APPLIED MATHEMATICS AND SCIENTIFIC COMPUTING

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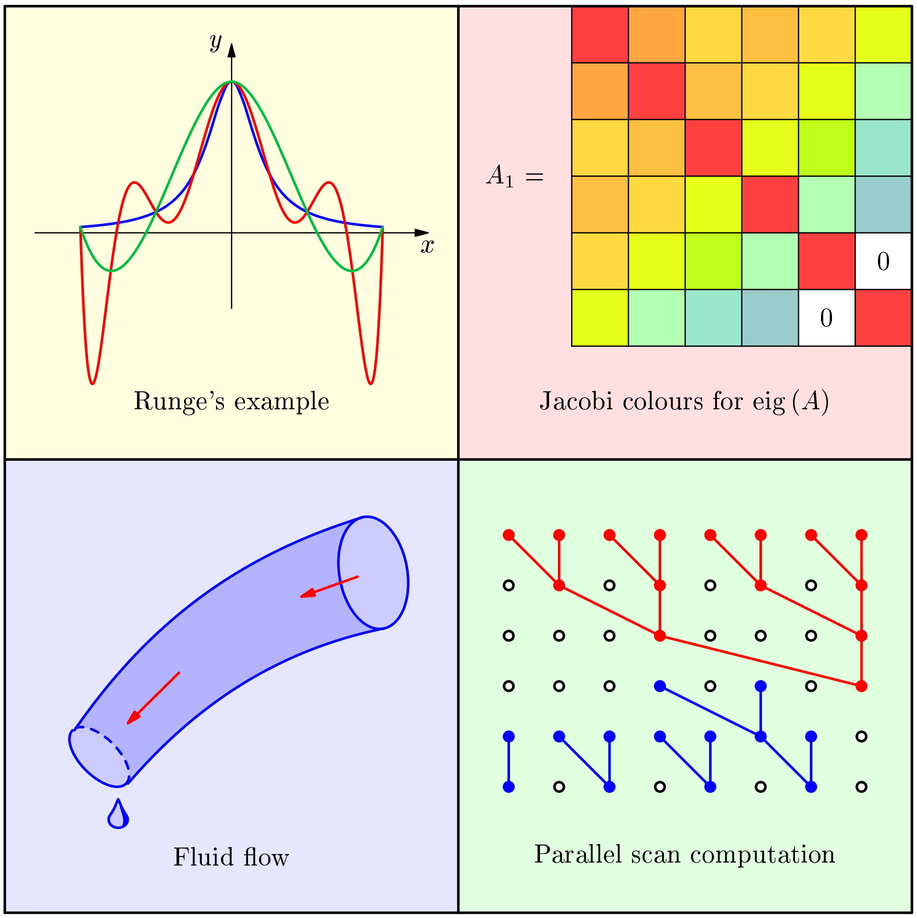

SCIENTIFIC PROGRAM

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Monday, June 23

Time	**Section A** chair: M. Marušić
8:45– 9:00	**Conference opening**
9:00–10:00	Boris I. Kvasov On Interpolating Biharmonic Tension Splines

Time	**Section A** chair: M. Rogina
10:30–11:00	Jernej Kozak and Gašper Jaklič On the Dimension of Bivariate Spline Space $S_3^1(\triangle)$
11:00–11:30	Jernej Kozak and Emil Žagar Geometric Interpolation of Space Data

Time	**Section A** chair: J. Kozak
11:40–12:10	Mladen Rogina An Algebraic Proof of the B–Spline Derivative Formula
12:10–12:40	Tina Bosner Knot Insertion Algorithms for Weighted Splines

Monday, June 23

Time	**Section A** chair: I. Aganović
16:00–16:30	Josip Tambača Derivation of a Model of Leaf Springs
16:30–17:00	Mladen Jurak and Žarko Prnić Heating of Oil Well by Hot Water Circulation

Time	**Section A** chair: E. Marušić–Paloka
17:30–18:00	Nenad Antonić and Marko Vrdoljak On Some Properties of Homogenised Coefficients for Stationary Diffusion Problem
18:00–18:30	Nenad Antonić and Krešimir Burazin On Certain Properties of Spaces of Locally Sobolev Functions

Tuesday, June 24

Time	**Section A** chair: P. Chenin
9:00–10:00	Paolo Costantini A General Frame for the Construction of Constrained Curves

Time	**Section A** chair: B. I. Kvasov	**Section B** chair: V. Hari
10:30–11:00	Dragan Jukić, Rudolf Scitovski and Kristian Sabo Total Least Squares Problem for the Hubbert Function	Daniel Kressner Structure Preservation: A Challenge in Computational Control
11:00–11:30	Patrick Chenin Interpolary Spline Methods for Metrics and Application for Meshing	Sanja Singer Skew–Symmetric dqd Algorithm

Time	**Section A** chair: R. Scitovski	**Section B** chair: K. Veselić
11:40–12:10	Saša Singer, Emil Coffou and Mladen Rogina "On-Demand" Computation of Gaussian Quadrature Formulæ for Tension Powers	Ninoslav Truhar Optimizing the Solution of the Ljapunov Equation
12:10–12:40	Aleksandra Čižmešija On a Class of Integral Hardy-type and Geometric Mean Operators	Luka Grubišić On Ritz Approximations for Positive Definite Operators

Tuesday, June 24

Time	Section A chair: B. Vrdoljak	Section B chair: S. Baran
16:00–16:30	John Guardiola and Antonia Vecchio A Model of Viral Dynamics Based on a System of Integral Equations	Alma Omerspahić and Božo Vrdoljak On Parameter Classes of Solutions for System of Quasi-linebreak linear Differential Equations
16:30–17:00	Zvonimir Šipuš, Željka Milin Šipuš and Siniša Škokić Analysis of Conformal Antennas Using Integral Approach and Moment Method	Nermina Mujaković 1–D Flow of a Compressible Viscous Micropolar Fluid: Stabilization of the Solution

Time	Section A chair: Z. Tutek
17:30–18:00	Lechoslaw Hacia Galerkin Type Methods for the Mixed Integral Equation
18:00–18:30	Hüseyin Kaya, Murat Kaplan and Hasan Saygın A Recursive Algorithm for Finding HDMR Terms for Sensitivity Analysis

Wednesday, June 25

Time	**Section A** chair: P. Costantini
9:00–10:00	John J. H. Miller and G. I. Shishkin Robust Numerical Methods for the Singularly Perturbed Black–Scholes Equation

Time	**Section A** chair: J. Miller	**Section B** chair: N. Truhar
10:30–11:00	Ivo Beroš and Miljenko Marušić Solving Parabolic Singularly Perturbed Problems by Collocation with Tension Splines	Vjeran Hari On Block Jacobi-type Methods
11:00–11:30	Gisbert Stoyan and Ágnes Baran Crouzeix–Velte Decompositions for Higher Order Finite Element Families	Vjeran Hari and Josip Matejaš Accuracy of Kogbetliantz Method for Triangular Matrices

