THE NUMBER OF COMMON ORDINARY AND SCHNEIDER'S p-ADIC CONVERGENTS

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Abstract. Fix a prime p. For a positive rational number α which is a p-adic unit with finite Schneider's expansion, let $C_p(\alpha)$ denote the number of Schneider's convergents that are also ordinary continued fraction convergents of α. Let $M_p(H)$ be the maximum of $C_p(\alpha)$ for such α with height $H(\alpha) \leq H$. We prove that, for $H \geq 2p+1$,

$$\left\lfloor \log_2 \left(\frac{4(\log_p H + 3)}{5} \right) \right\rfloor \leq M_p(H) \leq \left\lfloor \log_2 \left(\frac{4(2\log_p H + 3)}{5} \right) \right\rfloor .$$

The two bounds differ by at most one. The upper bound follows from a sharp growth condition for the partial sums of the exponents in Schneider's expansion at successive common convergents. The lower bound is obtained from an explicit recursively defined family.

Keywords: continued fractions, p-adic continued fractions, common convergents

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1. Introduction

Let p be a prime number. Schneider's continued fraction is one of the classical continued fraction algorithms over $\mathbb{Q}_p$. See [5], [2], [4] for its definition and background. In [3], finite Schneider's expansions of positive rational p-adic units were compared with their ordinary real continued fractions.

For a reduced positive rational number $\alpha = a/b$, put $H(\alpha) = \max\{a, b\}$. Throughout, the ordinary convergents of a rational number are those of its normalized finite simple continued fraction, whose final partial quotient is at least 2, unless the number is an integer. Let $C_p(\alpha)$ denote the number of rational numbers that occur both as ordinary convergents and as Schneider's convergents of α, and define

$$M_p(H) = \max\{C_p(\alpha) : \alpha \in \mathbb{Q}_{>0} \cap \mathbb{Z}_p^\times,\ H(\alpha) \leq H,\ \alpha \text{ has a finite Schneider's expansion}\}.$$

In [3, Section 4], these functions were introduced and the bounds

$$M_p(H) \gg_p \log\log H, \qquad M_p(H) \ll \sqrt{\log_p H}$$

were obtained. For $p = 2$, the same paper [3, Proposition 4.6] introduced the recursively defined family used below and proved that its m-th member has at least m common convergents. We show that this construction works for every prime and determine the exact number of common convergents in the family. A new growth lemma then gives the following nearly exact bounds for $M_p(H)$.

Theorem 1.1. *Let p be a prime. For every $H \geq 2p+1$,*

$$\left\lfloor \log_2 \left(\frac{4(\log_p H + 3)}{5} \right) \right\rfloor \leq M_p(H) \leq \left\lfloor \log_2 \left(\frac{4(2\log_p H + 3)}{5} \right) \right\rfloor .$$

The two displayed integers differ by at most one. Consequently,

$$M_p(H) = \log_2 \log_p H + O(1)$$

as $\log_p H \to \infty$, where the implied constant in $O(1)$ is absolute.

The double logarithm in Theorem 1.1 comes from the fact that the partial sums of the exponents at successive common convergents grow exponentially, while the total sum of the exponents is bounded by a constant multiple of $\log_p H(\alpha)$.

We also show that, for every $m \geq 2$, a rational number with m common convergents must have height at least $p^{5\cdot 2^{m-2}-2m}$, while our construction gives one with height less than $p^{5\cdot 2^{m-2}-m}$. Thus these two bounds differ by a factor of at most p^m.

2. A growth lemma and the upper bound

Let

$$\alpha = [b_0, p^{a_1} : b_1, \ldots, p^{a_n} : b_n]_p = b_0 + \cfrac{p^{a_1}}{b_1 + \cfrac{p^{a_2}}{\ddots + \cfrac{p^{a_n}}{b_n}}}, \qquad 1 \leq b_j \leq p-1, \quad a_j \geq 1,$$

be a finite Schneider's expansion. Every finite expression of this form is recovered by Schneider's algorithm from its value.

