- $\delta = \delta_0$
- $\mathrm{rect}_{(\alpha,\beta)}(x) = \chi_{[\alpha,\beta]}(x)$
- $\mathrm{pulse}_\alpha(x) = \mathrm{rect}_{(-\alpha,\alpha)}(x)$
- $\mathrm{step}(x) = \mathrm{rect}_{(0,+\infty)}(x) = H(x)$

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- $\mathrm{sinc}(x) = \dfrac{\sin x}{x}$
- $D^k f(x) = f^{(k)}(x)$
- $\mathrm{tri}(x) = \begin{cases} 1+x, & -1 < x \le 0 \\ 1-x, & 0 < x < 1 \\ 0, & \text{inače} \end{cases}$
- $\mathrm{pole}(x) = \mathrm{pv}\left(\frac{1}{x}\right)$

OPREZ :

1) $a \in \mathbb{R}$, $\dfrac{1}{x-a} = \tau_a \mathrm{pv}\left(\frac{1}{x}\right)$

2) $a \in \mathbb{C}$, $\mathrm{Im}\, a \neq 0$

$$\frac{1}{x-a} \neq \tau_a \mathrm{pv}\left(\frac{1}{x}\right)$$

- $T_\alpha f = \tau_\alpha f$
- $\mathrm{ramp}(x) = \begin{cases} 0, & x < 0 \\ x, & x \ge 0 \end{cases}$
- $\mathrm{comb}_\alpha(x) = \sum_{k=-\infty}^{\infty} \delta_{\alpha t}$

TABLE A.1
Fourier Transforms of Some Common Functions

In the following:

*α, β, and γ all denote **real** numbers.*

$f(t)$	$F(\omega) = \mathcal{F}[f(t)]\vert_\omega$	Restrictions
$\mathrm{pulse}_\alpha(t)$	$2\alpha\ \mathrm{sinc}(2\pi\alpha\omega)$	$0 < \alpha$
$\mathrm{rect}_{(\alpha,\beta)}(t)$	$\frac{i}{2\pi\omega}\left[e^{-i2\pi\beta\omega} - e^{-i2\pi\alpha\omega}\right]$	$\alpha \le \beta$
$\mathrm{tri}(t)$	$\mathrm{sinc}^2(\pi\omega)$	none
$\cos\left(\frac{\pi}{2\alpha}t\right)\mathrm{pulse}_\alpha(t)$	$\alpha\left[\mathrm{sinc}\left(2\pi\alpha\omega + \frac{\pi}{2}\right) + \mathrm{sinc}\left(2\pi\alpha\omega - \frac{\pi}{2}\right)\right]$	$0 < \alpha$
$\mathrm{sinc}(2\pi\alpha t)$	$\frac{1}{2\alpha}\mathrm{pulse}_\alpha(\omega)$	$0 < \alpha$
$\mathrm{sinc}^2(2\pi\alpha t)$	$\frac{1}{2\alpha}\mathrm{tri}\left(\frac{\omega}{2\alpha}\right)$	$0 < \alpha$
$e^{-(\alpha+i\beta)t}\ \mathrm{step}(t)$	$\frac{1}{\alpha + i\beta + i2\pi\omega}$	$0 < \alpha$
$t^k e^{-(\alpha+i\beta)t}\ \mathrm{step}(t)$	$\frac{k!}{(\alpha + i\beta + i2\pi\omega)^{k+1}}$	$0 < \alpha$ $k = 0, 1, 2, \ldots$
$e^{(\alpha+i\beta)t}\ \mathrm{step}(-t)$	$\frac{1}{\alpha + i\beta - i2\pi\omega}$	$0 < \alpha$
$t^k e^{(\alpha+i\beta)t}\ \mathrm{step}(-t)$	$(-1)^k\frac{k!}{(\alpha + i\beta - i2\pi\omega)^{k+1}}$	$0 < \alpha$ $k = 0, 1, 2, \ldots$
$\frac{1}{\alpha + i\beta + i\gamma t}$	$\frac{2\pi}{\gamma}e^{2\pi(\alpha+i\beta)\omega/\gamma}\ \mathrm{step}(-\omega)$	$0 < \alpha, 0 < \gamma$
$\frac{1}{(\alpha + i\beta + it)^{k+1}}$	$\frac{2\pi}{k!}(2\pi\omega)^k e^{2\pi(\alpha+i\beta)\omega}\ \mathrm{step}(-\omega)$	$k = 0, 1, 2, \ldots$

