Rastav na parcijalne razlomke

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1. korak

(Npr. dijeljenjem polinoma s ostatkom) svaka racionalna funkcija r može se zapisati u obliku

$$r(x) = \text{polinom} + \underbrace{\frac{p(x)}{q(x)}}_{\substack{\textbf{prava}\text{ racionalna funkcija,}\\ \text{tj. } \deg p < \deg q.}}$$

Primjer

$$\frac{x^5 + x^3 - x^2 + 1}{x^3 - 1} = x^2 + 1 + \frac{2}{x^3 - 1}.$$

2. korak

Faktoriziramo nazivnik funkcije $\frac{p(x)}{q(x)}$, tj. zapišemo

$$\frac{p(x)}{q(x)} = \frac{p(x)}{A(x+B_1)^{k_1}\cdots(x+B_n)^{k_n}(x^2+C_1x+D_1)^{l_1}\cdots(x^2+C_mx+D_m)^{l_m}}$$

tako da vrijedi:

- $A \in \mathbb{R} \setminus \{0\}$.
- $x+B_1, \ \dots, \ x+B_n$ su međusobno različiti linearni polinomi.
- $x^2+C_1x+D_1, \ \dots, \ x^2+C_mx+D_m$ su međusobno različiti kvadratni polinomi bez realnih nultočaka, tj. diskriminante < 0.
- $k_1, \dots, k_n, l_1, \dots, l_m \in \mathbb{N}$.

Primjer

$$\frac{2}{x^3-1} = \frac{2}{(x-1)(x^2+x+1)}.$$

3. korak: Teorem

$\dfrac{p(x)}{q(x)}$ se rastavlja na **parcijalne razlomke**:

$$
\begin{aligned}
\frac{p(x)}{q(x)} =& \frac{*}{x+B_1} + \frac{*}{(x+B_1)^2} + \ldots + \frac{*}{(x+B_1)^{k_1}} \\
&+ \frac{*}{x+B_2} + \frac{*}{(x+B_2)^2} + \ldots + \frac{*}{(x+B_2)^{k_2}} \\
&+ \ldots \\
&+ \frac{*}{x+B_n} + \frac{*}{(x+B_n)^2} + \ldots + \frac{*}{(x+B_n)^{k_n}} \\
&+ \frac{*x+*}{x^2+C_1x+D_1} + \frac{*x+*}{(x^2+C_1x+D_1)^2} + \ldots + \frac{*x+*}{(x^2+C_1x+D_1)^{l_1}} \\
&+ \ldots \\
&+ \frac{*x+*}{x^2+C_mx+D_m} + \frac{*x+*}{(x^2+C_mx+D_m)^2} + \ldots + \frac{*x+*}{(x^2+C_mx+D_m)^{l_m}}
\end{aligned}
$$

za jedinstven izbor realnih koeficijenata $*$.

Primjer

Po Teoremu je

$$\frac{2}{x^3-1} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

za neke $A, B, C \in \mathbb{R}$. Odavde se lako izračuna da je

$$A = \frac{2}{3}, \ B = -\frac{2}{3}, \ C = -\frac{4}{3}.$$

Primjer

Sve zajedno:

$$\begin{aligned}\frac{x^5+x^3-x^2+1}{x^3-1} &= x^2+1+\frac{2}{x^3-1}\\ &= x^2+1+\frac{2}{(x-1)\left(x^2+x+1\right)}\\ &= x^2+1+\frac{2}{3}\cdot\frac{1}{x-1}-\frac{2}{3}\cdot\frac{x+2}{x^2+x+1}.\end{aligned}$$

Malo kompliciraniji primjer

$$\frac{x}{2x^2(x-1)(x+2)^3\left(x^2-x+1\right)^4\left(x^2+2x+7\right)(x^2+1)^2} =$$

$$\begin{aligned}
&= \frac{A}{x} + \frac{B}{x^2} \\
&\quad + \frac{C}{x-1} \\
&\quad + \frac{D}{x+2} + \frac{E}{(x+2)^2} + \frac{F}{(x+2)^3} \\
&\quad + \frac{Gx+H}{x^2-x+1} + \frac{Ix+J}{\left(x^2-x+1\right)^2} + \frac{Kx+L}{\left(x^2-x+1\right)^3} + \frac{Mx+N}{\left(x^2-x+1\right)^4} \\
&\quad + \frac{Ox+P}{x^2+2x+7} \\
&\quad + \frac{Rx+S}{x^2+1} + \frac{Tx+U}{\left(x^2+1\right)^2}
\end{aligned}$$

za jedinstven izbor koeficijenata $A, B, C, \ldots, U \in \mathbb{R}$.