

# Effective decompositions of continua with embedded arcs\*

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A continuum (compact and connected metric space) is said to be *decomposable* if it can be written as a union of two proper subcontinua. While it may seem that all nontrivial continua are decomposable, this is not the case; in fact, “most” continua are indecomposable [1]. Indecomposable continua naturally arise in various contexts in dynamical systems [2]. On the other hand, decomposability arises as a condition in the investigation of computable subcontinua of semicomputable chaniable continua [3].

In this talk, we consider continua endowed with the structure of a computable metric space and we investigate when a continuum is not merely decomposable, but expressible as a union of two *computable* proper subcontinua. We will call such continua *effectively decomposable*. Note that any effectively decomposable continuum is necessarily effectively compact. However, given an effectively compact continuum which is (topologically) decomposable, it is not obvious whether it is effectively decomposable, or in fact whether it contains a nontrivial computable proper subcontinuum.

Classically, a decomposable continuum can be characterised as a continuum which contains a proper subcontinuum with non-empty interior [4]. In the computable setting, an analogous proof gives the following result:

*An effectively compact continuum which contains a computable proper subcontinuum whose interior is non-empty and c.e. open is effectively decomposable.*

We are interested in versions of the above result where effective properties of the subcontinuum are replaced by purely topological ones. Particularly, we focus on continua which contain a copy of the unit segment  $[0, 1]$  as a subcontinuum with non-empty interior (or, equivalently, a copy of  $\mathbb{R}$  as an open subset).

In [5], it was shown that any point in a semicomputable set which has an open neighbourhood homeomorphic to  $\mathbb{R}$  also has a computable neighbourhood which is an arc with computable endpoints. Here, we build on this result to show that, whenever an effectively compact continuum contains (a homeomorphic copy of)  $\mathbb{R}$  as an open subset, it must be effectively decomposable. More precisely, we prove the following:



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**Theorem 1.** *Let  $(X, d, \alpha)$  be an effectively compact computable metric space. Suppose  $(X, d)$  is a continuum and there is an open subset of  $X$  homeomorphic to  $\mathbb{R}$ . Then  $X$  is effectively decomposable.*

Furthermore, we show that effectively compact, (classically) decomposable chainable continua are also effectively decomposable.

## References

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