Asocijacijske sheme

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22.1.2024.

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Moraju li Kreinovi parametri biti racionalni brojevi?

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Zadatak 1.35. (Patrikov zadatak)

Neka je $\mathscr{D}$ simetrični $(v, \Bbbk, \lambda)$ dizajn. Definiramo bipartitan graf G kojem su vrhovi točke i blokovi od $\mathscr{D}$, a susjedni su ako točka pripada bloku. Dokažite da je G distancijsko regularan graf dijametra $d = 3$, odredite mu presječni niz $\{b_0, b_1, b_2; c_1, c_2, c_3\}$ i presječne brojeve p_{ij}^k odgovarajuće asocijacijske sheme. Dolazi li svaka (metrička) asocijacijska shema s tim presječnim brojevima od simetričnog $(v, \Bbbk, \lambda)$ dizajna?

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Presječni niz:

$$\{k, k-1, k-\lambda; 1, \lambda, k\}$$

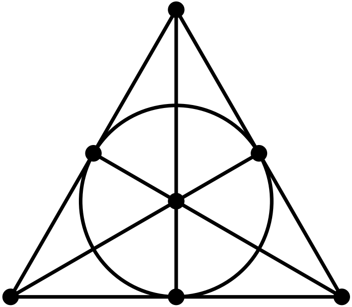
Teovo pitanje

Presječni brojevi:

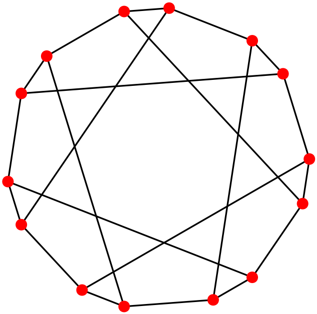
$$[p_{ij}^0] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & v-1 & 0 \\ 0 & 0 & 0 & v-k \end{bmatrix} \qquad [p_{ij}^1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & k-1 & 0 \\ 0 & k-1 & 0 & v-k \\ 0 & 0 & v-k & 0 \end{bmatrix}$$

$$[p_{ij}^2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \lambda & 0 & k-\lambda \\ 1 & 0 & v-2 & 0 \\ 0 & k-\lambda & 0 & v-2k+\lambda \end{bmatrix} \qquad [p_{ij}^3] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & k & 0 \\ 0 & k & 0 & v-k-1 \\ 1 & 0 & v-k-1 & 0 \end{bmatrix}$$

Heawoodov graf



Fanova ravnina



Heawoodov graf

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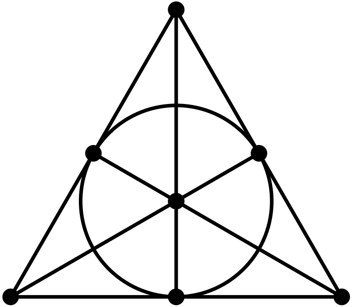
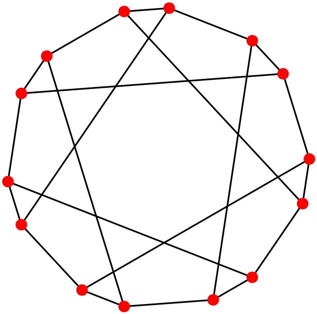


Fanova ravnina

Simetrični $(7, 3, 1)$ dizajn



Heawoodov graf

DRG s $n = 14$ i presječnim nizom $\{3, 2, 2;\ 1, 1, 3\}$

Heawoodov graf

Schurove idempotente:

$$A_0 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

Heawoodov graf

Schurove idempotente:

$$A_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Heawoodov graf

Schurove idempotente:

$$A_2 = \begin{bmatrix} 0&1&1&1&1&1&1&0&0&0&0&0&0&0\\ 1&0&1&1&1&1&1&0&0&0&0&0&0&0\\ 1&1&0&1&1&1&1&0&0&0&0&0&0&0\\ 1&1&1&0&1&1&1&0&0&0&0&0&0&0\\ 1&1&1&1&0&1&1&0&0&0&0&0&0&0\\ 1&1&1&1&1&0&1&0&0&0&0&0&0&0\\ 1&1&1&1&1&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&1&1&1&1&1\\ 0&0&0&0&0&0&0&1&0&1&1&1&1&1\\ 0&0&0&0&0&0&0&1&1&0&1&1&1&1\\ 0&0&0&0&0&0&0&1&1&1&0&1&1&1\\ 0&0&0&0&0&0&0&1&1&1&1&0&1&1\\ 0&0&0&0&0&0&0&1&1&1&1&1&0&1\\ 0&0&0&0&0&0&0&1&1&1&1&1&1&0 \end{bmatrix}$$