The convergents $P_k/Q_k = [b_0, p^{a_1} : b_1, \ldots, p^{a_k} : b_k]_p$, $0 \leq k \leq n$, are given by

$$P_{-2} = 0, \quad P_{-1} = 1, \qquad Q_{-2} = 1, \quad Q_{-1} = 0,$$

$$P_k = b_k P_{k-1} + p^{a_k} P_{k-2}, \qquad Q_k = b_k Q_{k-1} + p^{a_k} Q_{k-2},$$

where $a_0 = 0$. Put

$$A_0 = 0, \qquad A_k = a_1 + \cdots + a_k \quad (k \geq 1).$$

Then

$$P_kQ_{k-1} - P_{k-1}Q_k = (-1)^{k+1}p^{A_k}. \tag{2.1}$$

For $k \geq 1$,

$$P_k \equiv b_kP_{k-1} \pmod p, \qquad Q_k \equiv b_kQ_{k-1} \pmod p.$$

Since $P_0 = b_0$ and $Q_0 = 1$, induction gives $p \nmid P_kQ_k$ for all $k \geq 0$. Any common divisor of P_k and Q_k divides the left-hand side of (2.1), and hence every P_k/Q_k is reduced. The numerators satisfy

$$P_{-1} \leq P_0 < P_1 < P_2 < \cdots ,$$

while the denominators $Q_0, Q_1, \ldots$ are nondecreasing.

Let

$$\alpha = [c_0; c_1, \ldots, c_N] = c_0 + \cfrac{1}{c_1 + \cfrac{1}{\ddots + \cfrac{1}{c_N}}},$$

be the normalized ordinary continued fraction. Write $u_j/v_j = [c_0; c_1, \ldots, c_j]$ for its convergents and put $U_j = (u_j, v_j)^T$. We use the usual initial vectors $U_{-2} = (0,1)^T$ and $U_{-1} = (1,0)^T$. The standard determinant identity for ordinary continued fractions gives $\det(U_j, U_{j-1}) = (-1)^{j+1}$. For $i < j$ there are positive coprime integers R, S such that

$$U_j = RU_i + SU_{i-1}, \qquad \frac{R}{S} = [c_{i+1}; \ldots, c_j], \qquad R \geq S. \tag{2.2}$$

Lemma 2.1. *Suppose that $P_r/Q_r = u_i/v_i$ and $P_s/Q_s = u_j/v_j$ are common convergents, where $r < s$ and $i < j$. Then $s \geq r + 2$ and $a_{r+2} \geq A_{r+1}$. If $r \geq 1$, then the stronger inequality $a_{r+2} \geq A_{r+1} + 1$ holds. Consequently,*

$$A_s \geq 2 \quad \textit{if } r = 0, \qquad A_s \geq 2A_r + 3 \quad \textit{if } r \geq 1.$$

Proof. The recurrence for Schneider's convergents gives positive integers K, K' such that

$$\begin{pmatrix} P_s \\ Q_s \end{pmatrix} = K \begin{pmatrix} P_r \\ Q_r \end{pmatrix} + p^{a_{r+1}} K' \begin{pmatrix} P_{r-1} \\ Q_{r-1} \end{pmatrix}. \tag{2.3}$$

If $s = r + 1$, then $K/K' = b_{r+1}$. If $s \geq r + 2$, then

$$\frac{K}{K'} = [b_{r+1}, p^{a_{r+2}} : b_{r+2}, \ldots, p^{a_s} : b_s]_p \leq p - 1 + p^{a_{r+2}}. \tag{2.4}$$

Since the two common fractions are reduced and positive,

$$P_r = u_i, \quad Q_r = v_i, \qquad P_s = u_j, \quad Q_s = v_j.$$

By (2.3) and (2.1),

$$\begin{aligned} P_s Q_r - P_r Q_s &= p^{a_{r+1}} K' (P_{r-1} Q_r - P_r Q_{r-1}) \\ &= (-1)^r p^{A_{r+1}} K'. \end{aligned}$$

Thus this number has sign $(-1)^r$. On the other hand, (2.2) shows that $u_j v_i - u_i v_j$ has sign $(-1)^i$. Hence $r \equiv i \pmod 2$.

