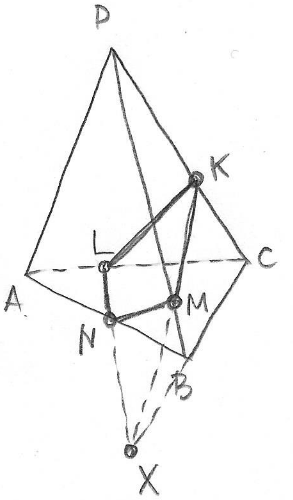
4.



Tetraedar = trostrana piramida
(ne nužno pravilna!)

Om.: ravnina π: KLM

K i L u bočnoj ravnini ACD:

$\overline{KL}$

K i M u bočnoj BCD:

$\overline{KM}$

(Tražimo još jednu točku ravnine π na osnovi ABC i bočnoj ABD.)

(KM je pravac u π, BC pravac u ravnini ABC:
$KM \cap BC$ je u $\pi \cap ABC$. Tu sijeku se)

$KM \cap BC =: X$

X i L u baznoj ravnini ABC:

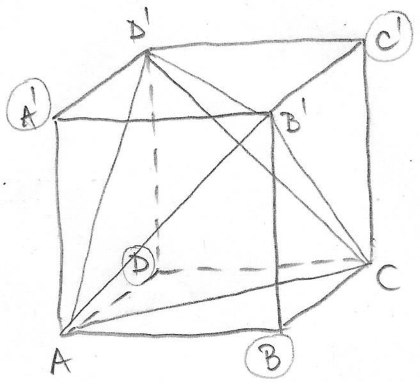
$XL \cap \overline{AB} =: N$

$\overline{LN}$

$\overline{NM}$

Presjek je četverokut KLNM.

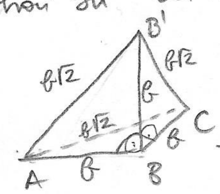
8.



stranica kocke je duljine b

PRAVILNI
Tetraedar AB'CD'.

Preostala četiri vrha kocke vrhovi su četiri tetraedra.



Kutovi pokraj u vrhu B su pravi.

Sva četiri tetraedra su sukladna.

Volumen tetraedra AB'CD' = volumen kocke $-$ 4 $\cdot$ Volumen tetraedra ABCB'

Volumen tetraedra ABCB' $= \frac{1}{3}$ površina baze ABC $\cdot$ visina $|BB'|$ $= \frac{1}{3} \cdot P(\triangle) \cdot b = \frac{1}{3} \cdot \frac{1}{2} b^2 \cdot b = \frac{1}{6} b^3$

(Odabrali smo ABC za bazu jer je tako lakše odrediti visinu na bazu.)

Dakle, Volumen tetraedra AB'CD' $= b^3 - 4 \cdot \frac{1}{6} b^3 = \frac{1}{3} b^3$ ← Trećina volumena kocke.

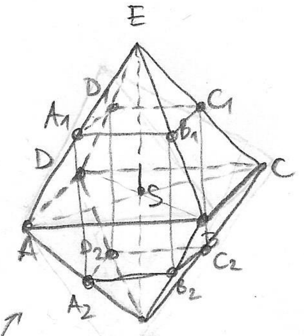
9.

Pravilni oktaedar

↓

dvije četverostrane piramide spojenih osnovki

jednaki svi bridovi



brid duljine a

Vrhovi A,B,C,D,E,F kao na slici. ABCD je kvadrat.
Označimo polovišta A_1,B_1,C_1,D_1, A_2,B_2,C_2,D_2 kao na slici.

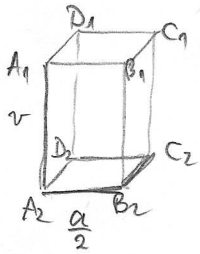
- U $\triangle ABE$ $\overline{A_1B_1}$ je srednjica $\Longrightarrow |A_1B_1| = \frac{1}{2}|AB|$ & $A_1B_1 \parallel AB$.

Analogno $\overline{B_1C_1}, \overline{C_1D_1}, \overline{D_1A_1}$,
$\overline{A_2B_2}, \overline{B_2C_2}, \overline{C_2D_2}, \overline{D_2A_2}$

- Slijedi: $A_1B_1C_1D_1$ je kvadrat duljine stranice $\frac{a}{2}$ (jer je ABCD kvadrat, a $A_1B_1C_1D_1$ ima paralelne stranice stranicama od ABCD)
i paralelan je ravnini ABCD.
Analogno i $A_2B_2C_2D_2$.

- $\overline{A_1A_2}$ je srednjica u trokutu AEF pa je
$|A_1A_2| = \frac{1}{2}|EF|$ i $A_1A_2 \parallel EF$.
Analogno zaključujemo za $\overline{B_1B_2}, \overline{C_1C_2}, \overline{D_1D_2} \parallel EF$.
Budući da je $EF \perp$ ravnini ABCD, zaključujemo:

$A_1B_1C_1D_1A_2B_2C_2D_2$ je kvadar.

