# Computable type of certain quotient spaces

**Matea Čelar[1], Zvonko Iljazović[2]**

[1,2] *University of Zagreb, Faculty of Science, Department of Mathematics*
*Bijenička cesta 30, Zagreb, Croatia*
*E-mail:* [1] `matea.celar@math.hr`, [2] `zilj@math.hr`

**Keywords**:
Computable topological space, Computable set, Semicomputable set, Quotient space.

Topology plays an important role in determining the relationship between different levels of computability of sets in computable topological spaces. In particular, semicomputable sets with certain topological properties are necessarily fully computable. This is expressed in the notion of *computable type:* a space $A$ is said to have computable type if every semicomputable set homeomorphic to $A$ must be computable. Some known examples of spaces with computable type are topological manifolds, chainable and circulary chainable continua and finite graphs ([3, 2, 4]).

We explore computable type of quotients of Euclidean spaces, motivated by the known fact that both the pair $(B^n, S^{n-1})$ of the unit ball and its boundary and the quotient space $B^n/S^{n-1} \cong S^n$ have computable type ([1]). Our aim is to, given a (locally Euclidean) space $A$ with computable type, describe a subset $B$ (or, more generally, an equivalence relation on $A$) such that the corresponding quotient space has computable type. We will present some positive results related to this, as well as some interesting counterexamples.

## Acknowledgment

This work has been fully supported by Croatian Science Foundation under the project 7459 CompStruct.

## References

[1] Miller, J.S., *Effectiveness for Embedded Spheres and Balls*, Electronic Notes in Theoretical Computer Science 66:127–138, 2002.

[2] Čičković, E., Iljazović, Z. and Validžić, L., *Chainable and circularly chainable semicomputable sets in computable topological spaces*, Archive for Mathematical Logic 58:885–897, 2019.

[3] Iljazović, Z. and Sušić, I., *Semicomputable manifolds in computable topological spaces*, Journal of Complexity 45:83–114, 2018.

[4] Iljazović, Z., *Computability of graphs*, Mathematical Logic Quarterly 66:51–64, 2020.