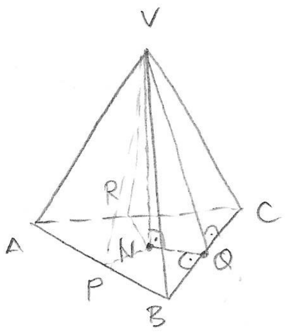
(14)



- N = nožište visine iz vrha V na osnovku ABC

  Q = nožište visine iz N na BC

  $\left.\begin{array}{l} VN \perp ABC \supseteq BC \\ NQ \perp BC \end{array}\right\}$ VNQ je ravnina okomita na BC

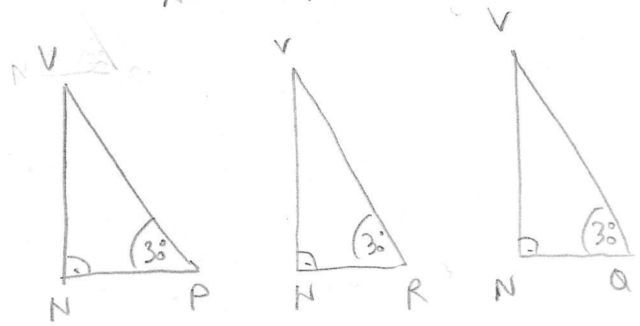
  $\Downarrow$

  $VQ \perp BC$

  (Pogledaj i teorem o tri normale.)

Dakle, $\overline{VQ}$ je visina u trokutu VBC: ⟵ To u ovom zadatku nije bitno, ali može biti pa zato spominjemo.

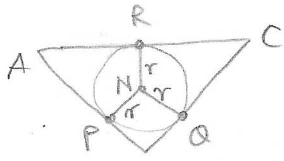
- Ozn. s P, R nožišta visine iz N na AB i CA.



Tri sukladna trokuta! ⟹ $|NP| = |NR| = |NQ|$

tj.

N je središte trokutu ABC upisane kružnice!

- 

stranice trokuta ABC su duljina 13, 20, 21 ⟶ $r = \frac{P}{s}$

površina trokuta $P = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{27 \cdot 14 \cdot 7 \cdot 6} = 9 \cdot 2 \cdot 7 = 126$

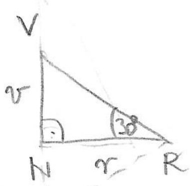
HERONOVA FORMULA

poluopseg $s = \frac{13+20+21}{2} = 27$

$P = 126$

$r = \frac{126}{27} = \frac{14}{3}$

Dakle, $|NP| = |NQ| = |NR| = r = \frac{14}{3}$.

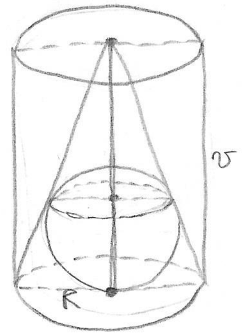
- 

$r = \frac{(2v) \cdot \sqrt{3}}{2} \rightarrow v = \frac{r}{\sqrt{3}} = \frac{14}{3\sqrt{3}}$ $\quad v = \frac{14}{3\sqrt{3}}$

(r je visina jednakostraničnog trokuta duljine stranice 2v)

- Sad možemo izračunati obujam: $V = \frac{1}{3} P \cdot v = \frac{1}{3} \cdot 126 \cdot \frac{14}{3\sqrt{3}} = \frac{196\sqrt{3}}{3}$.

17.



$R=6,\ v=8$

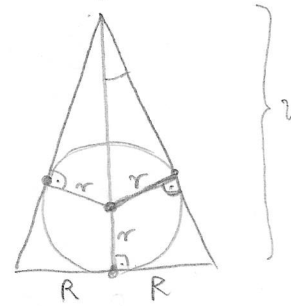
Omjer volumena kugle i valjka?

• Volumen valjka:

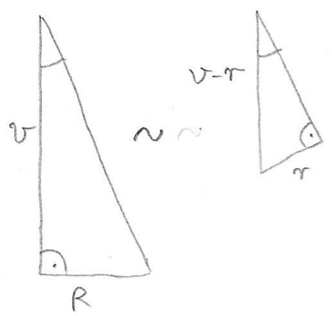
$$V = R^2\pi \cdot v = 36\pi \cdot 8 = 288\pi$$

• Stožac: $R=6,\ v=8$

OSNI PRESJEK STOŠCA:



Možemo izračunati r iz sličnosti trokuta:



$$\frac{r}{v-r} = \frac{R}{\sqrt{v^2+R^2}}$$

$R=6,\ v=8,\ \sqrt{v^2+R^2} = \sqrt{6^2+8^2} = 2\sqrt{9+16} = 10$

$$\frac{r}{8-r} = \frac{6}{10}$$

$$10r = 48-6r$$

$$r = \frac{48}{16}$$

$$r=3$$

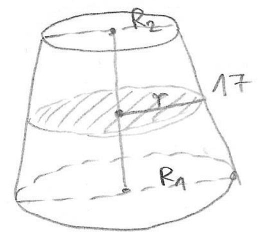
• Volumen kugle:

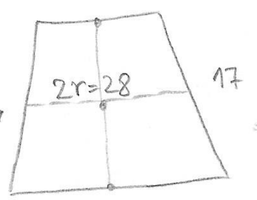
$$V_\ominus = \frac{4}{3}r^3\pi = \frac{4}{3}\cdot 3^3\pi = 36\pi$$

• Omjer volumena:

$$\frac{V_\ominus}{V} = \frac{36\pi}{288\pi} = \frac{1}{8}.$$

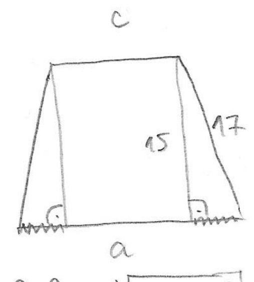
18.





Osni presjek krnjeg stošca je trapez

srednjica trapeza je $s=2r=28$



$P=196\pi$

$r^2\pi = 196\pi$

$r=14$

$P_\triangle = 420 = 2r \cdot v$ ← visina; $2r$ ↑ srednjica trapeza

$420 = 28\cdot v$

$v=15$

$$\frac{a-c}{2} = \sqrt{17^2-15^2} = \sqrt{(17-15)(17+15)} = \sqrt{2\cdot 32} = 8$$

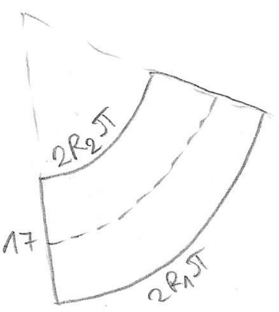
$a = s+8 = 28+8 = 36$

$c = s-8 = 28-8 = 20$

$2R_1 = 36 \Rightarrow R_1 = 18$

$2R_2 = 20 \Rightarrow R_2 = 10$

Oplošje krnjeg stošca:



$R_1 = 18$
$R_2 = 10$

$O = R_1^2\pi + R_2^2\pi +$ površina „krivolinijskog trapeza"

↑

Možemo ju izračunati kao umnožak „srednjice" i „visine"

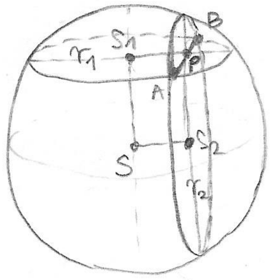
$$\frac{2R_1\pi + 2R_2\pi}{2} \cdot v = (R_1 + R_2)\pi \cdot v$$

Dakle,

$$O = 18^2\pi + 10^2\pi + 2\cdot(18+10)\pi\cdot 17 = (324+100+476)\pi = 900\pi$$

$$O = 900\pi \text{ cm}^2$$

19.



ozn. vrhove tetive s A, B

$|AB| = 16$

ozn. radijuse presjeka s r_1, r_2 i središta s S_1, S_2

$P_1 = 185\pi$ cm

$r_1^2\pi = 185\pi$

$r_1 = \sqrt{185}$

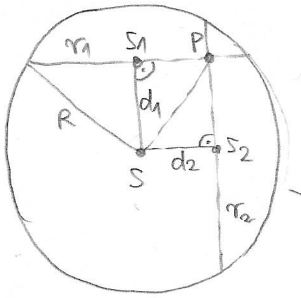
$P_2 = 320\pi$

$r_2^2 = 320$

$r_2 = \sqrt{320}$

ozn. polovište od $\overline{AB}$ s P

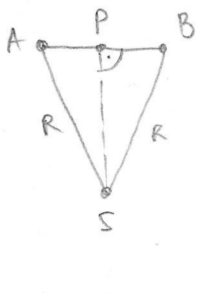
ozn. udaljenost presjeka od središta kugle s d_1 i d_2



$$r_1^2 + d_1^2 = R^2 = r_2^2 + d_2^2$$

$$|SP|^2 = d_1^2 + d_2^2$$

$r_1^2 = 185$
$r_2^2 = 320$
$|AB| = 16$



$$\left(\frac{|AB|}{2}\right)^2 = R^2 - |SP|^2$$

Dakle,

$$\left.\begin{aligned} r_1^2 + d_1^2 &= R^2 \\ r_2^2 + d_2^2 &= R^2 \\ \left(\frac{|AB|}{2}\right)^2 &= R^2 - d_1^2 - d_2^2 \end{aligned}\right\} \quad \left.\begin{aligned} 185 + d_1^2 &= R^2 \\ 320 + d_2^2 &= R^2 \\ 64 &= R^2 - d_1^2 - d_2^2 \end{aligned}\right\}$$

$\Rightarrow d_1^2 = R^2 - 185$, $d_2^2 = R^2 - 320$ uvrstimo u treću:

$64 = \cancel{R^2} - \cancel{R^2} + 185 - R^2 + 320$

$R^2 = 441$

$R = 21$

Polumjer kugle je 21 cm.