The vectors U_i, U_{i-1} form a basis of $\mathbb{Z}^2$. Write

$$\begin{pmatrix} P_{r-1} \\ Q_{r-1} \end{pmatrix} = cU_i + dU_{i-1}.$$

Taking determinants with $U_i = (P_r, Q_r)^T$ as the first column and using the parity just obtained gives $d = p^{A_r}$. Moreover, $c \leq 0$. If $c \geq 1$, comparison of the first coordinates gives

$$P_{r-1} = cu_i + p^{A_r} u_{i-1} > u_i = P_r,$$

contrary to $P_{r-1} \leq P_r$. Thus $c = -h$ for some integer $h \geq 0$. If $r \geq 1$, then $h \neq 0$. Otherwise, we would have

$$\begin{pmatrix} P_{r-1} \\ Q_{r-1} \end{pmatrix} = p^{A_r} U_{i-1},$$

which contradicts $\gcd(P_{r-1}, Q_{r-1}) = 1$ because $A_r \geq 1$.

Substitution in (2.3) gives

$$U_j = (K - hp^{a_{r+1}} K') U_i + p^{A_r + a_{r+1}} K' U_{i-1}.$$

Comparison with (2.2), whose first coefficient is at least as large as its second one, yields

$$\frac{K}{K'} \geq p^{a_{r+1}} (h + p^{A_r}) \geq p^{A_{r+1}}. \tag{2.5}$$

If $s = r + 1$, this contradicts $K/K' = b_{r+1} \leq p - 1$. Hence $s \geq r + 2$.

If $A_{r+1} = 1$, then $a_{r+2} \geq 1 = A_{r+1}$. Assume that $A_{r+1} \geq 2$. If $a_{r+2} \leq A_{r+1} - 1$, then by (2.4)

$$\frac{K}{K'} \leq p - 1 + p^{A_{r+1} - 1} < p^{A_{r+1}},$$

contrary to (2.5). Thus $a_{r+2} \geq A_{r+1}$.

If $r \geq 1$, then $h \geq 1$, and (2.5) gives

$$\frac{K}{K'} \geq p^{A_{r+1}} + p^{a_{r+1}}.$$

The assumption $a_{r+2} \leq A_{r+1}$ would give

$$\frac{K}{K'} \leq p - 1 + p^{A_{r+1}} < p^{A_{r+1}} + p^{a_{r+1}},$$

which is impossible. Therefore $a_{r+2} \geq A_{r+1} + 1$ when $r \geq 1$.

If $r = 0$, then

$$A_s \geq A_2 = A_1 + a_2 \geq 2A_1 \geq 2.$$

If $r \geq 1$, then

$$A_s \geq A_{r+2} = A_{r+1} + a_{r+2} \geq 2A_{r+1} + 1 \geq 2A_r + 3. \qquad \square$$

Proposition 2.2. *Let $\alpha = [b_0, p^{a_1} : b_1, \ldots, p^{a_n} : b_n]_p$ be a positive rational p-adic unit with finite Schneider's expansion. Then*

$$C_p(\alpha) \leq \left\lfloor \log_2 \left(\frac{4(A_n + 3)}{5} \right) \right\rfloor.$$

In particular, for every $H \geq 1$,

$$M_p(H) \leq \left\lfloor \log_2 \left(\frac{4(2 \log_p H + 3)}{5} \right) \right\rfloor.$$

Proof. List the common convergents in their ordinary order as

$$\frac{u_{i_1}}{v_{i_1}}, \ldots, \frac{u_{i_t}}{v_{i_t}} = \alpha, \qquad i_1 < \cdots < i_t,$$

and let $r_1, \ldots, r_t$ be the corresponding indices in Schneider's expansion. The numerator sequences in both expansions are strictly increasing at nonnegative indices, so

$$r_1 < \cdots < r_t = n.$$

If $t = 1$, then the claimed bound follows from $A_n \geq 0$. Suppose $t \geq 2$. Lemma 2.1 gives $A_{r_2} \geq 2$. Since $r_2 \geq 2$, the stronger part of the lemma applies at every subsequent step. Induction yields

$$A_{r_\ell} \geq 5 \cdot 2^{\ell - 2} - 3 \qquad (2 \leq \ell \leq t). \tag{2.6}$$

In particular,

$$2^t \le \frac{4(A_n+3)}{5},$$

which proves the estimate.