$f(t)$	$F(\omega) = \mathcal{F}[f(t)]\|_\omega$	Restrictions
$\dfrac{1}{\alpha + i\beta - i\gamma t}$	$\dfrac{2\pi}{\gamma} e^{-2\pi(\alpha+i\beta)\omega/\gamma}\ \text{step}(\omega)$	$0 < \alpha, 0 < \gamma$
$\dfrac{1}{(\alpha + i\beta - it)^{k+1}}$	$\dfrac{2\pi}{k!}(-2\pi\omega)^k e^{-2\pi(\alpha+i\beta)\omega}\ \text{step}(\omega)$	$0 < \alpha$ $k = 0, 1, 2, \ldots$
$e^{-\alpha\|t\|}$	$\dfrac{2\alpha}{\alpha^2 + 4\pi^2\omega^2}$	$0 < \alpha$
$\dfrac{1}{\alpha^2 + \gamma^2 t^2}$	$\dfrac{\pi}{\alpha\gamma} e^{-2\pi\alpha\|\omega\|/\gamma}$	$0 < \alpha, 0 < \gamma$
$e^{-\alpha t^2 + \beta t}$	$\sqrt{\dfrac{\pi}{\alpha}} \exp\left(-\dfrac{1}{4\alpha}(2\pi\omega + i\beta)^2\right)$	$0 < \alpha$
$e^{-\lambda t^2}$	$\left[\sqrt{\|\lambda\| + \alpha} - i\,\text{sgn}(\beta)\sqrt{\|\lambda\| - \alpha}\right] \times \dfrac{1}{\|\lambda\|}\sqrt{\dfrac{\pi}{2}} \exp\left(-\dfrac{\pi^2}{\lambda}\omega^2\right)$	$\lambda = \alpha + i\beta,$ $0 \leq \alpha$
$e^{-\alpha t^2}\cos\left(\beta t^2\right)$	$\dfrac{1}{\|\lambda\|}\sqrt{\dfrac{\pi}{2}} \exp\left(-\dfrac{\pi^2\alpha}{\|\lambda\|^2}\omega^2\right) \times \left[\sqrt{\|\lambda\| + \alpha}\cos\left(\dfrac{\pi^2\beta}{\|\lambda\|^2}\omega^2\right) + \sqrt{\|\lambda\| - \alpha}\sin\left(\dfrac{\pi^2\beta}{\|\lambda\|^2}\omega^2\right)\right]$	$\lambda = \alpha + i\beta,$ $0 \leq \alpha, 0 < \beta$
$e^{-\alpha t^2}\sin\left(\beta t^2\right)$	$\dfrac{1}{\|\lambda\|}\sqrt{\dfrac{\pi}{2}} \exp\left(-\dfrac{\pi^2\alpha}{\|\lambda\|^2}\omega^2\right) \times \left[\sqrt{\|\lambda\| - \alpha}\cos\left(\dfrac{\pi^2\beta}{\|\lambda\|^2}\omega^2\right) - \sqrt{\|\lambda\| + \alpha}\sin\left(\dfrac{\pi^2\beta}{\|\lambda\|^2}\omega^2\right)\right]$	$\lambda = \alpha + i\beta,$ $0 \leq \alpha, 0 < \beta$
$e^{\pm i\pi\alpha t^2}$	$\dfrac{1}{\sqrt{\alpha}} \exp\left(\mp i\pi\left[\dfrac{\omega^2}{\alpha} - \dfrac{1}{4}\right]\right)$	$\alpha > 0$
$\cos\left(\pi\alpha t^2\right)$	$\dfrac{1}{\sqrt{\alpha}} \cos\left(\pi\left[\dfrac{\omega^2}{\alpha} - \dfrac{1}{4}\right]\right)$	$\alpha > 0$
$\sin\left(\pi\alpha t^2\right)$	$-\dfrac{1}{\sqrt{\alpha}} \sin\left(\pi\left[\dfrac{\omega^2}{\alpha} - \dfrac{1}{4}\right]\right)$	$\alpha > 0$