Heawoodov graf

Schurove idempotente:

$$A_3 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

Heawoodov graf

Presjesječni brojevi $p_{ij}^k = \dfrac{1}{n\,n_k}\,\mathrm{tr}(A_i A_j A_k^t)$:

$$[p_{ij}^0] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad [p_{ij}^1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

$$[p_{ij}^2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 5 & 0 \\ 0 & 2 & 0 & 2 \end{bmatrix} \qquad [p_{ij}^3] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

Heawoodov graf

Svojstvene vrijednosti i dualne svojstvene vrijednosti:

$$P = \begin{bmatrix} 1 & 3 & 6 & 4 \\ 1 & -3 & 6 & -4 \\ 1 & \sqrt{2} & -1 & -\sqrt{2} \\ 1 & -\sqrt{2} & -1 & \sqrt{2} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 1 & 6 & 6 \\ 1 & -1 & 2\sqrt{2} & -2\sqrt{2} \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \end{bmatrix}$$

Heawoodov graf

Primitivne idempotente:

$$E_0 = \frac{1}{14} \begin{bmatrix} 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix}$$

Heawoodov graf

Primitivne idempotente:

$$E_1 = \frac{1}{14}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Heawoodov graf

Primitivne idempotente:

$$E_2 = \frac{1}{14}\begin{bmatrix}
6 & -1 & -1 & -1 & -1 & -1 & -1 & 2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\
-1 & 6 & -1 & -1 & -1 & -1 & -1 & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\
-1 & -1 & 6 & -1 & -1 & -1 & -1 & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} \\
-1 & -1 & -1 & 6 & -1 & -1 & -1 & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} \\
-1 & -1 & -1 & -1 & 6 & -1 & -1 & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} \\
-1 & -1 & -1 & -1 & -1 & 6 & -1 & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} \\
-1 & -1 & -1 & -1 & -1 & -1 & 6 & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} \\
2\sqrt{2} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 6 & -1 & -1 & -1 & -1 & -1 & -1 \\
2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -1 & 6 & -1 & -1 & -1 & -1 & -1 \\
2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -1 & -1 & 6 & -1 & -1 & -1 & -1 \\
-\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -1 & -1 & -1 & 6 & -1 & -1 & -1 \\
-\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -1 & -1 & -1 & -1 & 6 & -1 & -1 \\
-\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -1 & -1 & -1 & -1 & -1 & 6 & -1 \\
-\frac{3}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & 2\sqrt{2} & 2\sqrt{2} & -\frac{3}{\sqrt{2}} & -1 & -1 & -1 & -1 & -1 & -1 & 6
\end{bmatrix}$$

Heawoodov graf

Primitivne idempotente:

$$E_3 = \frac{1}{14}\begin{bmatrix}
6 & -1 & -1 & -1 & -1 & -1 & -1 & -2\sqrt{2} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\
-1 & 6 & -1 & -1 & -1 & -1 & -1 & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\
-1 & -1 & 6 & -1 & -1 & -1 & -1 & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} \\
-1 & -1 & -1 & 6 & -1 & -1 & -1 & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} \\
-1 & -1 & -1 & -1 & 6 & -1 & -1 & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} \\
-1 & -1 & -1 & -1 & -1 & 6 & -1 & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} \\
-1 & -1 & -1 & -1 & -1 & -1 & 6 & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} \\
-2\sqrt{2} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & 6 & -1 & -1 & -1 & -1 & -1 & -1 \\
-2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -1 & 6 & -1 & -1 & -1 & -1 & -1 \\
-2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & -1 & -1 & 6 & -1 & -1 & -1 & -1 \\
\frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -1 & -1 & -1 & 6 & -1 & -1 & -1 \\
\frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -1 & -1 & -1 & -1 & 6 & -1 & -1 \\
\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -1 & -1 & -1 & -1 & -1 & 6 & -1 \\
\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -2\sqrt{2} & -2\sqrt{2} & \frac{3}{\sqrt{2}} & -1 & -1 & -1 & -1 & -1 & -1 & 6
\end{bmatrix}$$

