to semi-definite programming relaxations, i.e. to the minimization of a linear function of symmetric matrices. The implementation for the first time solved to some extent several large scale problems including nug30 and tho30 problems. The computations associated with these problems are among the largest ever performed in solving engineering problems numerically.

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Numerical Approximations of the One-Dimensional Sediment Transport Equations

Nelida Črnjarić-Žic, Senka Vuković and Luka Sopta

Abstract. The main goal of our work is to investigate and compare different mathematical formulations and numerical approximations for the one-dimensional bed-load sediment transport problem. We follow the work of Hudson and Sweby who described few different formulations describing the physical phenomena of one-dimensional bed-load sediment transport in channels and rivers and used experimentally obtained laws for sediment flux proposed by van Rijn. The resulting mathematical model consists of three equations. Two of them represent conservation laws for one-dimensional shallow water equations, and the third is the conservation law governing bed-load sediment transport.

We concentrate on the homogeneous and non-homogeneous formulation of the conservation laws and on the application of the 1st order Q-scheme (Berubez, Vazquez), 2nd order Hubbard scheme with flux limiters and high-order ENO and WENO with the source term decomposed (developed by the authors of this paper). We discuss and compare numerical results obtained on standard sediment transport test problems.

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Evolutionary Stability Analysis of Tree Based Decision Principles

A. Ekárt and S. Z. Németh

Abstract. In multicriteria decision problems any subjective decisions must be made, such as the importance of the different criteria and the values of the alternatives with respect to subjective criteria. Since subjective decisions are approximate, it is very important to analyze the sensitivity of results when small modifications of the subjective values are made.

When solving a multicriteria decision problem, it is desirable to choose a decision principle that conducts to a solution as stable as possible. We consider a class of decision principles that are constructed from elementary decision principles and we perform the sensitivity analysis of these decision principles with respect to weights and alternative values. In both cases we introduce a global stability index, which measures the stability of the order of alternatives with respect to small perturbations of alternative values and weights, respectively.

We then propose a method based on genetic programming that produces better decision principles than the commonly used ones. New decision principles of the defined class are evolved by genetic programming where the driving force of evolution is stability. The theoretical expectations are validated by two case studies.

References

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Computing accurate eigenvalues and eigenvectors of symmetric quasi-Cauchy matrices.

Froilán M. Dopico

Quasi-Cauchy matrices generalize Cauchy matrices via multiplication by diagonal matrices on both sides. These matrices appear frequently in the literature as test matrices and also in applications like rational interpolation problems. Demmel has shown in [1] that it is possible to compute singular value decompositions of quasi-Cauchy with high relative accuracy. We extend these results and give two algorithms to compute spectral decompositions of symmetric quasi-Cauchy matrices with high relative accuracy. The first algorithm consists just in assigning the correct signs to the singular values computed by Demmel's algorithm. Reference [2] has proved that this can be accurately done. The second algorithm begins by computing an accurate enough $G J G^T$ ($J = I_p \oplus (-I_{n-p})$) factorization of the quasi-Cauchy matrix, adapting an algorithm in [1] to compute accurate componentwise Schur complements of quasi-Cauchy matrices. Then the J-orthogonal Jacobi-like algorithm of Veselić and Slapničar [4], [3] is applied to the matrices G , J . Theory and numerical experiments of these algorithms are presented.

References

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- [3] I. SLAPNIČAR, *Accurate Symmetric Eigenreduction by a Jacobi Method*, PhD Thesis, Fachbereich Mathematik Fernuniversität, Gesamthochschule Hagen, Germany, 1992.
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Stabilization of Process of Random Walks in Graphes

Gerasin S. N., Ivaschenko V.V., Voskoboinic O. N.

Abstract. Modeling a lot of processes in economics, ecology, biology and techniques it is often easier to resort to the models connected with random walks in graphs. Walks in graphs are used in the research of rapidly evolving processes in economics in particular. The walks that become stabilize after a long period are of

peculiar interest. The last means that after a very long period the basic characteristics of walk process may be localized in the vicinity of preset values.