The final convergent in Schneider's expansion is $\alpha = P_n/Q_n$. By (2.1) and monotonicity,

$$p^{A_n} = |P_n Q_{n-1} - P_{n-1}Q_n| \le H(\alpha)^2.$$

Thus $A_n \le 2\log_p H(\alpha)$. Taking the maximum over all admissible α proves the second assertion. $\square$

Proposition 2.3. *Let $\alpha = [b_0, p^{a_1} : b_1, \ldots, p^{a_n} : b_n]_p$ be a positive rational p-adic unit with finite Schneider's expansion. If $t = C_p(\alpha) \ge 2$, then*

$$H(\alpha) \ge Q_n \ge p^{5\cdot 2^{t-2}-2t}.$$

Proof. Let $r_1 < \cdots < r_t = n$ and $i_1 < \cdots < i_t = N$ be the indices of the common convergents in Schneider's expansion and in the normalized ordinary continued fraction. For $1 \le \ell < t$, write

$$U_{i_{\ell+1}} = R_\ell U_{i_\ell} + S_\ell U_{i_\ell - 1}$$

as in (2.2), and let K_ℓ, K'_ℓ be the corresponding integers in (2.3). The calculation in the proof of Lemma 2.1 gives $S_\ell = p^{A_{r_\ell} + a_{r_\ell+1}} K'_\ell$. Since $R_\ell \ge S_\ell$ and $K'_\ell \ge 1$,

$$v_{i_{\ell+1}} \ge p^{A_{r_\ell+1}} v_{i_\ell}.$$

The exponent for $\ell = 1$ is at least 1. For $\ell \ge 2$, (2.6) gives $A_{r_\ell+1} \ge 5\cdot 2^{\ell-2} - 2$. Hence, with an empty sum when $t = 2$,

$$H(\alpha) \ge Q_n = v_{i_t} \ge p^{1+\sum_{\ell=2}^{t-1}(5\cdot 2^{\ell-2}-2)} v_{i_1} \ge p^{5\cdot 2^{t-2}-2t}. \qquad \square$$

3. An extremal family

We use Legendre's theorem in the following form, see for example [1, Theorem 8.26]. If a/b is a reduced positive rational number and

$$0 < \left| x - \frac{a}{b} \right| < \frac{1}{2b^2}, \tag{3.1}$$

then a/b is an ordinary convergent of x. A rational number has two finite ordinary continued fraction representations, which differ only at the final partial quotient. These representations have all proper convergents of the normalized expansion in common. Hence every ordinary convergent of a/b is also an ordinary convergent of x.

For $n \geq 1$, put

$$T_{p,n} = [1, p : 1, p^n : 1]_p = \frac{p^n + p + 1}{p^n + 1}.$$

The displayed numerator and denominator are coprime. If

$$\theta = [b_0, p^{a_1} : b_1, \ldots, p^{a_\ell} : 1]_p,$$

let $F_\theta(x)$ be the linear fractional expression obtained by replacing the final digit 1 by x. We retain the integral coefficients supplied by the convergent recurrence:

$$F_\theta(x) = \frac{P_{\ell-1}x + p^{a_\ell}P_{\ell-2}}{Q_{\ell-1}x + p^{a_\ell}Q_{\ell-2}} = \frac{Ax + B}{Cx + D}.$$

If $\theta = a/b$ is reduced, then

$$A + B = a, \qquad C + D = b. \tag{3.2}$$

For $\theta = 1$, set $F_\theta(x) = x$.