$f(t)$	$F(\omega) = \mathcal{F}[f(t)]\|_\omega$	Restrictions
1	$\delta(\omega)$	none
t^k	$\left(\dfrac{i}{2\pi}\right)^k D^k\delta(\omega)$	$k = 0, 1, 2, \ldots$
$e^{i2\pi(\alpha+i\beta)t}$	$\delta_{\alpha+i\beta}(\omega)$	none
$\sin(2\pi\alpha t)$	$\dfrac{1}{2i}\left[\delta_\alpha(\omega) - \delta_{-\alpha}(\omega)\right]$	none
$\cos(2\pi\alpha t)$	$\dfrac{1}{2}\left[\delta_\alpha(\omega) + \delta_{-\alpha}(\omega)\right]$	none
$\text{step}(t)$	$\dfrac{1}{i2\pi} T_{-i}\left[\dfrac{1}{\omega - i}\right]$	none
$\text{step}(t)$	$\dfrac{1}{i2\pi}\,\text{pole}(\omega) + \dfrac{1}{2}\delta(\omega)$	none
$\text{step}(-t)$	$\dfrac{-1}{i2\pi} T_i\left[\dfrac{1}{\omega + i}\right]$	none
$\text{step}(-t)$	$\dfrac{-1}{i2\pi}\,\text{pole}(\omega) + \dfrac{1}{2}\delta(\omega)$	none
$\text{ramp}(t)$	$\dfrac{-1}{4\pi^2} T_{-i}\left[(\omega - i)^{-2}\right]$	none
$\text{ramp}(t)$	$\dfrac{-1}{4\pi^2}\left[\text{pole}^2(\omega) - i\pi D\delta(\omega)\right]$	none
$t^k\,\text{step}(t)$	$\left(\dfrac{1}{i2\pi}\right)^{k+1} k!\, T_{-i}\left[(\omega - i)^{-k-1}\right]$	$k = 0, 1, 2, \ldots$
$t^k\,\text{step}(t)$	$\left(\dfrac{-i}{2\pi}\right)^{k+1}\left[k!\,\text{pole}^{k+1}(\omega) + i(-1)^k\pi D^k\delta(\omega)\right]$	$k = 0, 1, 2, \ldots$
$t^k\,\text{step}(-t)$	$-\left(\dfrac{1}{i2\pi}\right)^{k+1} k!\, T_i\left[(\omega + i)^{-k-1}\right]$	$k = 0, 1, 2, \ldots$
$t^k\,\text{step}(-t)$	$\left(\dfrac{-i}{2\pi}\right)^{k+1}\left[-k!\,\text{pole}^{k+1}(\omega) + i(-1)^k\pi D^k\delta(\omega)\right]$	$k = 0, 1, 2, \ldots$

$f(t)$	$F(\omega) = \mathcal{F}[f(t)]\|_\omega$	Restrictions
$\operatorname{sgn}(t)$	$\frac{1}{i\pi} D \ln \|\omega\|$	none
$\operatorname{sgn}(t)$	$\frac{1}{i\pi} \operatorname{pole}(\omega)$	none
$\operatorname{pole}(t)$	$-i\pi \operatorname{sgn}(\omega)$	none
$\operatorname{pole}^k(t)$	$\frac{(-i2\pi)^k}{2(k-1)!} \omega^{k-1} \operatorname{sgn}(\omega)$	$k = 1, 2, \ldots$
$t^k \operatorname{sgn}(t)$	$\left(\frac{-i}{2\pi}\right)^{k+1} 2k! \operatorname{pole}^{k+1}(\omega)$	$k = 0, 1, 2, \ldots$
$\|t\|^n$	$\left(\frac{-i}{2\pi}\right)^{n+1} 2n! \operatorname{pole}^{n+1}(\omega)$	$n = 1, 3, 5, \ldots$
$\operatorname{comb}_\alpha(t)$	$\frac{1}{\alpha} \operatorname{comb}_{1/\alpha}(\omega)$	$0 < \alpha$
$\operatorname{saw}_\alpha(t)$	$\frac{\alpha}{2}\delta(\omega) + \sum_{k=-\infty, k\neq 0}^{\infty} \frac{i\alpha}{2\pi k} \delta_{k/\alpha}(\omega)$	$0 < \alpha$