There is considered a walk process in a graph that constitutes a tree in the report. Enumerating the vertexes of a graph walks in graph may be reduced to the research of some Markov's process. This process may be considered as a discrete Markov's chain generated by Markov's process in incessant time. In application the case when process is nonuniform and has variable duration of conversions is of particular interest. An infinitesimal matrix $\Lambda(t) = (\lambda_{ij}(t))$ is used in researching of the process. Let's adduce the conditions that may stabilize the process during a finite period if they are realized.

Suppose the sequence of mutually disjoint periods $\{[s_k, t_k]\}_{k=1}^{\infty}$, $s_k < t_k < s_{k+1}$, $s_k \uparrow t_0 \leq \infty$ and the order of indexes $j_k, k = 1, 2, \dots$ (j_k enumerates the states) for which

$$\sum_{k=1}^{\infty} \inf_i \left| \int_{s_k}^{t_k} \lambda_{ij_k}(s) ds \right| = \infty,$$

moreover in the set $[s_0, t_0) \setminus \cup_{k=1}^{\infty} [s_k, t_k)$ the norm of matrix $\Lambda(s)$ is bounded with the same constant. Suppose $\Lambda(t)$ to be continuous on $[s_k, t_k)$ and exist the limit

$$\lim_{k \rightarrow \infty} \vec{p}(\tau_k) = \vec{p},$$

where $\tau_k \in [s_k, t_k)$, $\vec{p}(\tau_k)$ - null eigenvector of the matrix $\Lambda(\tau_k)$. Then for any j and any initial probability distribution specified in the point $s \in [s_0, t_0]$

$$\lim_{t \uparrow t_0} p_j(s, t) = p_j.$$

Here p_j - j component of the vector \vec{p} . If the series convergence but its sum is quite big then t_0 is its point of stabilization.

Iterative-Collocation Methods with Corrections for Volterra-Fredholm Integral Equations

Lechoslaw Hącia

Abstract. We consider the following integral equation

$$u(x, t) = f(x, t) + \int_0^t \int_M K(x, t, y, s) u(y, s) dy ds,$$

where u is unknown function in D , $D = M \times [0, T]$, M is a closed subset of Euclidean space R^m and K is defined in $\Omega = \{(x, t, y, s) : 0 \leq s \leq t \leq T; x, y \in M\}$.

Volterra-Fredholm integral equations play very important role in the theory of parabolic initial-boundary value problems (heat conduction problems) and various physical, technological and biological problems.

These equations arise in the mathematical modelling of the spatio-temporal development of an epidemic. Spread of the disease in a given population can be described by Volterra-Fredholm equations.

In this paper we propose the iterative-collocation method

$$\begin{aligned} u_k(x, t) &= f(x, t) + \int_0^t \int_M K(x, t, y, s) [u_{k-1}(y, s) + p_k^n(y, s)] dy ds, & k = 1, 2, \dots, \\ u_0(x, t) &= f(x, t) \end{aligned} \quad (1)$$

with corrections p_k^n based on the interpolation polynomial of the Lagrange type with given collocation points.

Presented methods lead to a system of linear algebraic equations or a system of Volterra integral equations.

The convergence is proved and an error estimate is established. The considered theory is illustrated by numerical examples.

References

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Convergence of Generalized Jacobi-type Methods

Vjeran Hari

Abstract. A general form of Jacobi-type processes for solving matrix eigenvalue and singular value problems is considered. This form includes the two extremes: “sequential” and “parallel” methods. Pivot strategies are also generalized to suite the new frame. Sufficient conditions for the convergence to diagonal form of such a general process is proved. These conditions can be viewed as generalized Forsythe-Henrici conditions. They can be applied equally well to simple methods such as standard Hermitian Jacobi methods, Kogbetliantz methods for triangular or butterfly matrices and to more complex methods such as the norm-reducing ones.