Heawoodov graf

Kreinovi parametri $q_{ij}^k = \dfrac{n}{m_k}\,\mathrm{tr}((E_i \circ E_j)E_k)$:

$$[q_{ij}^0] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix} \qquad [q_{ij}^1] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

$$[q_{ij}^2] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & \frac{10+\sqrt{2}}{4} & \frac{10-\sqrt{2}}{4} \\ 0 & 1 & \frac{10-\sqrt{2}}{4} & \frac{10+\sqrt{2}}{4} \end{bmatrix} \qquad [q_{ij}^3] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & \frac{10-\sqrt{2}}{4} & \frac{10+\sqrt{2}}{4} \\ 1 & 0 & \frac{10+\sqrt{2}}{4} & \frac{10-\sqrt{2}}{4} \end{bmatrix}$$

Domaća zadaća

Zadatak 2.23.

Dokažite da su Kreinovi parametri jako regularnog grafa (asocijacijske sheme s dvije klase) racionalni brojevi. Moraju li biti cijeli brojevi?

Dualnost u Bose-Mesnerovoj algebri

Schurove idempotente:

- $\{A_0, \ldots, A_d\}$
- $A_i \circ A_j = \begin{cases} A_i, & i = j \\ 0, & \text{inače} \end{cases}$
- $A_0 = I$
- $\sum_{i=0}^{d} A_i = J$
- $A_i \cdot A_j = \sum_{k=0}^{d} p_{ij}^k A_k$

Primitivne idempotente:

- $\{E_0, \ldots, E_d\}$
- $E_i E_j = \begin{cases} E_i, & i = j \\ 0, & \text{inače} \end{cases}$
- $E_0 = \frac{1}{n} J$
- $\sum_{i=0}^{d} E_i = I$
- $E_i \circ E_j = \frac{1}{n} \sum_{k=0}^{d} q_{ij}^k E_k$

Dualnost u Bose-Mesnerovoj algebri

Stupnjevi:

- $n_i = p^0_{ii'} = \frac{1}{n} \operatorname{sum} A_i$
- $n_0 = 1$
- $n_0 + \ldots + n_d = n$
- $n_i = n_{i'}$

Kratnosti:

- $m_i = q^0_{i\hat{i}} = \operatorname{tr} E_i$
- $m_0 = 1$
- $m_0 + \ldots + m_d = n$
- $m_i = m_{\hat{i}}$

Dualnost u Bose-Mesnerovoj algebri

Stupnjevi:

- $n_i = p^0_{ii'} = \frac{1}{n} \operatorname{sum} A_i$
- $n_0 = 1$
- $n_0 + \ldots + n_d = n$
- $n_i = n_{i'}$
- $p^k_{ij} = \dfrac{1}{n\, n_k} \operatorname{tr}(A_i A_j A_k^t)$

Kratnosti:

- $m_i = q^0_{i\,\hat{i}} = \operatorname{tr} E_i$
- $m_0 = 1$
- $m_0 + \ldots + m_d = n$
- $m_i = m_{\hat{i}}$
- $q^k_{ij} = \dfrac{n}{m_k} \operatorname{tr}((E_i \circ E_j) E_k)$

Dualnost u Bose-Mesnerovoj algebri

Presječni brojevi:

- $p_{i0}^k = \delta_{ik}$
- $p_{0j}^k = \delta_{jk}$
- $p_{ij}^0 = n_i \delta_{ij'}$
- $p_{ij}^k = p_{j'i'}^{k'}$
- $\sum_{j=0}^{d} p_{ij}^k = n_i$
- $n_k p_{ij}^k = n_j p_{i'k}^j = n_i p_{kj'}^i$
- $\sum_{a=0}^{d} p_{ij}^a p_{ka}^b = \sum_{a=0}^{d} p_{ki}^a p_{aj}^b$