Time	**Section A** chair: L. Sopta	**Section B** chair: A. Čižmešija
11:40–12:10	Krešimir Veselić Exponential Decay of Damped Systems	W. H. Laverty and Milivoj J. Miket Simulated Annealing for Hidden Markov Models
12:10–12:40	Ivica Nakić Optimal Damping of Infinitedimensional Vibrational Systems	Pavle Goldstein Multiple Alignments of Biological Sequences Using Hidden Markov Models

Wednesday, June 25

Time	**Section A** chair: S. Vuković	**Section B** chair: J. Tambača
16:00–16:30	Sándor Baran and Ágnes Baran An Application of Stochastic Optimization in Earth Sciences	Eduard Marušić–Paloka Modeling of an Underground Waste Disposal Site by Upscaling
16:30–17:00	Rózsa Horváth Bokor Strong Consistency of One–Step Approximations of Solutions of Stochastic Ordinary Differential Equations	Ivan Veselić Basic Spectral Properties of the Quantum Percolation Model

Time	**Section A** chair: Z. Drmač	**Section B** chair: D. Jukić
17:30–18:00	Senka Vuković, Nelida Črnjarić–Žic and Luka Sopta Order of Accuracy of Extended WENO Schemes	Emine Can Baran Determination of an Unknown Parameter in a Parabolic Equation
18:00–18:30	Nelida Črnjarić–Žic, Senka Vuković and Luka Sopta Balanced Central NT scheme for the Shallow Water Equations	Afet Golayoglu Fatullayev Numerical Procedure for the Simultaneously Determining of the Hydraulic Properties of Porous Media

Thursday, June 26

Excursion

Friday, June 27

Time	**Section A** chair: I. Slapničar
9:00– 9:30	Zlatko Drmač New Fast Implementation of the Jacobi SVD Algorithm
9:30–10:00	Nela Bosner and Zlatko Drmač On Numerical Properties of One–Sided Bidiagonalization Algorithm
10:00–10:30	Belkacem Kebli and Mohamed Ouadjaout Exact Solution of an Axisymmetric Elastic Deformation of a Cylinder in Axial Compression

INVITED LECTURES

TUESDAY, 9:00–10:00, SECTION A

A General Frame for the Construction of Constrained Curves

Paolo Costantini

Abstract. During the recent years, a conspicuous part of the research on mathematical methods for the construction of curves (and surfaces) has been devoted to develop new algorithms which satisfy, along with the classical interpolation or approximation conditions, also other constraints given by the context where the curve is looked for. Typical examples are given by data interpolation in industrial applications or by data approximation in the context of reverse engineering, where it is often required that the shape of the curve reproduces the shape of the data. Typically, the solution is provided by "ad-hoc" methods, specifically constructed for a single class of problems.

On the other hand we notice that, in the case of *piecewise* (polynomial, exponential, rational etc.) curves, very often these schemes follow a common procedure. First, a suitable set of parameters is chosen and each piece of the function is expressed using some of these parameters; second, the constraints are also rewritten in terms of these parameters and a set of *admissible domains* is derived, and, third, a theory is developed for checking the feasibility of the problem and, eventually, an algorithm for computing a solution is provided.

An unified algorithmic approach was proposed some years ago in the so called *abstract schemes*, which provide a general purpose practical theory that can give a common framework in which various and different problems and targets can be dealt with. It can be proven indeed that *any* problem regarding piecewise defined interpolating or approximating functions subject to *any* kind of local (i.e. piecewise defined) constraints can be modelled by means of abstract schemes and therefore solved with a general algorithmic procedure. In general a constrained curve construction problem admits a set of possible solutions; abstract schemes can be easily linked with many *optimization functionals* which can be used to select the best solution among the admissible ones.

Aim of this talk is to explain the basic structure of this schemes, to discuss some recent improvements and to show some applications in practical problems.

MONDAY, 9:00–10:00, SECTION A

On Interpolating Biharmonic Tension Splines

Boris I. Kvasov

Abstract. This paper addresses a new approach in solving the problem of multidimensional spline interpolation. Based on the formulation of the latter problem as a differential multipoint boundary value problem for biharmonic tension spline we consider its finite-difference approximation. The resulting system of linear equations can be efficiently solved by successive over-relaxation iterative method or using finite-difference schemes in fractional steps. We consider the basic computational aspects and illustrate the main advantages of this original approach.

WEDNESDAY, 9:00–10:00, SECTION A

Robust Numerical Methods for the Singularly Perturbed Black–Scholes Equation

John J. H. Miller and G. I. Shishkin

Abstract. We discuss a number of formulations of the Black–Scholes equation for the value of an option and we show that these may lead to singularly perturbed problems. We demonstrate that, for such cases, standard numerical methods do not produce robust numerical solutions. We construct a new numerical method which generates numerical approximations to the exact solution and its derivatives that converge parameter-uniformly in the maximum norm.

CONTRIBUTED TALKS

MONDAY, 18:00–18:30, SECTION A

On Certain Properties of Spaces of Locally Sobolev Functions

Nenad Antonić and Krešimir Burazin

Abstract. In recent years the locally Sobolev functions got quite popular in works on applications of partial differential equations. However, the properties of those spaces have not been systematically studied and proved in the literature, resulting in many particular proofs by reduction to classical Sobolev spaces.

Following some hints of general theory scattered through classical literature (where famous authors probably new the properties, but never found the time to write them down), as well as some proofs of special cases (namely, the Hilbert case), we systematically present the main results regarding the properties of $\mathrm{W}^{\mathrm{m,p}}_{\mathrm{loc}}$ spaces, their duality, imbedings, density, weak topologies, etc., with particular emphasis to application in studying partial differential equations of mathematical physics.

MONDAY, 17:30–18:00, SECTION A

On Some Properties of Homogenised Coefficients for Stationary Diffusion Problem

Nenad Antonić and Marko Vrdoljak

Abstract. We consider optimal design of stationary diffusion problems for two-phase materials. Speaking in the context of thermal conductivity, it consists in finding the best arrangement of two given materials in a fixed domain that maximises some functional, expressed in terms of temperature for some fixed source term on the right-hand side.

Since problems of this kind usually have no solution, a relaxation (there are also some other approaches) consists in introducing the notion of composite materials, as fine mixtures of different phases, mathematically described by the homogenisation theory. Denoting the set of all possible composite materials with given local proportion θ of the first material by $\mathcal{K}(\theta)$ the problem can be written as an optimisation problem over this set. In their paper, Tartar and Murat (1985) described the set $\mathcal{K}(\theta)e$, for some vector e, and used this result to replace the optimisation over the complicated set $\mathcal{K}(\theta)$ by a much simpler one. Analogous characterisation even holds for the case of mixing more than two materials (possibly anisotropic), where the set $\mathcal{K}(\theta)$ is not effectively known (Tartar, 1995).