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Microlocal energy density for hyperbolic systems

Martin Lazar

Abstract. Starting from the method for computing microlocal energy density, which was developed independently by Gérard, and Francfort and Murat, we want to compute that very density for the hyperbolic system

$$A^0 \partial_0 v + \sum_1^d A^k \partial_k v + Bv = G.$$

The energy connected to the hyperbolic system is given by the relation

$$E := \frac{1}{2} \langle A^0 v, v \rangle.$$

We want to express the energy limit of the sequence of initial problems with the energy of initial conditions. The basic calculus tools are H-measures (also known as microlocal defect measures). We associate an H-measure to the sequence of gradients of solutions to our system and it represents the desired microlocal energy density.

We have determined the equation satisfied by the corresponding H-measure. In the case of the constant coefficients it reduces to a hyperbolic system similar to the initial one.

Rewriting the wave equation as a hyperbolic system, we calculated the associated H-measure for the oscillating sequence of the initial conditions. The result is analogous to the one obtained by the direct calculus of H-measure from the D'Alembert's formula for the solution of the wave equation.

This is a joint work with Nenad Anđić.

On using L_p -norms in estimating distances

Toislav Marošević

Abstract. The problem of determining or estimating the distances among the objects in space appears in many areas of application (e.g. road distances for continuous location models, various distances in Geographical Information Systems, etc., see in: *Facility Location, Survey of Application and Methods*, Z. Drezner (Ed.), (Chapt. 1. *Estimating Distances*), Springer Series in Operations Research, Springer-Verlag New York, 1995).

The so-called distance prediction function may be used to estimate the distance between the points with given coordinates. As a distance predicting function one can use the weighted L_2 -norm and generally the weighted L_p -norm ($p \geq 1$). In such weighted L_p -norm models or τL_p models, it is necessary to estimate parameters

τ and p . In that problem we also look at some special properties of L_p -norms, particularly the so-called directional bias of norms.

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Derivation of a non-linear filtration law by homogenization

Eduard Marušić-Paloka

Abstract. We consider the stationary viscous incompressible flow through a rigid porous medium. For a periodic porous medium with a period ε the homogenization method gives different results depending on the relationship between the Reynolds number, the Freude's number and ε . If both, the Reynolds number and the inverse of the Freude's number are of order ε^{-1} the filtration law is nonlocal and nonlinear. We call that, the critical case. In the subcritical case the fast and slow scales are well separated. We can find the sequence of linear auxiliary problems and with their solutions we can form an asymptotic expansion of the solution. In the critical case, we prove the existence of a unique solution for nonlinear homogenized problem. Moreover, we separate the fast from the slow scale and we find a Taylor's expansion for the filtration velocity as a function of the pressure gradient. Finally, we prove the convergence of the homogenization process as well as the error estimates. (Work in collaboration with A.Mikelić and A.Bourgeat)

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On derivatives of orthonormal bases of invariant subspaces of Hermitian matrices

Froilán M. Dopico and Julio Moro

Abstract. Explicit formulas are given for the derivatives of orthonormal bases of invariant subspaces of linear Hermitian perturbations $A(t) = A + tE$ of a Hermitian matrix A . A characterization is provided for all possible differentiable orthonormal bases via Sylvester equations, together with a procedure allowing to construct differentiable orthonormal bases starting from arbitrary ones.

As a direct consequence of these results, derivatives of bases of left and right singular subspaces are obtained for arbitrary matrices using the Jordan-Wielandt formula.

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Wielandt and Ky–Fan theorem for matrix pairs

Ivica Nakić and Krešimir Veselić

Abstract. The classical Wielandt and Ky–Fan theorems are widely used in matrix theory. Using these results one can obtain various eigenvalue perturbation bounds.

The classical Wielandt theorem states that

$$\sup_{\mathcal{M}_1 \subset \mathcal{M}_k, \dim \mathcal{M}_j = i_j} \inf_{X = [x_1 \dots x_k], x_j \in \mathcal{M}_j, X^* X = I_k} \sum_{j=1}^k X^* A X$$

equals the sum $\lambda_{i_1} + \dots + \lambda_{i_k}$, where $\lambda_1 \geq \dots \geq \lambda_n$ are the eigenvalues of the Hermitian matrix A , and $1 \leq i_1 < \dots < i_k \leq n$, and the classical Ky–Fan theorem states that

$$\inf_{X \in \mathbf{C}^{n \times p}, X^* X = I_p} \text{Tr}(X^* A X)$$

equals the sum of the p smallest eigenvalues of the Hermitian matrix $A \in \mathbf{C}^{n \times n}$.

