Seminar on Numerical Analysis and Geometric Integration

NAday07

SCIENTIFIC PROGRAM

Zagreb, September 7th, 2007

Morning session

9:00–9:30 **Registration**

9:30–9:50 **Opening ceremony**

Time	chairperson: Mladen Rogina
9:50–10:25	**Tina Bosner** Algorithms for Non-uniform Exponential Tension Splines
10:25–11:00	**Emil Žagar** Deviations of Polynomial and Cubic Periodic Spline Interpolant from its Data Polygon

11:00–11:30 **Coffee break**

Time	chairperson: Saša Singer
11:30–12:05	**Gašper Jaklič** Lattices on Simplicial Partitions
12:05–12:40	**Bojan Orel** Step-size Control in Numerical ODEs

12:40–15:00 **Lunch break**

Afternoon session

Time	chairperson: Jernej Kozak
15:00–15:35	**Roman Kozlov** Conservative Discretizations of the Kepler Motion
15:35–16:10	**Bor Plestenjak** Harmonic Rayleigh-Ritz for the Multiparameter Eigenvalue Problem

16:10–16:40 **Coffee break**

Time	chairperson: Bojan Orel
16:40–17:15	**Vjeran Hari** Accelerating Block-Jacobi Methods
17:15–17:50	**Sanja Singer** Advances in Speedup of the Indefinite One–Sided Block Jacobi Method

19:00 **Dinner**

List of Abstracts

Algorithms for Non-uniform Exponential Tension Splines

Tina Bosner and Mladen Rogina
University of Zagreb

Abstract. We describe explicitly each stage of a numerically stable algorithm for calculating with exponential tension B-splines with non-uniform choice of tension parameters. These splines are piecewisely in the kernel of $D^2(D^2 - p^2)$, where D stands for ordinary derivative, defined on arbitrary meshes, with a different choice of the tension parameter p on each interval. The algorithm provides values of the associated B-splines and their generalized and ordinary derivatives by performing positive linear combinations of positive quantities, described as lower-order exponential tension splines. We show that nothing else but the knot insertion algorithm and good approximation of a few elementary functions is needed to achieve machine accuracy. The underlying theory is that of splines based on Chebyshev canonical systems which are not smooth enough to be ECC-systems. First, by de Boor algorithm we construct exponential tension spline of class C^1, and then we use quasi-Oslo type algorithms to evaluate classical non-uniform C^2 tension exponential splines.

Accelerating Block-Jacobi Methods

Vjeran Hari
University of Zagreb

Abstract. We consider a way how the one- and two-sided block-Jacobi methods can be accelerated using a block version of fast scaled rotations. Our approach exploits the computer memory hierarchy and is not prone to underflow or overflow exceptions. We explain the ideas for the case of a one-sided block-Jacobi method for computing SVD of rectangular matrices.

At each step of that method, an orthogonal matrix U is applied to the two block-columns of the iterated matrix, $[G_i', G_j'] = [G_i, G_j]U$. Partitioning U appropriately, we make the CS decomposition of U, $U = V\Gamma W^T$, where V, W are orthogonal and block-diagonal while Γ is a direct product of at most $n/2$ simple rotations. To reduce the flop count we observe that post-multiplying $[G_i, G_j]$ with V, followed by Γ, is cheaper than post-multiplying $[G_i, G_j]$ with the full matrix U. The multiplication with W^T can be postponed and combined with a V-multiplication at a later stage.

The additional cost coming from book-keeping of the matrices V, Γ, W and from computing the CS decomposition is expected to be small since it involves small

matrices and the computation can be done in the fast (cache) memory. Preliminary numerical tests show that the CPU time of one sweep of the block-Jacobi method can be, substantially (by more than 40%) reduced. It can be shown that the block-Jacobi method and its accelerated modification are relatively accurate.

As yet, the main obstacle in developing a full implementation of the modified method, is the lack of a reliable an accurate code for computing the CS decomposition of orthogonal matrices. In our tests we have computed CS decomposition via two SVDs of the diagonal (or off-diagonal) blocks of U. To this end, we have used QR with column pivoting followed by the Kogbetliantz method for triangular matrices.

Lattices on Simplicial Partitions

Gašper Jaklič, Jernej Kozak, Marjetka Krajnc, Vito Vitrih and Emil Žagar
University of Ljubljana

Abstract. In contrast to the univariate case, uniqueness of the solution of a multivariate Lagrange polynomial interpolation problem depends not only on the fact that interpolation points should be distinct but also on their geometry. Lattices are perhaps the most often used configurations of prescribed interpolation points on simplices in the domain.

In this talk, $d+1$-pencil lattices on simplicial complexes in $\mathbb{R}^d$ will be considered. The explicit representation of a lattice on a simplex, based upon barycentric coordinates, will be presented. This enables us to construct lattice points in a simple way and carries over to simplicial partitions in a natural way.

Conservative Discretizations of the Kepler Motion

Roman Kozlov
Universitetet i Bergen

Abstract. Modified vector fields are used to construct high-order conservative discretizations of the three-dimensional Kepler motion. The numerical integrators preserve the Hamiltonian function, the angular momentum and Runge-Lenz vector. In particular, the exact integrator of the Kepler motion is found. The proposed numerical schemes permit explicit implementation.

References

[1] R. Kozlov, *Conservative discretizations of the Kepler motion*, J. Phys. A: Math. Theor., vol. 40, no. 17 (2007), pp. 4529-4539.

