Nizovi realnih brojeva

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Nizovi realnih brojeva

$:=$ funkcije $a : \mathbb{N} \to \mathbb{R}$.

Uobičajeno je pisati

$$a_n := a(n), \qquad n \in \mathbb{N},$$

a_n zvati n-**tim članom niza** i niz a označavati sa (a_n).

Primjer:

(a) $a_n := 2$

$$\begin{array}{cccccc} 2, & 2, & 2, & 2, & \dots \\ \| & \| & \| & \| & \\ a_1 & a_2 & a_3 & a_4 & \dots \end{array}$$

$\rightsquigarrow$ **konstantan niz**.

(b) $a_n := (-1)^n$

$$\begin{array}{cccccc} -1, & 1, & -1, & 1, & \dots \\ \| & \| & \| & \| & \\ a_1 & a_2 & a_3 & a_4 & \dots \end{array}$$

(c) $\boxed{a_n := s + (n-1)d}$ $\quad (s, d \in \mathbb{R}$ zadani)

$\rightsquigarrow$ **aritmetički niz**. Vrijedi:

$$\boxed{a_{n+1} - a_n = d} \quad \text{za sve } n \in \mathbb{N}.$$

Npr. $s = -1$, $d = 4$:

$$\begin{array}{cccccc} -1, & 3, & 7, & 11, & 15, & \dots \\ \| & \| & \| & \| & \| & \\ a_1 & a_2 & a_3 & a_4 & a_5 & \dots \end{array}$$

(d) $\boxed{a_n := s\, q^{n-1}}$ $\quad (s, q \in \mathbb{R} \setminus \{0\}$ zadani)

$\rightsquigarrow$ **geometrijski niz**. Vrijedi:

$$\boxed{\frac{a_{n+1}}{a_n} = q} \quad \text{za sve } n \in \mathbb{N}.$$

Npr. $s = 1$, $q = \frac{1}{2}$:

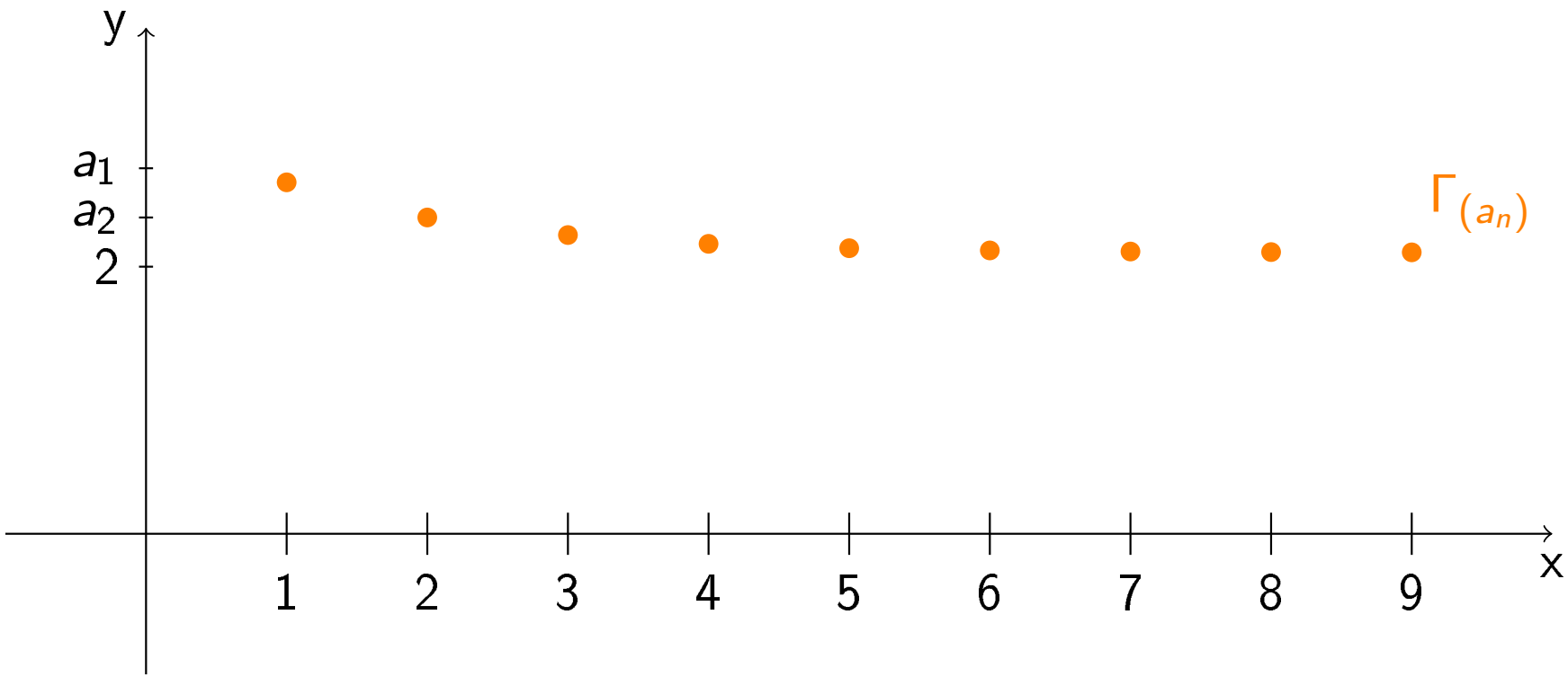
$$\begin{array}{ccccccc} 1, & \frac{1}{2}, & \frac{1}{4}, & \frac{1}{8}, & \frac{1}{16}, & \dots \\ \| & \| & \| & \| & \| & \\ a_1 & a_2 & a_3 & a_4 & a_5 & \dots \end{array}$$