Some generalizations of these theorems to the class of Hermitian matrix pairs are already known. The authors obtained a generalization to a wider class of matrices. Especially, the definite matrix pairs satisfy our assumptions.

For example, the generalized Ky–Fan theorem states that

$$\inf_{X \in \mathbf{C}^{N \times p}, X^* B X = I_p} \text{Tr}(X^* A X)$$

equals the sum of the first p "good" eigenvalues of the positive type of the matrix pair (A, B) .

Also, some new eigenvalue perturbation estimates will be presented. The application is made on a class of partially overdominated quadratic matrix pencils.

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Approximate solutions to some second order linear recurrences

Katalin Balla and Vilmos Horvat

Abstract. The second order linear recurrence

$$y_{n+1} + b_n y_n + h_n y_{n-1} = e_n \quad (2)$$

with real coefficients was considered in an earlier paper assuming that

$$\lim_{n \rightarrow \infty} b_n = b_\infty, \quad \lim_{n \rightarrow \infty} h_n = h_\infty \quad \text{and} \quad \lim_{n \rightarrow \infty} e_n = e_\infty$$

hold with constants $b_\infty, h_\infty, e_\infty$ such that the quadratic equation $q^2 + b_\infty q + h_\infty = 0$ has a pair of (real) roots q_1, q_2 such that $|q_1| > 1 > |q_2| > 0$. Under these conditions it was stated that a solution of (2) is bounded, i.e.

$$|y_n| = O(1), \quad n \rightarrow \infty \quad (3)$$

iff for any sufficiently large n it satisfies the first order recurrence

$$y_{n+1} = \omega_n y_n + \psi_n$$

where the pair ω and ψ is the solution of the problem

$$\begin{aligned} \omega_{n+1} \omega_n + b_{n+1} \omega_n + h_{n+1} &= 0, & \omega_\infty &:= \lim_{n \rightarrow \infty} \omega_n = q_2 \\ \psi_{n+1} + (\omega_{n+1} + b_{n+1}) \psi_n - e_{n+1} &= 0, & \psi_\infty &:= \lim_{n \rightarrow \infty} \psi_n = -\frac{e_\infty}{q_1 - 1}. \end{aligned}$$

When the coefficients in (2) have asymptotic series representation with respect to nonpositive powers of n , so have the functions ω and ψ .

The aim of this paper is to show that given an error bound ε , one can compute a value $N(S, \varepsilon)$ such that for $n \geq N(S, \varepsilon)$ the approximation errors of partial sums $\tilde{\omega}_n$ and $\tilde{\psi}_n$ for ω_n and ψ_n with S terms do not exceed ε .

Provided the problem (2), (3) is supplied with a left boundary condition

$$y_0 = \bar{y}_0, \quad (4)$$

the approximate boundary value problem consisting of (2), (4) and

$$y_{N+1} = \tilde{\omega}_N y_N + \tilde{\psi}_N$$

given on the finite interval $[0, N + 1]$ furnishes us with an approximate solution of the exact boundary value problem (2), (3), (4) posed on the infinite interval $[0, \infty)$.

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Calculation of non-stationary temperature field in multilayer under boundary conditions of the third sort

H. Hovhanissyan

Abstract. Is considered the unlimited plate with known initial distribution of temperature. On borders between layers the ideal thermal contact is carried out (i.e. the boundary conditions of the 4-th sort are fulfilled). On external boundary surfaces the boundary conditions of the third sort are given.

For the description of a temperature field in multilayer plate the system of the differential equations of heat conductivity, written down in cartesian coordinates is used. This system is joining initial and boundary conditions, which are giving the complete mathematical description of the task.

The system of the equations with initial and boundary conditions is solved certainly by the method of finite differences.