Kreinovi parametri:

- $q_{i0}^k = \delta_{ik}$
- $q_{0j}^k = \delta_{jk}$
- $q_{ij}^0 = m_i \delta_{i\hat{j}}$
- $q_{ij}^k = q_{\hat{i}\hat{j}}^{\hat{k}}$
- $\sum_{j=0}^{d} q_{ij}^k = m_i$
- $m_k q_{ij}^k = m_j q_{\hat{i}k}^j = m_i q_{k\hat{j}}^i$
- $\sum_{a=0}^{d} q_{ij}^a q_{ka}^b = \sum_{a=0}^{d} q_{ki}^a q_{aj}^b$

Zadatak 2.15.

Iz svojstva Schurovih idempotenta $A_i^t = A_{i'}$ dobivamo involuciju $i \mapsto i'$ (permutaciju stupnja 2) na skupu svih indeksa $\{0, \ldots, d\}$. Dualno, iz svojstva primitivnih idempotenta $E_i^t = E_{\hat{i}}$ dobivamo involuciju $i \mapsto \hat{i}$. Dokažite da možemo numerirati Schurove idempotente $A_0, \ldots, A_d$ i primitivne idempotente $E_0, \ldots, E_d$ tako da se te dvije involucije podudaraju, tj. da obje involucije imaju isti broj fiksnih točaka i transpozicija.

Svojstvene vrijednosti

$V_i = \operatorname{Im} E_i, \quad m_i = \operatorname{tr} E_i = \operatorname{rk} E_i = \dim V_i$

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Propozicija.

$$PQ = QP = nI$$

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Lema.

- $A_i E_j = P_i(j) E_j$
- $A_i \circ E_j = \frac{1}{n} Q_j(i) A_i$

Svojstvene vrijednosti

Skalarni produkt: $\langle A, B\rangle = \mathrm{tr}(AB^*) = \mathrm{sum}(A \circ \overline{B})$

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Teorem.

$$\frac{Q_j(i)}{m_j} = \frac{\overline{P_i(j)}}{n_i}$$

Svojstvene vrijednosti

$$N = \begin{bmatrix} n_0 & & 0 \\ & \ddots & \\ 0 & & n_d \end{bmatrix}, \quad M = \begin{bmatrix} m_0 & & 0 \\ & \ddots & \\ 0 & & m_d \end{bmatrix}$$

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Propozicija.

$Q = N^{-1} P^* M, \quad P = M^{-1} Q^* N$

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Relacije ortogonalnosti:

1. $$\sum_{k=0}^{d} \frac{1}{n_k} P_k(i) \overline{P_k(j)} = \frac{n}{m_i} \delta_{ij}$$
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Označimo s $\widehat{A}$ ortogonalnu projekciju matrice $A \in M_n(\mathbb{C})$ na Bose-Mesnerovu algebru $\mathscr{A}$.

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$$\widehat{A} = \sum_{i=0}^{d} \alpha_i A_i, \qquad \alpha_i = \frac{\langle A_i, A\rangle}{\langle A_i, A_i\rangle} = \frac{\langle A_i, A\rangle}{n\, n_i}$$

$$= \sum_{i=0}^{d} \lambda_i E_i, \qquad \lambda_i = \frac{\langle E_i, A\rangle}{\langle E_i, E_i\rangle} = \frac{\langle E_i, A\rangle}{m_i}$$

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Tablice karaktera

$$P = \begin{bmatrix} 1 & n_1 & \cdots & n_d \\ 1 & & & \\ \vdots & & & \\ 1 & & & \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & m_1 & \cdots & m_d \\ 1 & & & \\ \vdots & & & \\ 1 & & & \end{bmatrix}$$

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Za Heawoodov graf:

$$P = \begin{bmatrix} 1 & 3 & 6 & 4 \\ 1 & -3 & 6 & -4 \\ 1 & \sqrt{2} & -1 & -\sqrt{2} \\ 1 & -\sqrt{2} & -1 & \sqrt{2} \end{bmatrix} \qquad Q = \begin{bmatrix} 1 & 1 & 6 & 6 \\ 1 & -1 & 2\sqrt{2} & -2\sqrt{2} \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -\frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \end{bmatrix}$$