We address the question of describing the set $\{(Ae, Af) : A \in \mathcal{K}(\theta)\}$, for given vectors e and f, which is important for optimal design problems with multiple state equations (different right-hand sides in stationary diffusion equation). In other words we are interested in describing two columns of matrices in $\mathcal{K}(\theta)$. In two dimensions we describe this set in appropriate coordinates and give some geometric interpretation. For the three dimensional case we consider the set $\{Af : A \in \mathcal{K}(\theta), Ae = t\}$, for a fixed t, and show how it can be reduced to a two dimensional one, although the solution involves tedious computations.

WEDNESDAY, 16:00–16:30, SECTION A

An Application of Stochastic Optimization in Earth Sciences

Sándor Baran and Ágnes Baran

Abstract. We describe a version of the simulated annealing optimization algorithm that is designed to find the global maximum points of multimodal functions defined on a continuous region. This algorithm is applied to estimate the transition matrices of partially observed Markov chains where the likelihood functions very often have several local maxima (see [1]). The estimates of the transition matrices are used in modeling discrete geological structures as Markov random fields (see [2]) and we would like to show the results of this approach applied to real geological data.

References

[1] S. Baran, Á. Szabó, *An application of simulated annealing to ML-estimation of a partially observed Markov chain*, Proceedings of the 3^{rd} International Conference on Applied Informatics, Eger–Noszvaj, Hungary, August 25–28, 1997, pp. 85–95.

[2] T. Norberg, L. Rosén, Á. Baran and S. Baran, *On modeling discrete geological structures as Markov random fields*, Math. Geol., 34 (2002), no. 1, pp. 63–77.

WEDNESDAY, 10:30–11:00, SECTION A

Solving Parabolic Singularly Perturbed Problems by Collocation with Tension Splines

Ivo Beroš and Miljenko Marušić

Abstract. Tension spline is a function that, for given partition $x_0 < x_1 < \cdots < x_n$, on each interval $[x_i, x_{i+1}]$ satisfies differential equation $(D^4 - \rho_i^2 D^2)u = 0$, where ρ_i's are prescribed nonnegative real numbers. In many articles, tension splines are used in collocation methods applied to two-points singularly perturbed boundary value problems with Dirichlet boundary conditions.

We apply collocation method to parabolic singularly perturbed problem

$$\varepsilon\frac{\partial^2 u}{\partial x^2} - c(x,t)u - p(x,t)\frac{\partial u}{\partial t} = f(x,t)$$

with Dirichlet boundary conditions. We observe the solution on the uniform mesh with $N_x \times N_t$ elements. Numerical results show ε-uniformly convergence of the method.

FRIDAY, 9:30–10:00, SECTION A

On Numerical Properties of One–Sided Bidiagonalization Algorithm

Nela Bosner and Zlatko Drmač

Abstract. The singular value decomposition (SVD) of a general matrix is the fundamental theoretical and computational tool in numerical linear algebra. The most efficient way to compute the SVD is to reduce the matrix to bidiagonal form in a finite number of orthogonal (unitary) transformations, and then to compute the bidiagonal SVD. We analyze a new modification of one–sided bidiagonalization method, and prove that it is numerically backward stabile. We offer detailed error analysis, and show that in special cases this approach possesses high relative accuracy property. We also propose some improvements in order to avoid loss of orthogonality and to obtain higher accuracy. Even without this improvements, proposed bidiagonalization method can be applied in solving symmetric definite eigenvalue problem and linear least squares problem, again in numerically stabile manner. For the conclusion, we offer some numerical examples, which ratify theoretical results.

MONDAY, 12:10–12:40, SECTION A

Knot Insertion Algorithms for Weighted Splines

Tina Bosner

Abstract. We develop a technique to calculate with weighted splines of arbitrary order, i.e., with the splines from the kernel of the operator $D^k w D^2$, with w piecewisely constant, based on a knot insertion type algorithm. The algorithm is a generalization of de Boor algorithm for polynomial splines, and it inserts the evaluation point in the knot sequence with maximal multiplicity. To achieve this, we use general form of knot insertion matrices, and an Oslo type algorithm for calculating integrals of B-splines in reduced Chebyshev systems. We use the fact that the space of weighted splines is a subspace of the polynomial spline space. The algebraic complexity of proposed algorithm can be reduced to the computationally reasonable size. Now we can calculate weighted splines, and the splines associated with their reduced system, in a stable and efficient manner.

References

[1] P. J. Barry and R. N. Goldman, *Knot insertion algorithms*, in Knot Insertion and Deletion Algorithms for B-Spline Curves and Surfaces, R. N. Goldman and T. Lyche (eds.), SIAM, 1993, pp. 89–133.

[2] C. de Boor, *A Practical Guide to Splines*, Springer, New York, 1978.

[3] T. Bosner, *Polar Forms of Splines and Knot Insertion Algorithms*, Master's thesis, Department of Mathematics, University of Zagreb, 2002, (in Croatian), pp. 58–78.

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WEDNESDAY, 17:30–18:00, SECTION B

Determination of an Unknown Parameter in a Parabolic Equation

Emine Can Baran

Abstract. Numerical procedures are considered for the solution of the following inverse problem of determining unknown source parameter in a parabolic equation.

Find $u(x,t)$ and $p = p(t)$ which satisfy

$$s\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} + q\frac{\partial u(x,t)}{\partial x} + p(t)u(x,t) + f(x,t),$$
$$(x,t) \in Q_0 = (0,1)\times(0,T), \qquad (1)$$

$$u(x,0) = \varphi(x), \qquad x \in (0,1), \qquad (2)$$

$$u(0,t) = \mu_1(t), \quad u(1,t) = \mu_2(t), \qquad t \in (0,T), \qquad (3)$$

subject to the overspecifications

$$u(x^*,t) = E(t), \quad 0 \le t \le T \qquad (4)$$

where $f(x,t)$, $\varphi(x)$, $\mu_1(t)$, $\mu_2(t)$ and $E(t)$ are known functions, and x^* is a fixed prescribed interior point in $(0,1)$. If u is a temperature then (1)–(4) can be regarded

as a control problem finding the control $p(t)$ such that the internal constraint is satisfied. If $p(t)$ is known then direct initial boundary value problem (1)–(3) has a unique smooth solution $u(x,t)$ [1].

In this work, we study two type of numerical procedures for considered problem. One of them is proposed in [2, 3]. According to this procedure the term $p(t)u$ in (1) is eliminated by introducing some transformation and system (1)–(4) is written in the canonical, suitable form for the finite difference solution. Another procedure to the solution of the same problem is obtained by using trace type functional (TTF) formulation [4].