Define

$$\xi_1 = 1, \qquad n_1 = 1,$$
$$n_m = 5 \cdot 2^{m-2} - 1 \quad (m \geq 2), \qquad \xi_{m+1} = F_{\xi_m}(T_{p,n_m}).$$

For $m \geq 2$, equivalently,

$$\xi_m = [1, p : 1, p^{n_1} : 1, p : 1, p^{n_2} : 1, \ldots, p : 1, p^{n_{m-1}} : 1]_p.$$

Proposition 3.1. *For every prime p and every $m \geq 1$, the numbers $\xi_1, \ldots, \xi_m$ are common ordinary and Schneider's convergents of ξ_m, and $C_p(\xi_m) = m$.*

For $m \geq 2$, the sum of the exponents in Schneider's expansion of ξ_m is $5 \cdot 2^{m-2} - 3$, and

$$H(\xi_m) < 3p^{5 \cdot 2^{m-2} - m - 2} < p^{5 \cdot 2^{m-2} - m}.$$

Proof. The assertion is immediate for $m = 1$. Also,

$$\xi_2 = T_{p,1} = \frac{2p + 1}{p + 1} = [1; 1, p],$$

so ξ_1 and ξ_2 are common convergents of ξ_2.

Let $m \geq 2$ and write

$$F_{\xi_m}(x) = \frac{A_m x + B_m}{C_m x + D_m}, \qquad \Delta_m = A_m D_m - B_m C_m.$$

Since

$$[1, p : 1, p^n : x]_p = \frac{(p+1)x + p^n}{x + p^n},$$

composition gives

$$\Delta_{m+1} = p^{n_m+1}\Delta_m.$$

As $\Delta_2 = p^2$, induction gives

$$\Delta_m = p^{5\cdot 2^{m-2}-3}, \qquad p^{n_m} = p^2\Delta_m \quad (m \geq 2). \tag{3.3}$$

Write $\xi_m = \tilde{a}_m/\tilde{b}_m$ in lowest terms so that $\tilde{b}_m = C_m + D_m$. Since $T_{p,n_m} > 1$ and $C_m, D_m > 0$, we have $C_m T_{p,n_m} + D_m > C_m + D_m = \tilde{b}_m$. From

$$F_{\xi_m}(x) - F_{\xi_m}(1) = \frac{\Delta_m(x-1)}{(C_m x + D_m)(C_m + D_m)},$$

we obtain, by substituting T_{p,n_m} that

$$\begin{aligned}\xi_{m+1} - \xi_m = F_{\xi_m}(T_{p,n_m}) - F_{\xi_m}(1) &= \frac{p\Delta_m}{(p^{n_m}+1)(C_m T_{p,n_m} + D_m)\tilde{b}_m} \\ &< \frac{p\Delta_m}{p^{n_m}\tilde{b}_m^2} = \frac{1}{p\tilde{b}_m^2} \leq \frac{1}{2\tilde{b}_m^2},\end{aligned}$$

where we used (3.3).

The displayed difference is positive, so $\xi_{m+1} > \xi_m$. By (3.1), every ordinary convergent of ξ_m remains an ordinary convergent of ξ_{m+1}. Schneider's expansion of ξ_{m+1} is obtained by replacing the final digit 1 in the expansion of ξ_m by the block $[1, p : 1, p^{n_m} : 1]_p$. Hence every Schneider's convergent of ξ_m remains a Schneider's convergent of ξ_{m+1}. Since ξ_{m+1} itself is a common convergent, induction shows that $\xi_1, \ldots, \xi_m$ are distinct common convergents of ξ_m.

For $m \geq 2$, the sum of the exponents in Schneider's expansion of ξ_m is

$$2 + \sum_{j=2}^{m-1} 5 \cdot 2^{j-2} = 5 \cdot 2^{m-2} - 3.$$

Proposition 2.2 therefore gives

$$C_p(\xi_m) \leq \left\lfloor \log_2\left(\frac{4(5\cdot 2^{m-2})}{5}\right)\right\rfloor = m.$$

The reverse inequality was proved above, so $C_p(\xi_m) = m$.

