Formulation of Compressible Immiscible Two-Phase Flow Model by Means of Global Pressure

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Outline

1. Two-phase immiscible, compressible flow equations
2. Global pressure formulation
3. Compressible flow: Fully equivalent global pressure formulation
4. Compressible flow: Simplified global pressure formulation
5. Comparison of fully equivalent and simplified formulation
6. Conclusion
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Mass conservation: for $\alpha \in \{w, g\}$,

$$\Phi \frac{\partial}{\partial t} (\rho_\alpha S_\alpha) + \text{div}(\rho_\alpha \mathbf{q}_\alpha) = \mathcal{F}_\alpha,$$

The Darcy-Muscat law: for $\alpha \in \{w, g\}$ (gravity neglected),

$$\mathbf{q}_\alpha = -\lambda_\alpha (S_\alpha) K \nabla p_\alpha,$$

Capillary law: $p_c(S_w) = p_g - p_w$,

No void space: $S_w + S_g = 1$.

Water is incompressible $\rho_w = \text{cte}$, gas is compressible $\rho_g(p_g) = c_g p_g$. 
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Goal: Reformulate flow equations in order to

- Make coupling between the two differential equations less strong,
- Give to the system well defined mathematical structure.

We consider:

1. Compressible flow.
   - Fully equivalent formulation
   - Simplified formulation
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Rewrite two-phase flow equations as (keep conservative form):

**Total flow:** \( Q_t = \rho_w q_w + \rho_g q_g \)

\[
Q_t = -\lambda(S_w, p_g) \mathbb{K} (\nabla p_g - f_w(S_w, p_g)\nabla p_c(S_w)),
\]

**Total mass conservation:**

\[
\Phi \frac{\partial}{\partial t} (S_w \rho_w + (1 - S_w) \rho_g(p_g)) + \text{div} (Q_t) = \mathcal{F}_w + \mathcal{F}_g,
\]

**Water mass conservation:**

\[
\Phi \rho_w \frac{\partial S_w}{\partial t} + \text{div}(f_w(S_w, p_g)Q_t) = \text{div}(\mathbb{K} a(S_w, p_g)\nabla S_w) + \mathcal{F}_w.
\]
In total flow eliminate saturation gradient:

\[ Q_t = -\lambda(S_w, p_g) K (\nabla p_g - f_w(S_w, p_g)p'_c(S_w) \nabla S_w), \]

- Idea: introduce a new pressure-like variable that will eliminate \( \nabla S_w \) term (Chavent (1976), Antontsev-Monakhov (1978))

- Introduce a new pressure variable \( p \), called global pressure, such that \( p_g = \pi(S_w, p) \). Find functions \( \pi(S_w, p) \) and \( \omega(S_w, p) \) that satisfy:

\[
\nabla p_g - f_w(S_w, \pi(S_w, p))p'_c(S_w) \nabla S_w = \omega(S_w, p) \nabla p \quad (1)
\]
Compressible flow: Fully equivalent global pressure formulation

Compressible flow: global pressure

\begin{align*}
\frac{d\pi(S, p)}{dS} &= \frac{\rho_w \lambda_w(S)p'_c(S)}{\rho_w \lambda_w(S) + \rho_g(\pi(S, p))\lambda_g(S)}, \quad 0 < S < 1 \\
\pi(1, p) &= p.
\end{align*}

\[\omega(S_w, p) = \exp \left( \int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s)\lambda_g(s)}{\rho_w \lambda_w(s) + c_g \lambda_g(s)\pi(s, p)}^2 p'_c(s) \, ds \right),\]

\[p_w \leq p \leq p_g.\]
Compressible flow: transformed equations

Variables: $p$ (global pressure) and $S_w$ (water saturation).

Total flow:

$$Q_t = -\lambda^n(S_w, p)\omega(S_w, p)K\nabla p.$$ 

Total mass conservation:

$$\Phi \frac{\partial}{\partial t}(S_w\rho_w + c_g(1 - S_w)\pi(S_w, p)) + \text{div}Q_t = F_w + F_g.$$ 

Water mass conservation:

$$\Phi \rho_w \frac{\partial S_w}{\partial t} + \text{div}(f^w(S_w, p)Q_t) = \text{div}(Ka^n(S_w, p)\nabla S_w) + F_w.$$ 

These equations are fully equivalent to original equations.
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Coefficients in global formulations are “difficult” to calculate since they depend on $\pi(S_w, p)$: $\lambda^n(S_w, p) = \lambda(S_w, \pi(S_w, p))$ etc.

