

Seminar za funkcionalnu analizu

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The dual of the standard Hilbert C^* -module

Abstract

We will determine the dual $\ell^2(\mathbf{A})'$ of $\ell^2(\mathbf{A})$ for an arbitrary (not necessarily unital) C^* -algebra. It will be demonstrated that this dual is naturally embedded in a larger Hilbert C^* -module over a C^* -algebra that suitably extends \mathbf{A} .

In particular, when \mathbf{A} is a von Neumann algebra, this enables us to construct explicitly a self-dual Hilbert \mathbf{A} -module $\ell_{\text{strong}}^2(\mathbf{A})$ that is isometrically isomorphic to $\ell^2(\mathbf{A})'$, which contains $\ell^2(\mathbf{A})$, and whose \mathbf{A} -valued inner product extends the original inner product on $\ell^2(\mathbf{A})$. This serves as a concrete realization of a general construction of a self-dual structure on the dual X' for a Hilbert C^* -module X over a von Neumann algebra \mathbf{A} introduced by W. Paschke in the case $X = \ell^2(\mathbf{A})$.