$T_{n+1}(x) - 2xT_n(x) + T_{n-1}(x) = 0$ $\quad T_0(x)=1,\ T_1(x)=x$

nule od $T_n$ su $x_k = \cos\frac{(2k-1)\pi}{2n}$, $k=1,\dots,n$

$T_n(x) = \cos(n\arccos x)$, $x\in[-1,1]$

$U_{n+1}(x) - 2xU_n(x) + U_{n-1}(x) = 0$ $\quad U_0(x)=1,\ U_1(x)=2x$

$U_n(x) = \frac{\sin((n+1)\arccos x)}{\sin(\arccos x)}$, $x\in[-1,1]$

$$\underbrace{\begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 0 & 1 \\ & & & 1 & 0 \end{bmatrix}}_{C_n}\underbrace{\begin{bmatrix} T_0, U_0(x) \\ \vdots \\ T_{n-1}, U_{n-1}(x) \end{bmatrix}}_{z_n(x)} + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ T_n(x), U_n(x) \end{bmatrix}}_{[T_n(x),U_n(x)]\cdot e_n} = 2x\underbrace{\begin{bmatrix} T_0, U_0 \\ \vdots \\ T_{n-1}(x) \end{bmatrix}}_{2x\cdot z_n(x)}$$

(13_NM, p. 42)

$C_n \quad z_n(x) \quad + [T_n(x), U_n(x)]\cdot e_n = 2x\cdot z_n(x)$

Trebam još $p_{-1}(x)=0$,

$$\underbrace{\sin(n+2)t}_{u} + \underbrace{\sin nt}_{v} = 2\cdot\sin\frac{u+v}{2}\cdot\cos\frac{u-v}{2}$$
$$= 2\cdot\sin\frac{2(n+1)t}{2}\cos\frac{2t}{2}$$
$$= 2\cdot\sin(n+1)t\cdot\underbrace{\cos t}_{x} \quad \Big| :\sin t$$

$x=\cos t$, $t=\arccos x$,

$$\frac{\sin(n+2)t}{\sin t} + \frac{\sin nt}{\sin t} = 2\,\frac{\sin(n+1)t}{\sin t}\cdot\cos t$$
$$U_{n+1}(x) + U_{n-1}(x) = 2\cdot U_n(x)\cdot x$$

$U_{-1}(x) = \frac{\sin(0\cdot t)}{\sin t} = 0.$ $\quad (T_{-1}(x) = T_1(x) = x\ !!!)$

Dakle, MORAM na $U_n$, a ne $T_n$

Nultočke od $U_n$: $\frac{\sin(n+1)t}{\sin t} = 0 \Rightarrow \sin(n+1)t = 0 \Rightarrow (n+1)t_k = k\pi$, $k=0,\dots$

tj. $t_k = \frac{k\cdot\pi}{n+1}$, $k=0,1,\dots,n+1$

ali zbog $\sin t_k \neq 0$ mora $k\neq 0$, $k\neq n+1$

$\Rightarrow t_k = \frac{k\cdot\pi}{n+1}$, $k=1,\dots,n$

$\Rightarrow (x=\cos t)$

$$\boxed{x_k = \cos\frac{k\pi}{n+1},\ k=1,\dots,n}$$

~~Komp~~ Svojstv. vrij. su $2x_k$, $k=1,\dots,n$

Komp. sv. vekt.

$$z_j(k) = U_{j-1}(x_k) = \frac{\sin(j\cdot t_k)}{\sin t_k}\cdot c_{norm} = \boxed{\frac{c_{norm}}{\sin\frac{k\pi}{n+1}}}\cdot\sin\frac{j\cdot k\pi}{n+1},\quad k,j=1,\dots,n$$