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Variable Degree Polynomial Splines are Chebyshev Splines

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Abstract Variable degree polynomial (VDP) splines have recently proved themselves as a valuable tool in obtaining shape preserving approximations. However, some usual properties which one would expect of a spline space in order to be useful in geometric modeling, do not follow easily from their definition. This includes total positivity (TP) and variation diminishing, but also constructive algorithms based on knot insertion. We consider variable degree polynomial splines of order $k \geqslant 2$ spanned by $\{1, x, \dots x^{k-3}, (x-x_i)^{m_i-1}, (x_{i+1}-x)^{n_i-1}\}$ on each subinterval $[x_i, x_{i+1}\rangle \subset [0,1]$, $i = 0, 1, \dots l$. Most of the paper deals with non-polynomial case $m_i, n_i \in [4, \infty)$, and polynomial splines known as VDP–splines are the special case when m_i, n_i are integers. We describe VDP–splines as being piecewisely spanned by a Canonical Complete Chebyshev system of functions whose measure vector is determined by positive rational functions $p(x)$, $q(x)$. These functions are such that variable degree splines belong piecewisely to the kernel of the differential operator $\frac{d}{dx} p \frac{d}{dx} q \frac{d^{k-2}}{dx^{k-2}}$. Although the space of splines is not based on an Extended Chebyshev system, we argue that total positivity and variation diminishing still holds. Unlike the abstract results, constructive properties, like Marsden identity, recurrences for quasi-Bernstein polynomials and knot insertion algorithms may be more involved and we prove them only for VDP splines of orders 4 and 5.

Keywords Chebyshev splines · Total positivity · Variation diminishing · Marsden identity · Recurrence relations

1 Preliminaries

Variable or *non-uniform degree* splines have been introduced to computer aided geometric design in [3]; see also [4,6,9]. They represent a polynomial alternative to exponential

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