WORKSHOP "DIRAC OPERATORS AND REPRESENTATION THEORY"

ZAGREB, 18.-22.6.2018

Dan Barbasch: Admissible modules and closures of nilpotent orbits

In the case of classical complex groups, we find admissible $(\mathfrak{g}, K)$-modules which have K-structure equal to the structure of regular functions on the closure of a nilpotent orbit. This has consequences as to which nilpotent orbits have normal closures.

Leticia Barchini: Reducible characteristic cycle of Harish-Chandra modules for $U(p, q)$

The purpose of the talk is to describe the first examples of reducible characteristic cycle for irreducible modules of $U(p, q)$. In category $\mathcal{O}$, Kashiwara-Saito found an example of reducible characteristic cycle by analyzing the singularity of an appropriate Schubert variety. The relevant singularity is eight dimensional. The Kashiwara-Saito singularity also explains an example by Williamson of a highest weight module with reducible associated variety (and leading term cycle). Our examples are defined in terms of $\mathrm{GL}(p) \times \mathrm{GL}(q)$ orbits on the flag variety of $\mathrm{GL}(p + q, \mathbb{C})$. We identify a singular $\mathrm{GL}(4) \times \mathrm{GL}(4)$ orbit closure that leads to a $U(4, 4)$ module with reducible characteristic cycle. The relevant singularity controlling this example is a four-dimensional subvariety of $\mathbb{C}^8$. We find a related example for $(p, q) = (6, 6)$ having the more subtle property reducible leading term cycle. The talk is based on joint work with Petr Somberg and Peter Trapa

Dan Ciubotaru: Dirac cohomology for rational Cherednik algebra at t=0 and Calogero-Moser cells

I will explain the definition and main properties of the local Dirac operator for rational Cherednik algebras at $t = 0$ and the connection between Vogan's Dirac morphism in this setting and the partitioning of irreducible representations of a complex reflection group into Calogero-Moser families introduced by Gordon. I will then present some progress towards the verification of a conjecture by Gordon, Martino, and Rouquier, relating these cells to Kazhdan-Lusztig families. The talk is partly based on joint work with Marcelo De Martino.

Hendrik De Bie: Introduction to Bannai-Ito and Racah algebras

The Racah algebra was originally introduced as the algebra underlying the Racah polynomials. It turns out to have a strong connection with the generic superintegrable system on the unit sphere, which allowed to generalize the Racah algebra to arbitrary rank. Similarly, the Bannai-Ito algebra was introduced to describe the Bannai-Ito polynomials. Soon after, it was reinterpreted and generalized as the symmetry algebra of a deformed Dirac equation. In this talk I will review both algebras, focussing in particular on their construction and on their connection with quantum superintegrable models.

Marcelo De Martino: Dirac operators for rational Cherednik algebras

In this joint work with D. Ciubotaru, we introduce the notions of local and global indices of Dirac operators for a rational Cherednik algebra H, with underlying reflection group G. In the local theory, I will report on relations between the (local) Dirac index of a simple module in category $\mathcal{O}$, the graded G-character and the composition series polynomials for standard modules. In the global theory, we introduce an "integral-reflection" module over which we define and compute the index of a (global) Dirac operator and show that the index is independent of the parameters. If time permits, I will discuss some local-global relations.

Chaoping Dong: Spin norm: combinatorics and representations

We will introduce the following three preprints in 2017: arXiv:1702.01876 (joint with Jian Ding), arXiv:1707.01380 and arXiv:1708.00383. As a consequence, for a real reductive Lie group $G(\mathbb{R})$ in the sense of [Adams, van Leeuwen, Trapa and D. Vogan, arXiv:1212.2192], we will report a finiteness theorem for the classification of all the irreducible unitary Harish-Chandra modules (up to equivalence) for $G(\mathbb{R})$ with non-zero Dirac cohomology. In particular, when the rank of $G(\mathbb{R})$ is small, we should be able to obtain the complete classification via atlas.

Karmen Grizelj: Strengthening of the Dirac inequality

Dirac operators were introduced into representation theory by Parthasarathy. One of his results was "Dirac inequality", which connects the infinitesimal character and K-types of a unitary $(\mathfrak{g}, K)$-module. In 1997., Vogan introduced the algebraic notion of Dirac cohomology of Harish-Chandras modules and formulated a conjecture that Dirac cohomology, if not 0, determines the infinitesimal character of the representation. This conjecture was proved by Huang and Pandžić. We can think of it as a strengthening of the Dirac inequality: If M is a unitary $(\mathfrak{g}, K)$-module, and if in the tensor product of M with the spin module S there exists a K-tipe E on which the Dirac inequality becomes equality, then the infinitesimal character of M is the same as the K-infinitesimal character of the K-type E.

The Salamanca-Vogan conjecture states that the infinitesimal character of a unitary module M such that the tensor product of M and S contains E lies in the convex hull of the Weyl group translates of E. In this talk I will present a strategy for a possible proof of this conjecture in some cases, using an idea of Pandžić and Renard.