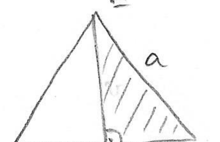


- Volumen kvadra:

$V = B \cdot v$

$B = P\left(\square \tfrac{a}{2}\right) = \left(\frac{a}{2}\right)^2$

$v = \frac{1}{2}|EF| = |ES|$ gdje je S središte oktaedra



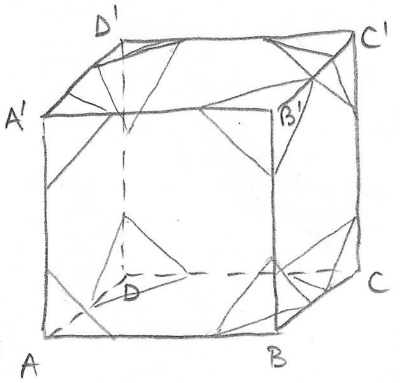
$|AC| = a\sqrt{2}$

$|ES| = \sqrt{a^2 - \left(\frac{a\sqrt{2}}{2}\right)^2} = \frac{a\sqrt{2}}{2}$

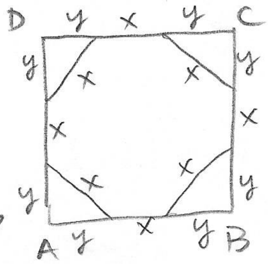
(Usput, iako se ne traži:
Volumen oktaedra je
$2 \cdot$ volumen piramide ABCDE $= 2 \cdot \frac{1}{3} a^2 \cdot v =$
$= \frac{2}{3} a^2 \cdot \frac{a\sqrt{2}}{2} = \frac{a^3\sqrt{2}}{3}$)

Dakle, $V = \frac{a^2}{4} \cdot \frac{a\sqrt{2}}{2} = \frac{a^3\sqrt{2}}{8}$.

(10.)



duljina stranice kocke je a



Eulerova formula:

$v - b + s = 24 - 36 + 14 = 2$ ✓

vrhova je: $v = 8 \cdot 3 = 24$

bridova je: $b = 12$ (od kocke) $+ 8 \cdot 3 = 36$

strana je: $s = 6$ (od kocke) $+ 8 \cdot 1 = 14$

Možemo odrediti duljinu y.

$y^2 + y^2 = x^2$, tj. $y\sqrt{2} = x$

$2y + x = a$

Sustav: $\left.\begin{matrix} y\sqrt{2} = x \\ 2y + x = a \end{matrix}\right\}$

$2 \cdot y + y\sqrt{2} = a$

$y = \frac{a}{2+\sqrt{2}}$

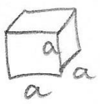
$y = \frac{a}{2} \cdot (2 - \sqrt{2})$

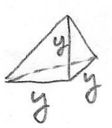
$x = \frac{y}{\sqrt{2}} = \frac{a}{2}(\sqrt{2} - 1)$

$x = \frac{a}{2}(\sqrt{2} - 1)$

- volumen dobivenog tijela:

V = Volumen kocke − 8 · Volumen tetraedra =





$= a^3 - 8 \cdot \frac{1}{3} \frac{y \cdot y}{2} \cdot y =$

$= a^3 - 8 \cdot \frac{1}{6} y^3 = a^3 - \frac{4}{3} \cdot y^3 =$

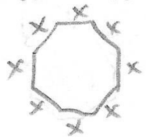
$= a^3 - \frac{4}{3} \cdot \frac{a^3}{8} \cdot (2 - \sqrt{2})^3 = a^3 \left(1 - \frac{1}{6} \cdot (8 - 12\sqrt{2} + 12 - 2\sqrt{2})\right) =$

$= a^3 \left(1 - \frac{10 - 7\sqrt{2}}{3}\right) = a^3 \cdot \frac{-7 + 7\sqrt{2}}{3} = \frac{7}{3} a^3 (\sqrt{2} - 1)$

Dakle, $V = \frac{7}{3}(\sqrt{2} - 1) a^3$.