As an example the results of calculation of a non-stationary field in three-layered preparation of footwear consisting from calf-leather of chrome tanning, polyurethane glue-solution and leather lining. In the initial moment of time all points of preparation of top of footwear have constant temperature. The leather lining enters the subsequent moment of time in thermal contact with footstep of the man, and the calf-leather of chrome tanning with an environment (boundary conditions of the first sort).

Curves of distributions of temperatures on thickness of preparation of top of footwear for the various moments of time are obtained. The obtained results will be well coordinated with available experimental and theoretical data.

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Qualitative Analysis of some Solutions of System of Quasilinear Differential Equations

Božo Vrdoljak and Alina Oštrpašić

Abstract. In this paper we study the existence and behaviour of some solutions of the system of quasilinear differential equations. The behaviour of solutions in

the neighborhood of an arbitrary curve is considered, with extraordinary attention on some special cases. The obtained results contain an answer to the question on approximation as well as stability of solutions whose existence is established. The errors of the approximation are defined by the functions that can be sufficiently small. The qualitative analysis theory of differential equations and the topological retraction method are used.

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Relaxation of some energy functionals related to formation of microstructure

Andrija Raguz

Abstract. In this paper we investigate asymptotic behaviour of certain energy functionals which appear in modelling of anisotropic materials having ability to perform microstructures. In particular, following [1], we study the form and the properties of the limiting relaxed energy associated with such a family of functionals, including a high-oscillatory lower order terms.

We will explain the physical background and the reasons why it is difficult to obtain a natural form of a relaxed functional at the limit.

Furthermore, we prove some preliminary results which assure that a limiting behaviour can essentially differ from expected one, and we will propose some new relaxations which are believed to be consistent with actual physical observations.

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Accurate computation of SVD with one-sided orthogonal transformations

Rui Ralha

Abstract. It has been known for quite some years that one-sided Jacobi's method can be significantly more accurate than QR for the computation of singular values

of certain matrices. The price to pay for using Jacobi's method is, of course, the increase in the computational complexity.

In this lecture we will discuss a method which is based on the transformation of a matrix A into A_{n-2} which is such that $A_{n-2}^T A_{n-2}$ is tridiagonal. This is one-sided transformation since it involves only post-multiplication of A with Householder matrices, thus preserving high accuracy of the singular values when $A = DX$ where D is diagonal and X is much better conditioned than A .

The way to proceed depends on how much one needs to know about the SVD of the given A . If only singular values are sought then a simple orthogonalization of adjacent columns of A_{n-2} delivers bidiagonal form to which the dqds method, or any alternative methods, can be applied. The proposed transformation is quite straightforward but there are, however, some less obvious aspects that need to be taken into account to guarantee the accuracy of the computed bidiagonal form and these will be addressed in the talk. Numerical examples carried out with Matlab's implementation of our algorithm will be presented to illustrate the main points of the error analysis.

We will also outline the main ideas behind another method which, like one-sided Jacobi's method, produces a matrix whose columns are sufficiently orthogonal to give not only accurate singular values but also accurate singular vectors.

A Bezier type model of Adriatic coast based on knot insertion and data reduction

Mladen Rogina and Vanja Cević

Abstract. We describe mathematical techniques needed to develop a vector model of a complicated geographical map, and apply those techniques to the modeling of the Adriatic coast. Cubic Bezier curves are used in the model, and Oslo-type algorithms are used in the control polygon description. Optimization questions based on data reduction software developed by Lyche *et al.* are considered. We discuss some real-world problems, like WGS-84 cartographic projections, GPS support and navigational problems in the above setting. A feedback of this practical problems to tangent-discontinuous parametric spline functions may be of interest.

We present an elaborate software NAV2001, which can accomplish untrivial navigational tasks, such as determination of routes and positions, as well as adding symbols and information to the map. Database software is employed in this context, containing detailed plans, historical and navigational information of various type, not to mention GSM, meteorological and Internet support as a *sine qua non* of the modern civilization.

The software itself has been extensively tested in the real world situations, and we give a brief description of auto-pilot, echo-sounder and similar NMEA-standard devices used in navigation, and touch some open problems including radar equipment and associated mathematical problems. Further information and so on freely

downloadable software is available on a web site <http://www.cvs.hr/Nav2001hr.html>.