Jing-Song Huang: Dirac cohomology, orbit method and unipotent representations

The method of coadjoint orbits for real reductive groups is divided into three steps in cooperation with the Jordan decomposition of a coadjoint orbit into hyperbolic part, elliptic part and nilpotent part. This is formulated in Vogan's 1986 ICM plenary speech. The hyperbolic step and elliptic step are well understood, while the nilpotent step to construct unipotent representations in correspondence with nilpotent orbits has been extensively studied in several different perspectives over the last thirty years. Still, the final definition of unipotent representations remains to be mysterious. The aim of this talk is to show that our recent work (joint with Pandžić and Vogan) on classifying unitary representations by their Dirac cohomology shed light on what kind of irreducible unitary representations should be defined as unipotent.

Denis Husadžić: Singular BGG complexes over isotropic 2-Grassmannian

We construct exact sequences of invariant differential operators acting on sections of certain homogeneous vector bundles in singular infinitesimal character, over the isotropic 2-Grassmannian. This space is equal to G/P, where G is $\mathrm{Sp}(2n,\mathbb{C})$, and P its standard parabolic subgroup having the Levi factor $\mathrm{GL}(2,\mathbb{C}) \times \mathrm{Sp}(2n-4,\mathbb{C})$. The constructed sequences are analogues of the Bernstein-Gelfand-Gelfand resolutions. We do this by considering the Penrose transform over an appropriate double fibration. The result differs from the Hermitian situation. This is joint work with Rafael Mrđen

Domagoj Kovačević: On unitary representations of disconnected real reductive groups.

Let G be a real reductive group and let G_0 be the identity component. Let us assume that the unitary dual $\widehat{G_0}$ is known. In this talk, the unitary dual $\widehat{G}$ is constructed. Automorphisms of G_0 generated by elements of G are the main ingredient of the construction. If the automorphism is outer, one has to consider the corresponding intertwining operators S. We analyze operators S and their properties. We also investigate automorphisms of the Lie algebra g_0.

Roman Lávička: Separation of variables and generalized Verma modules

Separation of variables is well-understood e.g. for scalar-valued polynomials in k variables on the m-dimensional Euclidean space with the underlying symmetry given by the orthogonal group $O(m)$. This is the well-known decomposition of polynomials into classical spherical harmonics. The hidden symmetry is given by the symplectic algebra $\mathfrak{sp}(2k)$ of invariant differential operators. The pair $(O(m), \mathfrak{sp}(2k))$ is an example of the so-called Howe duality.

It is well-known that the decomposition of polynomials into spherical harmonics is unique in the stable range, that is, when m is not less than $2k$. We explain that uniqueness of the decomposition is equivalent to irreducibility of generalized Verma modules for the Howe dual partner $\mathfrak{sp}(2k)$ generated by spherical harmonics. This simple observation leads to an alternative proof and an extension of the result to a semistable range when m is not less than $2k-1$.

This approach seems to be very flexible and to work well in other settings. We apply it to spinor valued polynomials that decompose into spherical monogenics.

Here spherical monogenics are polynomial solutions of the system of Dirac equations with respect to all vector variables.

The talk is based on results obtained jointly with V. Souček.

Dragan Miličić: A generalization of orthogonality relations

We will discuss a generalization of Harish-Chandra's character orthogonality relations for discrete series to arbitrary Harish-Chandra modules for real reductive Lie groups. This result is an analogue of a conjecture by Kazhdan for p-adic reductive groups proved by Bezrukavnikov, and Schneider and Stuhler.

This is a joint work with Jing-Song Huang and Binyong Sun.

Pierluigi Möseneder Frajria: Affine Dirac operator for the pair $(\mathfrak{g}, \mathfrak{h})$

We will describe the affine analogue of the Dirac operator introduced by Kac and Todorov. We will discuss its formal properties and give a proof of a statement analogous to Vogan's conjecture in the special case of the pair $(\mathfrak{g}, \mathfrak{h})$ where $\mathfrak{g}$ is a simple Lie algebra and $\mathfrak{h}$ a Cartan subalgebra.

Kyo Nishiyama: Functional equation of an enhanced zeta distribution

We consider a prehomogeneous vector space of direct sum of the space of symmetric matrices $V = Sym_n$ and its representation space $E = M_{n,d}$. As it is prehomogeneous, there are only finitely many orbits, and a b-function can be defined.

Over real, we take an open orbit whose first component consists of positive symmetric matrices, and call it an "enhanced symmetric cone".

In this talk, we introduce an enhanced zeta distribution which is a complex power of relative invariants restricted to the enhanced symmetric cone. Here we emphasize that there are two fundamental relative invariants so that our zeta distribution has two complex parameters. We discuss its analytic continuation, Fourier transform and functional equations.

This is on-going joint work with Bent Ørsted and Akihito Wachi.