TABLE A.2
Identities for the Fourier Transforms

In the following:

$\alpha =$ *any* ***real*** *number,* $F(\omega) = \mathcal{F}[f(t)]|_\omega$ *, and* $G(\omega) = \mathcal{F}[g(t)]|_\omega$

$h(t)$	$H(\omega) = \mathcal{F}[h(t)]\|_\omega$	Restrictions
$f(t)$	$\int_{-\infty}^{\infty} f(t)\, e^{-i2\pi\omega t}\, dt$	f in $\mathcal{A}$
$\int_{-\infty}^{\infty} F(\omega)\, e^{i2\pi\omega t}\, d\omega$	$F(\omega)$	F in $\mathcal{A}$
$f(\alpha t)$	$\frac{1}{\|\alpha\|} F\left(\frac{\omega}{\alpha}\right)$	$\alpha \neq 0$
$\frac{1}{\|\alpha\|} f\left(\frac{t}{\alpha}\right)$	$F(\alpha\omega)$	$\alpha \neq 0$
$f(t-\alpha)$	$e^{-i2\pi\alpha\omega} F(\omega)$	none
$\mathcal{T}_{\alpha+i\beta}[f(t)]$	$e^{-i2\pi(\alpha+i\beta)\omega} F(\omega)$	none
$e^{i2\pi\alpha t} f(t)$	$F(\omega - \alpha)$	none
$e^{i2\pi(\alpha+i\beta)t} f(t)$	$\mathcal{T}_{\alpha+i\beta}[F(\omega)]$	none
$\sin(2\pi\alpha t)\, f(t)$	$\frac{i}{2}[F(\omega+\alpha) - F(\omega-\alpha)]$	none
$\cos(2\pi\alpha t)\, f(t)$	$\frac{1}{2}[F(\omega+\alpha) + F(\omega-\alpha)]$	none
$\frac{1}{2}[f(t-\alpha) + f(t+\alpha)]$	$\cos(2\pi\alpha\omega)\, F(\omega)$	none
$\frac{i}{2}[f(t-\alpha) - f(t+\alpha)]$	$\sin(2\pi\alpha\omega)\, F(\omega)$	none
$\frac{df}{dt}$	$i2\pi\omega F(\omega)$	see chap. 22
$\frac{d^n f}{dt^n}$	$(i2\pi\omega)^n F(\omega)$	see chap. 22

$h(t)$	$H(\omega) = \mathcal{F}[h(t)]\|_{\omega}$	Restrictions
$tf(t)$	$\frac{i}{2\pi}\frac{dF}{d\omega}$	see chap. 22
$t^n f(t)$	$\left(\frac{i}{2\pi}\right)^n \frac{d^n F}{d\omega^n}$	see chap. 22
Df	$i2\pi\omega F(\omega)$	none
$D^n f$	$(i2\pi\omega)^n F(\omega)$	none
$tf(t)$	$\frac{i}{2\pi} DF$	none
$t^n f(t)$	$\left(\frac{i}{2\pi}\right)^n D^n F$	none
fg	$F * G$	yes, see chap. 24
$f * g$	FG	see chap. 24
$f^* g$	$F \star G$	see chap. 25
$f \star g$	$F^* G$	see chap. 25

Other Useful Identities:

For all transformable functions:

Near-equivalence:

$$\mathcal{F}^{-1}[\phi(x)]\|_y = \mathcal{F}[\phi(-x)]\|_y = \mathcal{F}[\phi(x)]\|_{-y}$$

and

$$\mathcal{F}[\phi(x)]\|_y = \mathcal{F}^{-1}[\phi(-x)]\|_y = \mathcal{F}^{-1}[\phi(x)]\|_{-y}$$

Under suitable conditions (see chap. 25):

Fundamental Identity:

$$\int_{-\infty}^{\infty} F(x)g(x)\,dx = \int_{-\infty}^{\infty} f(y)G(y)\,dy$$

Parseval's Identity:

$$\int_{-\infty}^{\infty} F(x)G^*(x)\,dx = \int_{-\infty}^{\infty} f(y)g^*(y)\,dy$$

Bessel's Identity:

$$\int_{-\infty}^{\infty} |F(x)|^2\,dx = \int_{-\infty}^{\infty} |f(y)|^2\,dy$$