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Mesoscale models of polymeric liquids

Jan Rosenzweig

Abstract. We shall outline two similar, although conceptually different computational and theoretical mesoscopic models of polymeric liquids, that are capable of describing the microstructures in most flow conditions. In both our models the polymers are treated as thin, semi-flexible Hookean cables immersed in a viscous liquid, and the models differ in the description of hydrodynamic coupling. In the first model the coupling is based on a multiscale dissipative particle dynamics scheme on a Lagrangian or a hybrid Eulerian-Lagrangian Voronoi lattice, and the second model employs boundary integral methods to reduce the hydrodynamic coupling to a potential-like contribution to the molecular forces between polymer chains. The complexity of the former model is at least $\mathcal{O}(n^3)$, where n is the size of discretisation of polymer chains, whereas the latter can be reduced to $\mathcal{O}(n \log n)$ using a treecode scheme and is thus better suited to large scale simulations. Some rheological equations of state corresponding to certain asymptotic limits of the latter model will also be derived.

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New Coarse Spaces and Schwarz Method with Harmonic Overlaps

Marcus Sarkis

Abstract. We introduce and analyze new Schwarz preconditioners for solving elliptic problems on unstructured grids. We construct the preconditioner by combining in one preconditioner the additive overlapping Schwarz method and the iterative

substructuring method. We use the iterative substructuring techniques to make functions harmonic in the overlapping regions. As a result, communication cost is smaller, PCG takes less number of iterations than the corresponding Schwarz preconditioners, and allows the use of inexact solvers in the nonoverlapping regions. We also enhance the preconditioner with a new, effective, easy-to-construct coarse space. Part of this research is in joint work with Xiao-Chuan Cai and Maksymilian Dryja. Numerical results and theory will be presented.

Factorization of Structured Symmetric Matrices

Sanja Singer and Saša Singer

Abstract. One-sided Jacobi-like algorithm can be the method of choice for accurate computation of eigenvalues of an indefinite symmetric matrix A . In some applications A is given implicitly by its factors. These factors may be unsuitable for the Jacobi process.

For example, $A = B^T C + C^T B$ is a such matrix with implicitly given factors, i.e.

$$A = \begin{bmatrix} B^T & C^T \end{bmatrix} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} := G^T \hat{J} G.$$

It is obvious that G^T is not a full column-rank matrix and, therefore, unsuitable for the one-sided Jacobi algorithm.

To avoid explicit computation of A , we can re-factorize A by using the indefinite QR factorization. Since the indefinite QR requires $J = \text{diag}(j_{11}, j_{22}, \dots, j_{mm})$, $j_{kk} \in \{-1, 1\}$, instead of \hat{J} , we first have to construct suitable unitary matrices U for the block-diagonalization of

\hat{J} . The initial problem is then transformed into

$$G^T \hat{J} G = (G^T U^T) J (UG)$$

and the indefinite QR factorization of UG gives the full column rank factor of A .

We describe the matrices U that can be used to block-diagonalize \hat{J} , and then analyze the perturbation and error bounds for the computed UG .

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Relative Perturbation Theory for Hyperbolic Singular Value Decomposition

Ivan Slapničar and Ninoslav Truhar

Abstract. We give relative perturbation bounds for singular values and perturbation bounds for singular subspaces of a hyperbolic singular value decomposition for the pair (G, J) , where G is a $m \times n$ full rank matrix and J is a $n \times n$ diagonal matrix of signs. The HSVD is defined as

$$G = U\Sigma V^{-1},$$

where U is unitary, Σ is diagonal with positive diagonal elements, and $V^*JV = P^TJP$ for some permutation matrix P . Diagonal entries of Σ are the hyperbolic singular values, and the columns of U and V are the corresponding singular vectors.

The HSVD is a sound numerical way to solve the downdating problem: the Hermitian eigenvalue problem for the matrix $AA^* - BB^*$ can be solved as the HSVD for the pair (G, J) , where

$$G = \begin{bmatrix} A & B \end{bmatrix}, \quad J = \begin{bmatrix} I & \\ & -I \end{bmatrix}.$$

The HSVD is also part of a highly accurate algorithm for the symmetric eigenvalue problem.