References

[1] O. A. Ladyzhenskaya, V. A. Solonnikov, N. N. Uralceva, *Linear and Quasi-linear Equations of Parabolic Type*, Nauka, Moscow, 1967.

[2] J. R. Cannon, Y. Lin, S. Wang, *Numerical procedures for determination of an unknown coefficient in semi-linear parabolic equation*, Inverse Problems, 10 (1994), pp. 227–243.

[3] A. G. Fatullayev, *Numerical procedure for determination of an unknown parameter in a parabolic equation*, Intern. J. Computer Math., 78 (2001), pp. 103–109.

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TUESDAY, 11:00–11:30, SECTION A

Interpolary Spline Methods for Metrics and Application for Meshing

Patrick Chenin

Abstract. The more recent methods for meshing surfaces and domains use definition of metrics (Paul–Louis Georges and Pascal Frey INRIA France): a metric on a surface allows natural criteria for meshing.

In the finite element method context, a metric is known in a finite set of points (depending on surfaces and a-posteriori estimation) and interpolation method is needed to define the metric everywhere.

For that purpose, we propose a new spline method, based on hilbertian reproducing kernel theory.

TUESDAY, 12:10–12:40, SECTION A

On a Class of Integral Hardy-type and Geometric Mean Operators

Aleksandra Čižmešija

Abstract. In this lecture we give criteria for boundedness of a general multidimensional Hardy-type integral operator with an Oinarov kernel and of the related limiting geometric mean operator between two Lebesgue spaces. The integrals are taken over cones in $\mathbf{R}^n$ with the origin as a vertex. We also give two-sided estimates, that is, lower and upper bounds for the $L^p_v \to L^q_u$ norms of these operators for all p and q satisfying $1 < p < \infty$, $0 < q < \infty$ for the Hardy-type operator case, and $0 < p, q < \infty$ for the geometric mean operator case.

The lecture presents a part of a joint research with Lars-Erik Persson and Anna Wedestig, Luleå University of Technology, Luleå, Sweden.

WEDNESDAY, 18:00–18:30, SECTION A

Balanced Central NT scheme for the Shallow Water Equations

Nelida Črnjarić–Žic, Senka Vuković and Luka Sopta

Abstract. The numerical method we consider is a generalization of the nonstaggered central scheme proposed by Jiang, Levy, Lin, Osher and Tadmor (SIAM J. Numer. Anal., 35, 2147 (1998)) that was obtained by conversion of the standard central NT scheme to the nonstaggered mesh. The presented scheme is applied to the nonhomogeneous shallow water system. Including an appropriate numerical treatment for the source term evaluation we obtain the scheme that could preserve quiescent steady-state for the shallow water equations exactly. We consider two different approaches that depend on the discretization of the riverbed bottom. The obtained schemes are well balanced and presents accurate and robust results in both steady and unsteady flow simulations.

FRIDAY, 9:00–9:30, SECTION A

New Fast Implementation of the Jacobi SVD Algorithm

Zlatko Drmač

Abstract. We present a new implementation of the Jacobi algorithm for accurate floating–point computation of the singular value decomposition (SVD) of matrices as well as of various forms of products (quotients) of two or three matrices. The main goal of such an algorithm is to compute all singular values to high relative accuracy. This means that we are seeking guaranteed number of accurate digits even in the smallest singular values.

The main goal of our recent efforts is to achieve computational efficiency, while maintaining high accuracy. We use preprocessing, preconditioning and a posteriori computation of one set of singular vectors, and properly organized computation that uses machine optimized BLAS. It is shown that the new Jacobi SVD matches the efficiency of the xGESVD procedure (QR method) from LAPACK, and, moreover, that it is not much slower then the xGESDD procedure (divide and conquer method).

WEDNESDAY, 18:00–18:30, SECTION B

Numerical Procedure for the Simultaneously Determining of the Hydraulic Properties of Porous Media

Afet Golayoglu Fatullayev

Abstract. A nonlinear parabolic equation arising in modeling flow in homogeneous and isotropic porous media is considered in which coefficients representing water capacity and hydraulic conductivity are unknown and to be determined from overspecified data measured on the boundary. The problem is described by the following inverse problem of simultaneously determining $u(x,t)$ and unknown coefficients $c(u)$ and $k(u)$ which satisfy

$$
\begin{aligned}
c(u(x,t))\frac{\partial u(x,t)}{\partial t} &= \frac{\partial}{\partial x}\Big(k(u(x,t))\Big(\frac{\partial u(x,t)}{\partial x}-1\Big)\Big), && (x,t)\in Q_0=(0,1)\times(0,T)\\
u(x,0) &= u_0, && x\in(0,1),\\
\frac{\partial u}{\partial x}\Big|_{x=0} -1 &= 0, && t\in(0,T),\\
u(1,t) &= f(t), && t\in(0,T),
\end{aligned}
$$

subject to the overspecifications

$$
p(t)=u(0,t),\quad q(t)=k(u)\Big(\frac{\partial u}{\partial x}\Big|_{x=1} -1\Big),\quad t\in(0,T),
$$

This problem arises in modeling flow in homogeneous and isotropic porous media. If we interpret $u(x,t)$ as pressure head, then $c(u)$ and $k(u)$ may be interpreted, respectively as water capacity and hydraulic conductivity for porous medium [1]. Uniqueness and solvability results for this inverse problem are presented in [2].

One approach to solving this problem (referred to in the literature as the method of output least squares) is to assume that the unknown coefficients are a specific functional form depending on some parameters and then seek to determine optimal parameter values so as to minimize an error functional based on the overspecified data. The approach of method presented in this paper is not of this type. The strategy used here is to approximate unknown coefficients by a piecewise linear functions and eliminate their in the equation, using overspecified data measured on the boundary. In so doing, the problem can be transformed into the standard nonlinear initial boundary value problem in which coefficients are functions depending on the values of unknown solution and theirs derivatives. Such problem can be solved by the finite difference method and the unknown coefficients can be determined from the numerically obtained solution.

References

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[2] P. DuChateau, *An inverse problem for the hydraulic properties of porous media*, SIAM J. Numer. Anal., 28 (1997), pp. 611–632.

WEDNESDAY, 12:10–12:40, SECTION B

Multiple Alignments of Biological Sequences Using Hidden Markov Models

Pavle Goldstein

Abstract. We consider a hidden Markov model associated to a multiple alignment as a probabilistic model of the protein family under consideration. A standard optimisation method for obtaining the model of the family from unaligned sequences is described, using globin alignment as an example. Finally, we discuss quantum Markov chains and their application in the above setting.

TUESDAY, 12:10–12:40, SECTION B

On Ritz Approximations for Positive Definite Operators

Luka Grubišić

Abstract. We present new bounds for the eigenvalues and eigenvectors of positive definite operators. An algorithm for the computation of rigorous bounds for finite-element approximations is outlined. An accuracy of these approximations is assessed on several model examples and compared with the existing bounds found in the literature.