Step-size Control in Numerical ODEs

Bojan Orel and Andrej Perne
University of Ljubljana

Abstract. In the beginning we will review some known results on how to select the step-size in order to be able to compute the solution of a system of ordinary differential equations with prescribed error tolerance. The step-size can be adjusted on the base of local or the global error estimates. At the end we will present a new approach for adaptive Lie group method with global error control.

Harmonic Rayleigh-Ritz for the Multiparameter Eigenvalue Problem

Bor Plestenjak
University of Ljubljana

Abstract. Harmonic extraction methods for the multiparameter eigenvalue problem will be presented. These techniques are generalizations of their counterparts for the standard and generalized eigenvalue problem. The methods aim to approximate interior eigenpairs, generally more accurately than the standard extraction does. The process can be combined with any subspace expansion approach, for instance a Jacobi-Davidson type technique, to form a subspace method for multiparameter eigenproblems of high dimension. We will focus on the two-parameter eigenvalue problem

$$A_1x_1 = B_1x_1 + C_1x_1,$$
$$A_2x_2 = B_2x_2 + C_2x_2,$$

for given $n_i \times n_i$ (real or complex) matrices A_i, B_i, C_i for $i = 1, 2$; we are interested in eigenpairs $((\lambda_1, \lambda_2), x_1 \otimes x_2)$ where x_1 and x_2 have unit norm.

Advances in Speedup of the Indefinite One–Sided Block Jacobi Method

Sanja Singer, Saša Singer, Vjeran Hari, Krešimir Bokulić*,
Davor Davidović*, Marijan Jurešić* and Aleksandar Uščumlić*
University of Zagreb

Abstract. Many authors have considered the Jacobi method for eigenvalue computation as an ideal for parallelization. But, if we respect the modern design of computer clusters with multi-level memory hierarchy, there appear many open questions how to implement it in an efficient way.

*undergraduate students

It is obvious that we have to reuse data locally when they reach the cache memory of a processor, and that the whole algorithm should be optimally decoupled so that workload of each processor is equal. To this end we have to develop a locally fast Jacobi algorithm which should be, when combined with the modulus block strategy, a method of choice for parallel computation.

Recent advances in fast implementation of the indefinite one-sided Jacobi method will be presented. Special attention is devoted to the block pivot strategies and to the column sorting which is embedded in the algorithm.

Deviations of Polynomial and Cubic Periodic Spline Interpolant from its Data Polygon

Emil Žagar
University of Ljubljana

Abstract. When fitting a parametric polynomial or spline curve through a sequence of points, it is important in applications that the curve should not exhibit unwanted oscillations. In this talk we study the local and global deviations of the polynomial interpolant and cubic periodic spline interpolant from the data polygon relative to the lengths of the polygon edges, focusing on the simple parameterization in which each parameter interval length is some power between 0 and 1 of the length of the chord between the two corresponding data points.

LIST OF PARTICIPANTS

Krešimir Bokulić, undergraduate student, Department of Mathematics, University of Zagreb, Croatia.
e–mail: kbokulic@gmail.com

Tina Bosner, Department of Mathematics, University of Zagreb, Croatia.
e–mail: tinab@math.hr

Ivan Ćatipović, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia.
e–mail: ivan.catipovic@fsb.hr

Davor Davidović, undergraduate student, Department of Mathematics, University of Zagreb, Croatia.
e–mail: davordavidov@email.t-com.hr

Damir Deković, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia.
e–mail: damir.dekovic@fsb.hr

Vjeran Hari, Department of Mathematics, University of Zagreb, Croatia.
e–mail: hari@math.hr

Gašper Jaklič, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: gasper.jaklic@fmf.uni-lj.si

Igor Jelaska, Department of Mathematics, University of Zagreb, Croatia.
e–mail: jelaska@math.hr

Maja Karaga, Department of Mathematics, University of Zagreb, Croatia.
e–mail: karaga@math.hr

Jernej Kozak, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: jernej.kozak@fmf.uni-lj.si

Roman Kozlov, Matematisk institutt, Universitetet i Bergen, Norway.
e–mail: Roman.Kozlov@math.uib.no

Marjetka Krajnc, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: marjetka.krajnc@fmf.uni-lj.si

Bojan Orel, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: bojan.orel@fri.uni-lj.si

Bor Plestenjak, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: bor.plestenjak@fmf.uni-lj.si

Mladen Rogina, Department of Mathematics, University of Zagreb, Croatia.
e–mail: rogina@math.hr

Sanja Singer, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia.
e–mail: ssinger@fsb.hr

Saša Singer, Department of Mathematics, University of Zagreb, Croatia.
e–mail: singer@math.hr

Gamze Tanoğlu, Department of Mathematics, IZTECH, Izmir, Turkey.
e–mail: gamzetanoglu@iyte.edu.tr

Nikola Vladimir, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Croatia.
e–mail: nikola.vladimir@fsb.hr

Aleksandar Ušćumlić, undergraduate student, Department of Mathematics, University of Zagreb, Croatia.
e–mail: sasa.uscumlic@gmail.com

Vito Vitrih, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: vito.vitrih@email.si

Emil Žagar, Department of Mathematics, University of Ljubljana, Slovenia.
e–mail: emil.zagar@fmf.uni-lj.si