Suma prvih n članova geometrijskog niza, $S_n := a_1 + a_2 + \dots a_n$, jednaka je

$$\boxed{S_n = s\, \frac{1 - q^n}{1 - q}} \qquad \text{ako } q \neq 1.$$

Limes niza

Graf niza: npr. $a_n := 2 + \left(\frac{1}{2}\right)^n$



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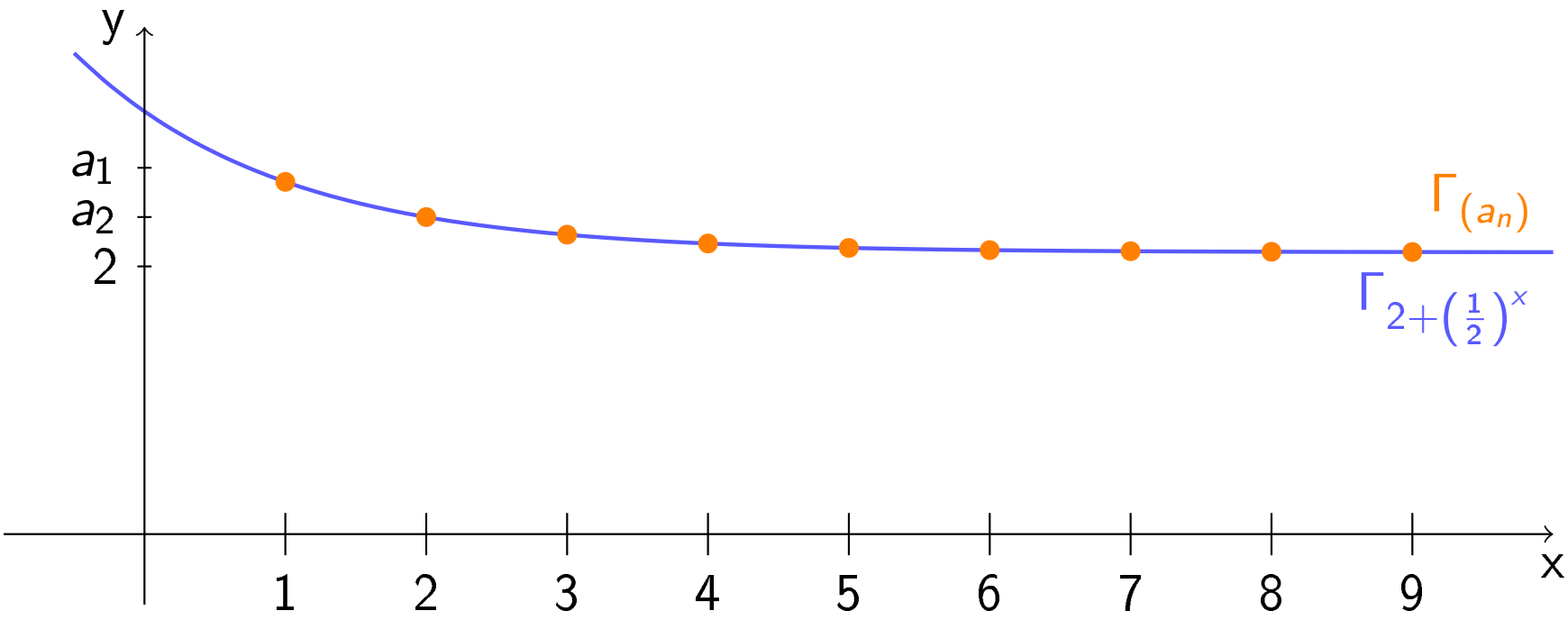

$$\lim_{x\to+\infty}\left(2+\left(\frac{1}{2}\right)^x\right)=2 \quad \rightsquigarrow \quad \lim_{n\to\infty}\left(2+\left(\frac{1}{2}\right)^n\right)=2.$$

Limese nizova računamo istim tehnikama kao limese funkcija

$$\lim_{x\to+\infty} f(x).$$

Nekoliko korisnih formula za računanje limesa nizova

Podsjetnik: $n! := 1 \cdot 2 \cdot 3 \cdots n$ za sve $n \in \mathbb{N}$.

$$\boxed{\lim_{n\to\infty} \frac{a^n}{n!} = 0} \quad \text{za sve } a \in \mathbb{R}.$$

$$\boxed{\lim_{n\to\infty} \sqrt[n]{n} = 1}$$

$$\boxed{\lim_{n\to\infty} \sqrt[n]{a} = 1} \quad \text{za sve } a \in \langle 0, \infty \rangle .$$

$$\boxed{\lim_{n\to\infty} \left(1 + \frac{a}{n}\right)^n = e^a} \quad \text{za sve } a \in \mathbb{R}.$$