TUESDAY, 16:00–16:30, SECTION A

A Model of Viral Dynamics Based on a System of Integral Equations

John Guardiola and Antonia Vecchio

Abstract. Recently there has been an increasing interest in developing mathematical models of dynamics of spread of a viral infection in the organism, in order to predict viral progression upon infections and viral decline after drug treatment. Here, we construct a general mathematical model which can account for viruses having different mechanisms of infection and spread. First of all, we describe the dynamics of the spread of the infection "in the most natural situation" that is in the absence of drugs or vaccines. The main purpose is to collect as much information as possible on the qualitative behavior of the analytical solution of the problem in order to use it as starting point in the study of more complete models. In this sense, we consider the following mathematical problem as our basic model for viral dynamics

$$P(t) = \int_0^t F(t-x)e^{-\delta(t-x)} k_i k_s V(x)S(x)\,dx$$

$$V(t) = V_0 e^{-ct} + \int_0^t e^{-c(t-x)} pP(x)\,dx, \quad t \in [0, t_f]$$

$$S(t) = S_0 e^{-\beta t} + \int_0^t e^{-\beta(t-x)}[\alpha - k_s V(x)S(x)]\,dx.$$

The model incorporates a continuously distributed intracellular delay and is a nonlinear system of three Volterra integral equations. We perform a complete analysis of the qualitative behavior of the solution by proving its positivity and boundedness and by explicitly giving its limiting values. The numerical resolution has also been

achieved. Based on this model we have next constructed a model which describes viral dynamics of human immunodeficiency virus-1(HIV) incorporating the effect of vaccines having different efficacies. A preliminary qualitative analysis of this more refined model is presently being performed.

TUESDAY, 17:30–18:00, SECTION A

Galerkin Type Methods for the Mixed Integral Equation

Lechoslaw Hacia

Abstract. The following mixed integral equation

$$u(x,t) = f(x,t) + \int_0^t \int_M k(x,t,y,s)u(y,s)\,dy\,ds$$

in space-time is considered, where f is given function in domain $D = M \times [0,T]$ (M – a compact subset of n-dimensional Euclidean space) and u is unknown function in D; given kernel k is defined in domain $\Omega = \{(x,t,y,s) : x,y \in M, 0 \le s \le t \le T\}$.

The general theory for the considered equations in weighted spaces was presented in [3]. Numerical solutions of these integral equations were studied in papers [1]–[7].

Presented equations arise in the heat conduction theory and diffusion theory and problems of electrotechnics. Their nonlinear counterparts play fundamental role in the mathematical modelling of the spatio-temporal development of an epidemic [1, 4]. Some initial-boundary problems for a number differential partial equations of the parabolic type in physics and technology are reducible to the considered integral equations [9]. In this paper the general theory of these equations [3] is used in the projection methods of the Galerkin type. Presented methods lead to a system of algebraic equations or to a system of Volterra integral equations. The convergence of studied algorithms is proved and an error estimates are established. The presented theory is illustrated by numerical examples. The comparison of analysed methods is made.

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WEDNESDAY, 10:30–11:00, SECTION B

On Block Jacobi-type Methods

Vjeran Hari

Abstract. One-sided Jacobi-type methods for solving eigenvalue and singular value problems are known for their efficiency on ordinary and parallel computers and for their relative accuracy. In order to further enhance efficiency, block versions of these methods can be designed. It is shown, on the example of one-sided block-Jacobi method for computing the singular value decomposition, how CS decomposition and some other tricks can be used to reduce the flop count in the slowest part of the algorithm — updating the block columns.

WEDNESDAY, 11:00–11:30, SECTION B

Accuracy of Kogbetliantz Method for Triangular Matrices

Vjeran Hari and Josip Matejaš

Abstract. Kogbetliantz method is a two-sided diagonalization method for computing the singular value decomposition of square matrices. If the initial matrix is triangular, the method becomes more efficient and gains improved convergence characteristics. It is proved that the Kogbetliantz method for triangular matrices computes the singular values and vectors to high relative accuracy. This conclusion holds for both the real and the complex algorithm. One can design efficient Kogbetliantz method which is relatively accurate, globally convergent and asymptotically at least quadratically convergent.

WEDNESDAY, 16:30–17:00, SECTION A

Strong Consistency of One–Step Approximations of Solutions of Stochastic Ordinary Differential Equations

Rózsa Horváth Bokor

Abstract. Stochastic ordinary differential equations (SODE) model physical phenomena driven by stochastic processes. Like for deterministic differential equations, various numerical schemes have been proposed for SODE.

The Itô SODEs are given by the formula

$$dX(t) = a(t, X(t))dt + \sum_{j=1}^{m} b_j(t, X(t))dW_j(t), \quad \text{for } 0 \leq t \leq T, \quad X(t) \in \mathbf{R}^d,$$

with functions

$$a : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d, \quad b_j : [0, T] \times \mathbf{R}^d \to \mathbf{R}^d, \quad X(0) = x_0 \ (\in \mathbf{R}^d)$$

and m independent 1–dimensional Brownian motion $\{W_j(t) : 0 \leq t \geq T, j = 1, \ldots, m\}$.

There are two main concepts in strong approximation: strong convergence and strong consistency. The strong consistency implies order of local errors. The local errors impact to the strong convergence are studied.

TUESDAY, 10:30–11:00, SECTION A

Total Least Squares Problem for the Hubbert Function

Dragan Jukić, Rudolf Scitovski and Kristian Sabo

Abstract. In this paper we consider the parameter estimation problem for the logistic model in case when it is not possible to measure its values. We show that the parameter estimation problem for the logistic function can be reduced to the parameter estimation problem for its derivative known as the Hubbert function. Our proposed method is based on finite differences and the total least squares method.

Given the data (p_i, t_i, y_i), $i = 1, \ldots, m$, $m > 3$, we give necessary and sufficient conditions which guarantee the existence of the total least squares estimate of parameters for the Hubbert function, suggest a choice of a good initial approximation and give some numerical examples.

MONDAY, 16:30–17:00, SECTION A

Heating of Oil Well by Hot Water Circulation

Mladen Jurak and Žarko Prnić

Abstract. When highly viscous oil is produced at low temperatures large pressure drops will significantly decrease production rate. One of the possible solutions to this problem is heating of oil well by hot water recycling.

We introduce and analyze the mathematical model of heating the oil-well. The model consists of three hyperbolic equations coupled with one Volterra integral equation. Under simplifying assumptions that specific heats and densities of various materials are independent of the temperature proposed model become linear. Further on we construct and examine numerical method for obtained model and present some simulations results.