We consider two types of relative perturbations: $G + \delta G = (B + \delta B)D$ and $G + \delta G = \bar{D}(\bar{B} + \delta \bar{B})$, depending whether G has full column or full row rank, respectively. In both cases we also consider relative element-wise perturbations of G which typically occur in numerical computations.

To get the flavor of our results, consider G with full column rank. Define

$$\beta = \|\delta BB^\dagger\|, \quad \beta_F = \|\delta BB^\dagger\|_F, \quad \psi = \frac{2\beta_F + \beta_F^2}{\sqrt{1 - 2\beta - \beta^2}},$$

where B^\dagger denotes the pseudo-inverse. In particular, for the element-wise perturbation of the form $|\delta G| \leq \epsilon|G|$ with diagonal D such that B has unit columns, we have $\beta \leq \epsilon\sqrt{n}/\sigma_{\min}(B)$. Assume the unperturbed and the perturbed hyperbolic singular values, σ_i and $\tilde{\sigma}_i$, respectively, are in the same order. A relative Wielandt–Hoffman-type perturbation bound is

$$\sqrt{\sum_{i=1}^n \left(\frac{\sigma_i^2 - \tilde{\sigma}_i^2}{\sigma_i \tilde{\sigma}_i} \right)^2} \leq \psi \left(\frac{1}{2}\psi + \sqrt{1 + \frac{1}{4}\psi^2} \right).$$

Thus, similarly to the classical SVD, the perturbation will be small if B is well-conditioned.

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Nonlinear Problems in Dynamics by the Finite Element Method in Time Domain

Nenad Kranjčević, Milenko Stegić and Nikola Vranković

Abstract. Exact solutions of nonlinear dynamic equations are very rare and almost all of the methods for solving nonlinear differential equations are only approximate. A general solution method of systems with complex and strong nonlinearity is the numerical time integration. Numerical integrations are usually very time-consuming and may encounter numerical difficulties when the nonlinearity becomes very strong. A very efficient method for solving nonlinear differential equations in the frequency domain is the harmonic balance method. When the assumption of dominance of primary resonance in the response is satisfied, the harmonic balance method is very accurate and numerically reliable method. If the influence of higher harmonics in the response is significant, this method becomes very unreliable. The incremental harmonic balance method is more convenient for determining frequency response characteristics, because a new solution can be sought, with the previous solution used as a very good approximation. Another approach for solving nonlinear problems in dynamics is the finite element in time method, which is based on a weak form of Hamilton principle of varying action [1]. Similar to the standard finite element technique, the time interval is divided into a finite number of time elements. The solution for all the spatial degrees of freedom at all time steps within a given time interval is sought through a set of algebraic equations. Furthermore, the straightforward determination of the stability of solution is the second advantageous feature of this method. The stability of periodic solution is investigated considering small perturbations of the solution. According to Floquet-Liapunov theorem the stability can be determined by the eigenvalues of the transition matrix that relates the initial and final perturbations. The model of a three-degree-of-freedom semi-definite system with clearances under harmonic excitations is used to study the feasibility of the finite element in time method. The stability of solutions is analyzed using Floquet-Liapunov theorem. Close agreement is found between obtained results and the published findings of a harmonic balance method [2].

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A model of irregular curved rods

Josip Tambača

Abstract. In this work we discuss the problem of the equilibrium of elastic curved rods for rods with the piecewise smooth middle curve.

The middle curve of the irregular rod can be approximated by the family of smooth curves. We consider the asymptotics of the solution of the one-dimensional model for rods with the smooth middle curve when the middle curve tends to the irregular one. The limit function is identified as the unique solution of a certain one-dimensional boundary value problem called a model of irregular curved rods.

The obtained model is compared to the model of the junction between two straight rods, meeting at the right angle, derived directly from the equations of the three-dimensional linearized elasticity as the thickness tends to zero.

