Computable type of certain quotient spaces*

Matea Čelar and Zvonko Iljazović

Topological space A is said to have *computable type* if any semicomputable set in a computable topological space which is homeomorphic to A is necessarily computable. More generally, topological pair (A, B) has computable type if, whenever A is embedded in a computable topological space, semicomputability of images of A and B implies that the image of A is computable. Some known examples of spaces with computable type are topological manifolds, chainable and circulary chainable continua and finite graphs ([2, 4, 1]).

In this work, we consider the effect of quotients on preserving computable type, focusing primarily on locally Euclidean spaces. This is motivated by the known fact that both the pair (B^n, S^{n-1}) of the unit ball and its boundary and the quotient space $B^n/S^{n-1} \cong S^n$ have computable type ([3]).

It can be shown that if A/B has computable type, then (A, B) has computable type whenever the interor of B in A is empty. The contrary generally does not hold even when the interior of B in A is empty and A is a compact manifold. Therefore, our next main goal is to, given a (locally Euclidean) space A with computable type, describe a subset B (or, more generally, an equivalence relation on A) such that the corresponding quotient space has computable type. We will present some positive results related to this, as well as some interesting counterexamples.

References

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