TUESDAY, 18:00–18:30, SECTION A

A Recursive Algorithm for Finding HDMR Terms for Sensitivity Analysis

Hüseyin Kaya, Murat Kaplan and Hasan Saygın

Abstract. High Dimensional Model Representation (HDMR) is a newly developed technique which decomposes a multivariate function into a constant, univariate, bivariate functions and so on. These functions are forced to be mutually orthogonal by means of an orthogonality condition. The technique which is generally used for high dimensional input-output systems can be applied to various disciplines including sensitivity analysis, differential equations, inversion of data and so on. In this article we present a computer program that computes individual components of HDMR resolution of a given multivariate function. The program also calculates the global sensitivity indices. Lastly the results of the numerical experiments for different set of functions are introduced.

FRIDAY, 10:00–10:30, SECTION A

Exact Solution of an Axisymmetric Elastic Deformation of a Cylinder in Axial Compression

Belkacem Kebli and Mohamed Ouadjaout

Abstract. In this work we give the exact solution of an axisymmetric elastic deformation of a cylinder in axial compression whose lateral surface is slipping. Using an appropriate finite Hankel integral transformation of the displacement vector, we reduce the Lamé equilibrium system to a system of an ordinary differential equations. The solution of the latter system is obtained explicitly by solving two differential equations of a fourth order. Using the inverse transformation we get the displacement vector in terms of Bessel function series. The solution of a compressed unbounded plate of thickness $2h$ is also given by using the infinite Hankel integral transformation and following the above scheme for the case of a cylinder.

MONDAY, 10:30–11:00, SECTION A

On the Dimension of Bivariate Spline Space $S_3^1(\triangle)$

Jernej Kozak and Gašper Jaklič

Abstract. In this work we tackle well known problem of determining dimension of bivariate spline space $S_3^1(\triangle)$ (space of C^1 cubic splines over regular planar triangulation). We use blossoming approach and the problem transforms into problem of computing ranks of certain matrices with special structure. Under some conditions on the degrees of vertices and collinearity of edges, it is possible to reduce the original problem into smaller problem on the subtriangulation. Therefore we can show, that for some special class of triangulations, the dimension of spline space equals Schumaker's lower bound. We present an algorithm which examines and reduces given triangulation, if it is possible.

MONDAY, 11:00–11:30, SECTION A

Geometric Interpolation of Space Data

Jernej Kozak and Emil Žagar

Abstract. The problem of interpolation of space data by cubic polynomial parametric curve is studied. Interpolating parameters are considered to be unknown which leads to nonlinear system of equations. The method using resultants for

global analysis of such a system is presented in case when five points have to be interpolated in $\mathbf{R}^3$ by a cubic parametric polynomial curve. Necessary and sufficient conditions for the existence and uniqueness of the interpolant are given in purely geometric sense.

TUESDAY, 10:30–11:00, SECTION B

Structure Preservation: A Challenge in Computational Control

Daniel Kressner

Abstract. In this talk some of the challenges that are related to the development of efficient and reliable numerical methods and numerical software for control problems will be addressed. These challenges include the demand for higher accuracy, robustness of the method with respect to uncertainties in the data or the model, and the need for methods to solve large-scale problems. To address these demands it is essential to preserve any underlying physical structure of the problem. At the same time, to obtain the required accuracy it is necessary to avoid all inversions or unnecessary matrix products. It will be demonstrated how these demands can be met to a great extent for some important tasks in control, model reduction, the linear-quadratic optimal control problem for first and second order systems as well as stability radius and H_∞-norm computations. All these problems are in some way related to Hamiltonian and skew-Hamiltonian eigenvalue problems.

HAPACK, a recently developed Fortran library, can be used to solve such eigenvalue problems. Structure exploitation and the use of new balancing and block factorization algorithms make HAPACK superior to standard eigenvalue solvers with respect to both accuracy and efficiency. The impact of these benefits on computational control will be illustrated by several examples. Moreover, new Arnoldi-like algorithms based on this library will be presented.

This is joint work with Peter Benner and Volker Mehrmann.

WEDNESDAY, 11:40–12:10, SECTION B

Simulated Annealing for Hidden Markov Models

W. H. Laverty and M. J. Miket

Abstract. One of the numerical difficulties associated with the application of hidden Markov models (see, for example, [5]) is the identification of a global maximum of the likelihood function. This difficulty has been addressed by Leroux and Puterman [7] by investigating several large classes of plausible initial values for the arguments of the likelihood function. However, this effort by Leroux and Putterman

has been criticized by MacDonald and Zucchini in their 1997 book [8]. The criticism by MacDonald and Zucchini is well-justified and is based on the fact that even when 'trying many sets of starting values of the parameters, there is no guarantee that this will succeed'.

The (usually unstated) objective of simulated annealing is to identify a global minimum of a function of many variables by gradual 'cooling', see [3] and [1]. The particular implementation of simulated annealing that was chosen for work on this project is due to Brooks [2]. An outstanding feature of this implementation is that it combines simulated annealing with a traditional algorithm for optimization. Hence the name: a hybrid optimization algorithm. It has been found by numerical experimentation, Brooks and Morgan [3], on examples taken from statistics, that this implementation performs significantly better than its alternatives.

Both, the book by MacDonald and Zucchini [8] and the article by Leroux and Puterman [7] come with a set of FORTRAN programs that could be used to verify the results of the author's numerical experiments. We chose to work with the programs designed by MacDonald and Zucchini. But the example for the present project is taken from the article by Leroux and Puterman [7]. The example is the sequence of counts of fetal lamb movements observed in 240 consecutive 5-second intervals.

The basic idea of the present attempt is to 'insert' the MacDonald and Zuchini program into the hybrid optimization algorithm due to Brooks in the place of a traditional minimizer, and then to verify the conclusion of Leroux and Puterman that a global maximum was reached for the example of fetal movements. There would still be no guarantee as demanded by MacDonald and Zucchini, but – under certain conditions – simulated annealing has been proved to converge to a global optimum [4]. The result is that our combined algorithm, i.e. hybrid optimization (of Brooks) and HMBASIC (of MacDonald and Zucchini), works much better than the EM algorithm (used by Leroux and Putterman, see [6]) on the example of fetal lamb movements. In addition, the combined algorithm provides – at no extra cost – the matrix of variances and covariances of estimated parameters.

A future project is to use simulated annealing for estimating the order of a hidden Markov model as suggested recently by Brooks, Friel and King [9] using some of the ideas from MacKay [10].

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WEDNESDAY, 16:00–16:30, SECTION B

Modeling of an Underground Waste Disposal Site by Upscaling

Eduard Marušić–Paloka

Abstract. The goal of this work is to give a mathematical model describing the global behavior of a nuclear waste disposal process. The disposal site can be described as an array made of high number of containers inside of a low permeability layer (e.g. clay) included between two larger layers with higher permeability (e.g. limestone). We study the worst possible scenario in which all containers start leaking at the same time, either due to some outer factor or due to simple aging of materials used to build the containers.