- Oplošje:

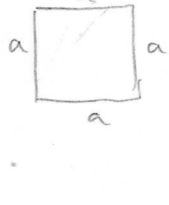
O = 6 · Površina osmerokuta + 8 · Površina trokuta



Površina trokuta $= \frac{x^2\sqrt{3}}{4} = \frac{a^2}{4}(\sqrt{2} - 1)^2 \cdot \frac{\sqrt{3}}{4} =$

$= \frac{\sqrt{3}}{4}(3 - 2\sqrt{2}) a^2$

Površina osmerokuta = Površina kvadrata − 4 · Površina trokutića $= a^2 - 4 \cdot \frac{y^2}{2} =$



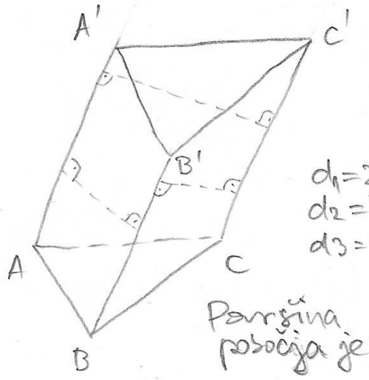
$= a^2 - 2 \cdot \frac{a^2}{4} \cdot (2 - \sqrt{2})^2 =$

$= a^2 \left(1 - \frac{1}{2}(4 - 4\sqrt{2} + 2)\right) =$

$= 2(\sqrt{2} - 1) a^2$

Dakle, $O = 6 \cdot 2(\sqrt{2} - 1) a^2 + 8 \cdot \frac{\sqrt{3}}{4}(3 - 2\sqrt{2}) a^2 = (12\sqrt{2} + 6\sqrt{3} - 4\sqrt{6} - 12) a^2$.

11.



$d_1 = 2\,cm$
$d_2 = 3\,cm$
$d_3 = 4\,cm$

Površina pobočja je 45cm.

bočni brid $b = |AA'| = |BB'| = |CC'|$

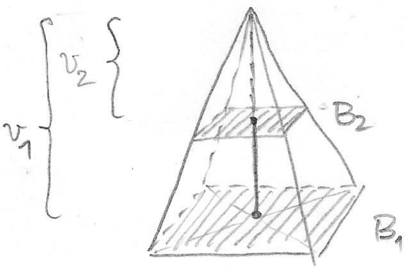
Površina pobočja je:

$$b \cdot d_1 + b \cdot d_2 + b \cdot d_3$$

Dakle, $b \cdot (2cm + 3cm + 4cm) = 45\,cm^2$

$$b = 5\,cm$$

15.



Ne znamo koliko strana ima piramida niti je li uspravna, no to nije bitno.

$B_1 = 245\,cm^2$

$B_2 = 80\,cm^2$

$v_1 = 35\,cm$

Označimo visinu male piramide s v_2.

Vrijedi: $B_1 : B_2 = v_1^2 : v_2^2$

(površine se odnose kao kvadrati duljina)

$$245 : 80 = 35^2 : v_2^2$$

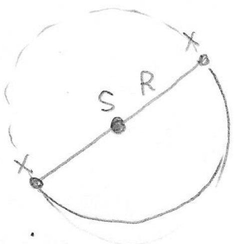
$$v_2 = \sqrt{\frac{80 \cdot 35^2}{245}} = \sqrt{\frac{16 \cdot \cancel{5} \cdot 35^2}{\cancel{5} \cdot 49}} = \frac{4 \cdot 35}{7} = 20$$

Dakle, $v_2 = 20.$

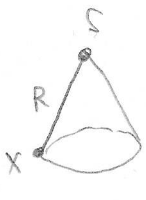
Volumen krnje piramide $V = \frac{1}{3} B_1 \cdot v_1 - \frac{1}{3} B_2 \cdot v_2$

$$V = \frac{1}{3} 245 \cdot 35 - \frac{1}{3} 80 \cdot 20 = \frac{1}{3}(8575 - 1600) = \frac{1}{3} \cdot 6975 = 2325\,cm^3$$

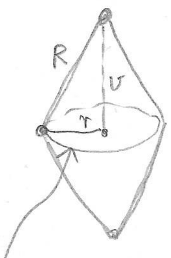
16.



$\rightarrow$



Od dva polukruga:



- površina kruga je $100\,cm^2$ $\Rightarrow$ polumjer kruga R je:
  $$100 = R^2 \pi$$
  $$R = \frac{10}{\sqrt{\pi}}$$

- opseg ovog kruga je $\frac{1}{2}$ opsega početnog kruga $= \frac{1}{\cancel{2}} \cdot \cancel{2} R\pi = 10\sqrt{\pi}$

  Dakle, $2r\pi = 10\sqrt{\pi}$

  $$r = \frac{5}{\sqrt{\pi}}$$

- 

  visina je: $v = \sqrt{R^2 - r^2} = \sqrt{\frac{10^2}{\pi} - \frac{5^2}{\pi}} = \frac{5\sqrt{3}}{\sqrt{\pi}} = v$

- VOLUMEN JE: $V = 2 \cdot \frac{1}{3} r^2 \pi \cdot v = \frac{2\pi}{3} \cdot \frac{25}{\pi} \cdot \frac{5\sqrt{3}}{\sqrt{\pi}} = \frac{250}{\sqrt{3\pi}}$