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Optimising the solution of a Ljapunov equation

Ninoslav Truhar and Krešimir Veselić

Abstract. We consider a second order damped-vibration equation $M\ddot{x} + C\dot{x} + Kx = 0$ where M, C, K (called mass, damping, stiffness matrix, respectively) are real, symmetric matrices of order n with M, K positive definite and C positive semidefinite, or equivalently $\dot{y} = \mathbf{A}y$, where \mathbf{A} is obtained from M, C, K .

In this case the Ljapunov equation

$$\mathbf{A}^T X + X \mathbf{A} = -B,$$

is uniquely solvable for any B . We optimize the solution X in the following sense. Let \mathbf{A} be written as $\mathbf{A} = \mathbf{A}_0 + \epsilon \mathbf{D}$, where \mathbf{A}_0 skew-symmetric, \mathbf{D} is positive semidefinite, $\epsilon > 0$. We call the parameter ϵ optimal if

$$\text{Tr}(QX) = \min,$$

where Q is a suitable chosen positive semidefinite matrix.

We obtain the formula

$$\text{Tr}(QX) = \frac{X_1}{\epsilon} - X_0 - \epsilon \hat{X}_1 + \epsilon \sum_{i=1}^s \frac{\lambda_i (\lambda_i X_i - \epsilon Y_i)}{\lambda_i^2 + \epsilon^2},$$

which is easy to minimize. The constants in this formula are obtained from solving two auxiliary symmetric eigenvalue problems. In particular, neither the eigenvalues nor the eigenvectors of \mathbf{A} are used in this approach. This optimization is sensibly more efficient from the standard methods (Bartels–Stewart), if r is small.

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On the convergence of the Newton iteration

K. Veselić

Abstract. We give a new convergence criterion for the classical Newton method for finding the zero of a vector field which is the gradient of a convex function. Our condition does not need the usual continuity of the derivative and it consists of only one bound with a constant, which is shown to be sharp. This enables us to extend the convergence result to the cases where the derivative is defined merely as a distribution.

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On principal eigenvalue of stationary diffusion problem with nonsymmetric coefficients

Marko Vrdoljak

Abstract. We consider the eigenvalue problem

$$\begin{cases} -\text{div}(\mathbf{A}\nabla u) = \lambda \rho u \\ u \in H_0^1(\Omega) \end{cases}$$

where $\Omega \in \mathbf{R}^d$ is open and bounded, $\rho \in L^\infty(\Omega)$ and $\mathbf{A} \in L^\infty(\Omega; M_{d \times d})$ satisfying

$$\mathbf{A}(x)\xi \cdot \xi \geq \alpha \xi \cdot \xi, \quad \rho(x) \geq c, \quad \xi \in \mathbf{R}^d, \text{ a.e. } x \in \Omega$$

for some $\alpha, c > 0$.

Using the strong maximum principle, obtained by Harnack's inequality, and Krein-Rutman's theorem, the existence of principle eigenvalue is proved. Moreover, under appropriate conditions, the principal eigenvalue depends continuously on coefficients with respect to H-topology for \mathbf{A} and L^∞ weak * topology for ρ .

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High-Order ENO and WENO Schemes with Flux Gradient and Source Term Balancing

Senka Vuković and Luka Sopta

Abstract. We developed a new set of numerical schemes particularly designed for hyperbolic conservation laws with significant source term. These schemes are based on one hand on the essentially non-oscillatory (ENO) schemes and weighted essentially non-oscillatory (WENO) schemes (authors Harten, Osher, Engquist, Chakravarthy, Shu, Balsara) and on the other hand on the concept of the flux gradient and source term balancing (Berdeuz, Vazquez, Hubbard, LeVeque). In this paper we present results of extended numerical testing of the original ENO and WENO schemes and the new schemes on one-dimensional shallow water equations. We perform computations using 2-step and 3-step Runge-Kutta time operator approximation and 3rd to 6th formal order reconstruction via primitive function for the space operator. On the basis of the obtained numerical results we examine effects of the scheme order increasing in interaction with the introduced improvement by the source term decomposition. We also discuss the time evolution of the numerical error due to variable bed depth in quiescent flow, steady state flow and unsteady flow for the original and for the newly developed schemes.

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