Some numerical simulations based on the derived model are given.

Work in collaboration with A. Bourgeat from Université Lyon 1 and ANDRA and G. B. Michel from CEA Seclay.

TUESDAY, 16:30–17:00, SECTION A

1–D Flow of a Compressible Viscous Micropolar Fluid: Stabilization of the Solution

Nermina Mujaković

Abstract. We consider nonstationary 1–D flow of a compressible and heat-conducting micropolar fluid, being in a thermodynamical sense perfect and polytropic. A corresponding initial-boundary value problem has a unique strong solution on $]0,1[\times]0,T[$, for each $T>0$. By using this result we obtain some a priori estimates of the solution not depending on T and we prove that the solution of the nonstationary problem converges to stationary in the norm of $(H^1(]0,1[))^4$ when $T\to\infty$.

WEDNESDAY, 12:10–12:40, SECTION A

Optimal Damping of Infinitedimensional Vibrational Systems

Ivica Nakić

Abstract. We introduce the notion of an abstract vibrational system in terms of a second–order differential equation involving sesquilinear forms, i.e.,

$$\mu(\ddot{x}(t), y) + \gamma(\dot{x}(t), y) + \kappa(x(t), y) = 0, \quad \forall y$$

where μ (mass form), γ (damping form) and κ (stiffness form) are symmetric and nonnegative.

Most mechanical vibrational systems can be written in this form.

Under some natural conditions, we solve this equation by the use of the semigroup theory technique.

An useful optimal damping criterion is

$$\min_{\gamma} \int_{\|u_0\|=1} \Big(\int_0^\infty E(t; u_0)\, \mathrm{d}t \Big) \, \mathrm{d}\sigma,$$

where $E(t; u_0)$ is the energy of the system with initial state u_0 at the moment t, and σ is some probability measure on the unit sphere. In other words, we minimize the average total energy of the system over all admissible damping forms.

We give a precise mathematical formulation of this criterion and show how to choose an appropriate measure σ.

Also, in the case of systems which posses an internal damping, we find the optimal damping forms.

TUESDAY, 16:00–16:30, SECTION B

On Parameter Classes of Solutions for System of Quasilinear Differential Equations

Alma Omerspahić and Božo Vrdoljak

Abstract. The paper presents some results on the existence and behaviour of some parameter classes of solutions for the system of quasilinear differential equations. The behaviour of integral curves in the neighbourhood of an arbitrary curve is considered, with particular treatment of some special cases. The obtained results contain the answer to the question on stability (stability (or instability) with the

function of stability (or instability), including auto-stability and stability along the coordinates of certain classes of solutions) as well as approximation of solutions whose existence is established. The errors of the approximation are defined by the functions that can be sufficiently small. The theory of qualitative analysis of differential equations and topological retraction method are used.

MONDAY, 11:40–12:10, SECTION A

An Algebraic Proof of the B–Spline Derivative Formula

Mladen Rogina

Abstract. We prove a well known formula for the generalized derivatives of Chebyshev B–splines:

$$L_1 B_i^k(x) = \frac{B_i^{k-1}(x)}{C_{k-1}(i)} - \frac{B_{i+1}^{k-1}(x)}{C_{k-1}(i+1)},$$

where

$$C_{k-1}(i) = \int_{t_i}^{t_{i+k-1}} B_i^{k-1}(x)\, d\sigma,$$

in a purely algebraic fashion, and thus show that it holds for the most general spaces of splines. The integration is performed with respect to a certain measure associated in a natural way to the underlying Chebyshev system of functions. Next, we discuss the implications of the formula for some special spline spaces, with an emphasis on those that are not associated with ECC-systems.

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TUESDAY, 11:00–11:30, SECTION B

Skew–Symmetric dqd Algorithm

Sanja Singer

Abstract. Differential qd algorithm could be a method of choice to compute eigenvalues of symmetric tridiagonal matrices with high relative accuracy.

This algorithm could be extended to skew-symmetric matrices by using the ordinary and the symplectic QR factorization of block-bidiagonal matrices. We shall present a direct convergence proof for the algorithm. Additionally, our algorithm avoid subtractions, so we can also guarantee high relative stability.

TUESDAY, 11:40–12:10, SECTION A

"On-Demand" Computation of Gaussian Quadrature Formulæ for Tension Powers

Saša Singer, Emil Coffou and Mladen Rogina

Abstract. We consider Gaussian quadrature formulæ which exactly integrate a system of tension powers

$$1, x, x^2, \ldots, x^{n-3}, \sinh(px), \cosh(px),$$

on $[a, b]$, where $n \geq 4$ is an even integer and $p > 0$ is a given tension parameter. The existence and uniqueness of such formulæ follows from the Chebyshev space theory, and it is "only" a numerical problem how to calculate the required nodes and weights for any given values of n and p.

These formulæ have a particularly important application in a recently constructed tension spline collocation method for singularly perturbed Volterra integro–differential and integral equations. In this context, n can be considered as given in advance, and only reasonably low values of n are of any practical importance. But it is essential that p can be changed dynamically, as it depends on the singular perturbation parameter ε, and can assume different values on different subintervals of integration. Since all possible values of p are not known in advance, it is impossible to precalculate or tabulate all the necessary data for the Gaussian quadrature.

Therefore, we need an efficient "on-demand" algorithm, that calculates the nodes and weights for a given value of p, at least for several low values of n. Unfortunately, no single approach can numerically achieve the required machine accuracy for all tension parameters of interest. By exploiting semianalytic techniques, translation invariance and known behaviour of tension powers and Gaussian quadrature formulæ in the limiting cases $p \to 0$ and $p \to \infty$, we can construct efficient and accurate algorithms for various ranges of p.

WEDNESDAY, 11:00–11:30, SECTION A

Crouzeix–Velte Decompositions for Higher Order Finite Element Families

Gisbert Stoyan and Ágnes Baran

Abstract. The Crouzeix–Velte decomposition is a decomposition into three orthogonal subspaces of the customary Sobolev space $(H_0^1(\Omega))^d$ of vector functions defined over a Lipschitz-continuous domain $\Omega \subset R^d$. This decomposition can be used to get more information about the inf-sup constant of the Stokes problem.

The decomposition and the knowledge of inf-sup constant are of interest for error estimation and numerical solution of the boundary value problems. For the same aim an algebraic and a discrete equivalent of the decomposition can be defined and is of advantage.

Here we consider the two-dimensional case and Scott–Vogelius elements of higher order supplemented by nonconforming bubbles and we show that they admit a discrete Crouzeix–Velte decomposition for even orders but not for odd orders. We also examine how many zeros are in the Crouzeix–Velte spectrum of the pressure space for the case of a velocity space with and without nonconforming bubbles.