Primjer 1.37. Schurova konstrukcija za grupu $G = \langle (1,2,3) \rangle$

Tablice karaktera

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Orbitale: $(1,1)^G = \{(1,1),\, (2,2),\, (3,3)\}$
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$$A_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

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$(n_0, n_1, n_2) = (m_0, m_1, m_2) = (1,1,1) \rightsquigarrow$ primjer **tanke** koherentne konfiguracije

Domaća zadaća

Zadatak 2.17.

Za koherentnu konfiguraciju kažemo da je tanka (eng. thin) ako su svi stupnjevi jednaki 1. Zbog $n_0 + \ldots + n_d = n$, to je ekvivalentno s $n = d + 1$. Dokažite da sve tanke koherentne konfiguracije nastaju Schurovom konstrukcijom od permutacijskih grupa koje djeluju strogo tranzitivno: $(\forall x, y \in X)(\exists! g \in G)\ x^g = y$. Dokažite da za strogo tranzitivne permutacijske grupe vrijedi: G je Abelova ako i samo ako je odgovarajuća Schurova koherentna konfiguracija komutativna. Nađite protuprimjer za tranzitivne grupe koje ne djeluju strogo i dokažite implikaciju koja vrijedi za sve tranzitivne permutacijske grupe.

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Schurove idempotente tankih koherentnih konfiguracija su permutacijske matrice.

Domaća zadaća

Zadatak 2.17.

Za koherentnu konfiguraciju kažemo da je tanka (eng. thin) ako su svi stupnjevi jednaki 1. Zbog $n_0 + \ldots + n_d = n$, to je ekvivalentno s $n = d + 1$. Dokažite da sve tanke koherentne konfiguracije nastaju Schurovom konstrukcijom od permutacijskih grupa koje djeluju strogo tranzitivno: $(\forall x, y \in X)(\exists! g \in G)\ x^g = y$. Dokažite da za strogo tranzitivne permutacijske grupe vrijedi: G je Abelova ako i samo ako je odgovarajuća Schurova koherentna konfiguracija komutativna. Nađite protuprimjer za tranzitivne grupe koje ne djeluju strogo i dokažite implikaciju koja vrijedi za sve tranzitivne permutacijske grupe.

Schurove idempotente tankih koherentnih konfiguracija su permutacijske matrice.

Zadatak 2.33.

Odredite svojstvene vrijednosti permutacijske matrice.

Svojstvene vrijednosti

Propozicija.

- $|P_j(i)| \leq n_j$
- $|Q_j(i)| \leq m_j$

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Teorem.

- $P_i(\ell)P_j(\ell) = \sum_{k=0}^{d} p_{ij}^k P_k(\ell)$
- $Q_i(\ell)Q_j(\ell) = \sum_{k=0}^{d} q_{ij}^k Q_k(\ell)$

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Teorem.

- $P_i(j)Q_j(\ell) = \sum_{k=0}^{d} p_{ik}^\ell Q_j(k)$
- $Q_i(j)P_j(\ell) = \sum_{k=0}^{d} q_{ik}^\ell P_j(k)$

Svojstvene vrijednosti

Teorem.

- $p_{ij}^k = \dfrac{1}{n\, n_k} \displaystyle\sum_{\ell=0}^{d} m_\ell P_i(\ell) P_j(\ell) \overline{P_k(\ell)} = \dfrac{n_i n_j}{n} \displaystyle\sum_{\ell=0}^{d} \dfrac{1}{m_\ell^2} Q_\ell(i) Q_\ell(j) \overline{Q_\ell(k)}$

- $q_{ij}^k = \dfrac{1}{n\, m_k} \displaystyle\sum_{\ell=0}^{d} n_\ell Q_i(\ell) Q_j(\ell) \overline{Q_k(\ell)} = \dfrac{m_i m_j}{n} \displaystyle\sum_{\ell=0}^{d} \dfrac{1}{n_\ell^2} P_\ell(i) P_\ell(j) \overline{P_\ell(k)}$