It remains to estimate the height. Since

$$\sum_{j=1}^{m-1} n_j = 5 \cdot 2^{m-2} - m - 2,$$

and $H(\xi_{j+1}) \leq (p^{n_j} + p + 1)H(\xi_j)$ is immediate for $j = 1$ and follows from (3.2) for $j \geq 2$, we have

$$H(\xi_m) \leq p^{5 \cdot 2^{m-2} - m - 2} \prod_{j=1}^{m-1} \left(1 + \frac{p+1}{p^{n_j}}\right).$$

The factor with $j = 1$ is at most $5/2$, and the factor with $j = 2$, if present, is at most $19/16$. Since $(p+1)/p^n \leq 3/2^n$ for $p \geq 2$ and $n \geq 2$, and since $n_j \geq 2j + 3$ for $j \geq 3$, we have

$$\sum_{j=3}^{\infty} \frac{p+1}{p^{n_j}} \leq \sum_{j=3}^{\infty} \frac{3}{2^{2j+3}} = \frac{1}{128}.$$

Hence

$$\prod_{j=3}^{m-1} \left(1 + \frac{p+1}{p^{n_j}}\right) \leq \frac{1}{1 - \sum_{j=3}^{m-1} \frac{p+1}{p^{n_j}}} \leq \frac{128}{127}.$$

Together with the factors corresponding to $j = 1, 2$, this gives

$$\prod_{j=1}^{m-1} \left(1 + \frac{p+1}{p^{n_j}}\right) \leq \frac{5}{2} \cdot \frac{19}{16} \cdot \frac{128}{127} = \frac{380}{127} < 3.$$

Consequently,

$$H(\xi_m) < 3p^{5 \cdot 2^{m-2} - m - 2} < p^{5 \cdot 2^{m-2} - m}.$$

This proves both stated height estimates. □

Proof of Theorem 1.1. The upper bound is Proposition 2.2. Put $x = \log_p H$ and

$$m = \left\lfloor \log_2 \left(\frac{4(x+3)}{5} \right) \right\rfloor.$$

The assumption $H \geq 2p + 1$ gives $m \geq 1$. If $m = 1$, use $\xi_1 = 1$. If $m = 2$, then $H(\xi_2) = 2p + 1 \leq H$. Suppose $m \geq 3$. The definition of m gives

$$5 \cdot 2^{m-2} - 3 \leq x.$$

By Proposition 3.1,

$$H(\xi_m) \leq p^{5 \cdot 2^{m-2} - m} \leq p^{5 \cdot 2^{m-2} - 3} \leq H,$$

and $C_p(\xi_m) = m$. This proves the lower bound.

Finally,

$$1 \leq \frac{2x+3}{x+3} < 2.$$

The logarithms of the two arguments in the displayed bounds of the theorem therefore differ by less than 1. Their floors differ by at most one, and the asymptotic assertion follows. □

Corollary 3.2. *Let $m \geq 2$ and $H \geq 1$. Then*

$$H < p^{5\cdot 2^{m-2}-2m} \implies M_p(H) \leq m-1, \qquad H \geq p^{5\cdot 2^{m-2}-m} \implies M_p(H) \geq m.$$

In particular, for $H \geq p^{32}$, we have $M_p(H) \leq \log_2 \log_p H$.

Proof. The function $5 \cdot 2^{t-2} - 2t$ is increasing for integers $t \geq 2$. Thus the first implication follows from Proposition 2.3 by contraposition, and the second follows from Proposition 3.1.

Put $t = M_p(H)$. Since $H \geq p^{32}$, we have $\log_2 \log_p H \geq 5$, so the desired estimate is immediate if $t \leq 5$. If $t \geq 6$, Proposition 2.3 gives $\log_p H \geq 5 \cdot 2^{t-2} - 2t \geq 2^t$, since $2^{t-2} \geq 2t$. Hence $t \leq \log_2 \log_p H$. □

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