TUESDAY, 16:30–17:00, SECTION A

Analysis of Conformal Antennas Using Integral Approach and Moment Method

Zvonimir Šipuš, Željka Milin Šipuš and Siniša Škokić

Abstract. Rapid growth in wireless communications, especially mobile communications, caused that the requirements on terminal antennas are more and more demanding. Arrays on cylindrical structures offer a possibility either to create directed beams in arbitrary direction in horizontal plane, or to create an omnidirectional pattern. Spherical arrays have possibility of directing single or multiple beams through complete hemisphere.

Conformal antennas and periodic surfaces are frequently analyzed by means of the electric field integral equation and the moment method. The kernel of the integral operator is a Green's function, which is different for different structures. Planar, circular cylindrical and spherical multilayer structures have one property in common: the structure is homogeneous in two dimensions, and varies in the third dimension. For example, the spherical structure varies in radial direction and is homogeneous in q and f directions. Thus, we can call planar, cylindrical and spherical structures one-dimensional structures since they vary only in one dimension. We simplify the problem of determining the Green's functions for one-dimensional

structures if we perform the two-dimensional (2D) Fourier transformation in the coordinates for which the structure is homogeneous (in the cylindrical case we perform the Fourier transformation in axial direction and the Fourier series in f direction, and in the spherical case we perform the vector-Legendre transformation). As a result, our original three-dimensional problem is transformed into a one-dimensional problem, which is much easier to solve.

There are two basic approaches for determining the Green's function of general multilayer structures: either to analytically derive an expression for it and then to code this expression, or to develop a numerical routine for the complete calculation. The analytic approach requires less computer time than the numerical approach. However, it is a very laborious process to analytically determine the Green's functions for substrates with more than two layers. Therefore, in such cases it is convenient to use a numerical algorithm that determines the Green's function directly. Another disadvantage of the analytic approach is that it is valid for a very specific geometry, so that a new derivation of the Green's functions is needed if the geometry is slightly different, such as for different locations of the patch antennas inside the layers. We will present the G1DMULT algorithm that calculates the spectral-domain Green's functions for planar, circular-cylindrical and spherical multilayer structures.

Sometimes it is complicate to rigorously calculate elements of the moment method matrix. In such cases approximate methods can be applied. The key point of applying the uniform theory of diffraction to conformal antenna analysis is to determine the geodesic on the antenna surface. We will describe a general ray-tracing method that is suitable for analyzing conformal antennas.

MONDAY, 16:00–16:30, SECTION A

Derivation of a Model of Leaf Springs

Josip Tambača

Abstract. In the linearized theory the spring at which the applied force F produces the displacement u is said to be of the stiffness $k = \frac{F}{u}$. The aim of this paper is to derive the stiffness coefficient k in terms of geometrical and mechanical properties of the leaf springs.

The starting point of the derivation is the linearized elastic body consisting of n straight rods (leaves of the spring) of lengths $\ell/n, 2\ell/n, \dots, \ell$ and thickness ε. The rods are connected by their lateral sides so they form a pseudo triangle (a one half of the spring). The one dimensional model for the behavior of the elastic body is then obtained by taking the thickness ε of rods to zero. The model is then solved and the stiffness coefficient of the spring is calculated.

TUESDAY, 11:40–12:10, SECTION B

Optimizing the Solution of the Ljapunov Equation

Ninoslav Truhar

Abstract. We consider a second order damped-vibration equation $M\ddot{x} + D\dot{x} + Kx = 0$, where M, D, K (called mass, damping, stiffness matrix, respectively) are real, symmetric matrices of order n with M, K positive definite and $D = C_u + C$, where C_u presents internal damping and C is positive semidefinite with $\text{rank}(\mathbf{C}) = r$.

We derive the trace $\text{Tr}(\mathbf{ZX})_\epsilon$ of the solution $\mathbf{X}$ of the Ljapunov equation $\mathbf{A}^T\mathbf{X} + \mathbf{XA} = -\mathbf{B}$, (where $\mathbf{A} = \mathbf{A}(\epsilon)$ is obtained from M, D, K and $\mathbf{B}$ is positive definite) and $\mathbf{Z}$ is a suitably chosen positive semidefinite matrix.

If r is small, our algorithm allows a sensibly more efficient optimization $\epsilon \to \text{Tr}(\mathbf{ZX})_\epsilon$, where parameter ϵ is optimal if $\text{Tr}(\mathbf{ZX})_\epsilon = \min$, than standard methods based on the Bartels–Stewart's Ljapunov solver.

WEDNESDAY, 16:30–17:00, SECTION B

Basic Spectral Properties of the Quantum Percolation Model

Ivan Veselić

Abstract. We consider the quantum percolation model on graphs with an amenable group action. This model is intermediate to two types of problems in physics of disordered media:

1.) The classical percolation problem concerns properties of clusters in an infinite graph — usually in the lattice $\mathbf{Z}^d$ — where vertices are deleted randomly. For instance, if too many vertices are deleted, there will be no infinite cluster almost surely.

2.) Random lattice Hamiltonians are a quantum mechanical model governing the motion of a particle in a disordered medium. The spectral properties of the random operator allow one to draw conclusions about transport properties of the modelled material.

To define the quantum percolation model fix a finite-range hopping operator on the original (non-random) graph. Each random configuration defines the subgraph of active (= undeleted) vertices. The quantum percolation model is the family of restrictions of the original hopping operator onto the random graph of active sites.

We derive basic spectral properties of these operators: non-randomness of the spectrum and its components, existence of an self-averaging integrated density of states and an associated trace-formula. We discuss some peculiar features of the quantum percolation model like the existence of eigenfunctions of finite support.

WEDNESDAY, 11:40–12:10, SECTION A

Exponential Decay of Damped Systems

Krešimir Veselić

Abstract. We present a new and simple bound for the exponential decay of second order systems using the spectral shift. This result is applied to finite matrices as well as to partial differential equations of Mathematical Physics. The type of the generated semigroup is shown to be the upper real part of the numerical range of the underlying quadratic operator pencil.

WEDNESDAY, 17:30–18:00, SECTION A

Order of Accuracy of Extended WENO Schemes

Senka Vuković, Nelida Črnjarić–Žic and Luka Sopta

Abstract. We developed an extension of WENO schemes for hyperbolic balance laws with spatially variable flux and geometrical source term. In the schemes we use high order ENO and WENO reconstruction for flux and source term characteristicwise components, therefore schemes give high-resolution results.

We analyze these high-resolution properties through three convergency tests: on a Burges equation test with geometrical source term, on a linear acoustics test, and on a shallow water test. We compare experimentally established orders of accuracy vs. the theoretically expected order for the new schemes as well as for the original WENO schemes combined with pointwise source term evaluation.

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