Eric Opdam: Unramified local Langlands parameters and the Plancherel measure

I will report on the ongoing work of several mathematicians (Solleveld, Feng, Opdam, building on work of Reeder, Ciubotaru, S. Kato, Hiraga, Ichino, Ikeda,...) to study local Langlands parameters for tempered representations "of unipotent reduction" from the perspective of the Plancherel formula. The goal is to show that there exists an essentially unique local Langlands parameterization for such representations for which the conjectures of Hiraga, Ichino and Ikeda on the Plancherel measure of a p-adic reductive group (split over an unramified extension) hold.

Pavle Pandžić: Dirac cohomology for D-modules

We study the category of B-equivariant D-modules on G/K, which is equivalent to the more usual category of K-equivariant D-modules on G/B. We introduce the notion of Dirac cohomology for modules in this category. We compute the Dirac cohomology for all irreducible modules in case when G is $SL(2, \mathbb{C})$, K is the diagonal torus, and B is the upper triangular matrices. This is joint work with Wolfgang Sörgel.

Ana Prlić: Classification of $A_q(\lambda)$ modules by their Dirac cohomology for type D and $\mathfrak{sp}(2n,\mathbb{R})$

Let G be a connected real reductive group with maximal compact subgroup K of the same rank as G. In the recent paper of Huang, Pandžić and Vogan, it was shown that the admissible Θ–stable parabolic subalgebras $\mathfrak{q}$ of $\mathfrak{g}$ are in one-to-one correspondence with the faces of $W\rho$ intersecting the $\mathfrak{k}$–dominant Weyl chamber and that $A_\mathfrak{q}(0)$–modules can be classified by their Dirac cohomology in geometric terms. We will describe faces corresponding to $A_\mathfrak{q}(0)$ for $\mathfrak{g}_0 = \mathfrak{sp}(2n, \mathbb{R}$ and for $\mathfrak{g}_0$ of type D.

David Renard: Arthur's packets for classical real groups

Arthur's packets were introduced to describe the discrete automorphic spectrum of reductive groups. They consist in unitary representations. For classical groups, they were characterized by endoscopic character identities. We will give an explicit construction of these packets in a three steps process, starting from unipotent packets, then using cohomological induction, and finally parabolic induction. This is joint work with Colette Moeglin.

Tomáš Salač: The domains of monogenicity for the n-Cauchy-Fueter operator and twistor theory

The n-Cauchy-Fueter operator is an overdetermined linear differential operator of first order which is natural to the flat almost quaternionic structure on $\mathbb{H}^n$ and which plays the role of the n-Cauchy-Riemann operator in the analysis of functions of several quaternionic variables. I will talk about the construction of a resolution of this operator and explain how this can be used to show that any pseudoconvex domain in $\mathbb{Q}^n$ is a domain of monogenicity.

Petr Somberg: Bernstein-Sato identities and conformal symmetry breaking operators

We present Bernstein-Sato identities for scalar-, spinor- and differential form-valued distribution kernels on Euclidean space associated to conformal symmetry breaking operators. The associated Bernstein-Sato operators lead to (partially) new formulas for conformal symmetry breaking differential operators on functions, spinors and differential forms.

Vladimír Souček: Example of a singular BGG complex in a non-Hermitian symmetric case.

The structure of BGG complexes in singular infinitesimal character is well understood for Hermitian symmetric cases due to the work of Enright and Shelton. The structure of the complex depends on the level of the singularity but not on a particular choice of the walls involved. However, specific properties of invariant differential operators involved in the complex varies significantly with the choice of the walls. Not too much is known outside the Hermitian symmetric case. In the lecture, a specific case of isotropic flag manifolds G/P, with $G = \mathrm{Spin}(2n)$ and P a maximal parabolic (different from the conformal case) will be discussed. It will be shown that using the ideas of the Penrose transform, it is possible to construct the full BGG complex in singular infinitesimal character of a highest possible level. The results are coming from the joint work with L. Krump and T. Salač.

Vít Tuček: Nilpotent cohomology of unitarizable highest weight modules

The unitarizable highest weight modules for a semisimple Lie algebra exist only in a very special case when G/K is a Hermitian symmetric space. We will review their classification and Enright's formula for their nilpotent cohomology, which in this case is isomorphic to the Dirac cohomology up to modular twists. We will present calculations in several cases and application in differential geometry.

Petr Zima: Killing equations and eigenvalues of Dirac operators

By a famous result of Thomas Friedrich, Killing spinors are closely related to certain eigenvalue estimate for the Dirac operator on compact spin manifolds. The estimate is based on the so called Bochner-Weitzenbock formula which relates suitable second order differential operators to the curvature of the underlying manifold. In this talk we show to some extent how these notions generalize to twisted and higher spin Dirac operators. In particular, we will focus on spinor-valued 1-forms and the Rarita-Schwinger operator. Unfortunately the situation is more complicated in this case and the eigenvalue estimate is not generally available. As the simplest example we consider the round sphere where explicit formulas are known.