



University of Zagreb
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS

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Completed Hopf algebroids

DOCTORAL THESIS

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Summary

Keywords: Hopf algebroid, formal completion, filtered vector space, cofiltered vector space, filtered-cofiltered vector space, strict ind-object, strict pro-object, strict ind-pro-object, formal sum, formal basis, universal enveloping algebra, completed tensor product, internal bialgebroid, dual Hopf algebra, Yetter-Drinfeld module, scalar extension, Heisenberg double

In this thesis a natural generalization of the definition of a Hopf algebroid is introduced, internal to any symmetric monoidal category with coequalizers that commute with the monoidal product. In this thesis we also construct a symmetric monoidal category $(\text{indproVect}, \tilde{\otimes}, k)$ of filtered-cofiltered vector spaces, whose morphisms are linear maps which in a weak sense respect the filtrations and cofiltrations, and whose monoidal product is the usual tensor product of vector spaces formally completed and with a corresponding filtration of cofiltrations. We prove that this category satisfies the above conditions for the existence of internal Hopf algebroids. It contains two dual subcategories, the category $(\text{indVect}, \otimes, k)$ of filtered vector spaces and the category $(\text{proVect}, \hat{\otimes}, k)$ of cofiltered vector spaces. The monoidal product in it combines the ordinary tensor product and a completed tensor product.

An important class of Hopf algebroids over a noncommutative base is comprised of smash products of a Hopf algebra H and a braided-commutative algebra A in the category of Yetter-Drinfeld modules over H . Such Hopf algebroids are called scalar extensions. In this thesis we prove that the smash products in which H and A are replaced by their analogues in the monoidal category of filtered-cofiltered vector spaces have the structure of Hopf algebroids in that monoidal category. By doing that, we set the basis for studying the Heisenberg doubles $A^*\sharp A$ in which A is an infinite-dimensional Hopf algebra instead of a finite-dimensional one, among other examples, and the existence of the Hopf algebroid structure on them internal to the category indproVect .

We then study Hopf pairings of a filtered Hopf algebra A and a cofiltered Hopf algebra H which are non-degenerate in the variable in H , and find sufficient conditions for A to be a braided-commutative Yetter-Drinfeld module algebra over H in the indproVect category. A smaller class of examples is also studied, for which A is a Hopf algebra countably filtered by

finite-dimensional vector spaces. Here we find necessary and sufficient conditions on Hopf algebras A and A^* , or A and H , in the form of finite dimensionality of the adjoint orbits of A and the existence of certain canonical elements in $H \sharp A$. Thus a construction of some filtered-cofiltered Hopf algebroids of scalar extension type is obtained. Important examples of such scalar extensions are the ones with A the universal enveloping algebra $U(\mathfrak{g})$ of a finite-dimensional Lie algebra \mathfrak{g} . When H is equal to its algebraic dual $U(\mathfrak{g})^*$ with induced cofiltration, the corresponding scalar extension, that is the Heisenberg double of $U(\mathfrak{g})$, can be identified as an algebra with the algebra of differential operators on the formal neighborhood of the unit of a Lie group integrating \mathfrak{g} , suggesting applications in geometry and mathematical physics.

I am currently translating this disertation to English language.

For original disertation in Croatian language, see

<http://web.math.pmf.unizg.hr/~stojic/Stojic-disertacija.pdf>

For a list of articles in English language written out of its content, see

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