Can we make reasonable approximation?

Yes, but we need a hypothesis:

- In all coefficients of differential equations gas density $\rho_g(\pi(S_w, p))$ can be replaced by $\rho_g(p)$ without introducing a significant error.
- Replace $\lambda^n(S_w, p) = \lambda(S_w, \pi(S_w, p))$ by $\lambda(S_w, p)$ etc.
Approximation

Replace gas pressure function

\[
\pi(S_w, p) = p - \int_{S_w}^{1} f_w(s, \pi(s, p)) p'_c(s) \, ds,
\]

by

\[
\pi(S_w, p) = p - \int_{S_w}^{1} f_w(s, p) p'_c(s) \, ds,
\]

(Chavent-Jaffré (1986))
Replace

\[
\omega(S_w, p) = \exp \left( \int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) \pi(s, p))^2} p'_c(s) \, ds \right),
\]

by

\[
\omega(S_w, p) = \exp \left( \int_{S_w}^{1} \frac{c_g \rho_w \lambda_w(s) \lambda_g(s)}{(\rho_w \lambda_w(s) + c_g \lambda_g(s) p)^2} p'_c(s) \, ds \right),
\]
The system written in unknowns $p$ and $S_w$.

**Total flow:**

$$Q_t = -\lambda(S_w, p)\omega(S_w, p)K\nabla p,$$

**Total mass conservation:**

$$\Phi \frac{\partial}{\partial t}(S_w\rho_w + c_g(1 - S_w)p) + \text{div}Q_t = \mathcal{F}_w + \mathcal{F}_g,$$

**Water mass conservation:**

$$\Phi\rho_w \frac{\partial S_w}{\partial t} + \text{div}(f_w(S_w, p)Q_t) = \text{div}(K_a(S_w, p)\nabla S_w) + \mathcal{F}_w.$$

Note that the equations are the same, only the coefficients are different.
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1D Flow simulations

Van Genuchten’s functions with parameters (Couplex test case):

\[ n = 2, \ Pr = 2 \text{ MPa}, \ \mu_w = 0.86 \cdot 10^{-3} \text{ Pas}, \ \mu_g = 9 \cdot 10^{-6} \text{ Pas}, \]
\[ \rho_w = 996.53 \text{ kg/m}^3, \ c_g = 0.808 \text{ and } T = 300K. \]

- Domain \( \Omega = (0, L), \ L = 100 \text{ m} \ K = 1 \text{ mD}, \ \Phi = 0.1; \)
- Discretization by vertex centered finite volume method;
- Simulation 1 - gas injection:

\[ p(0, t) = 1.0, \ p(L, t) = 0.1, \ S_w(0, t) = 0.4, \ \frac{\partial}{\partial x} S_w(L, t) = 0, \]
\[ S_w(x, 0) = 1.0, \ p(x, 0) = 0.1. \]

- Simulation 2 - water injection:

\[ p(0, t) = 4.0, \ p(L, t) = 0.5, \ S_w(0, t) = 1.0, \ \frac{\partial}{\partial x} S_w(L, t) = 0, \]
\[ S_w(x, 0) = 0.7, \ p(x, 0) = 0.5. \]
Comparison of fully equivalent and simplified formulation

Simulation 1

Gas injection

[Graphs showing comparisons between different formulations for gas injection]
Comparison of fully equivalent and simplified formulation

Simulation 2

Water injection

- $S_{w}^{\text{new}}$
- $S_{w}^{\text{simpl}}$
- $p_{g}^{\text{new}}$
- $p_{g}^{\text{simpl}}$
- $p^{\text{new}}$
- $p^{\text{simpl}}$
- $p_{w}^{\text{new}}$
- $p_{w}^{\text{simpl}}$
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Simplification of the system can safely be used:
- in applications with high mean field pressure.
- in applications with relatively small capillary pressure.

The difference between simplified and fully equivalent model becomes significant:
- for small global pressure.
- for relatively large capillary pressure.

The difference in computed saturations is always small, but the differences in gas pressure (global pressure) may be significant.
An extension to multiphase, multicomponent models is straightforward and it is at present in the course of study.

New and simplified formulation are well adapted for the mathematical analysis of the model. At present we study existence etc.

Implementation of FV method in two and three dimensions.

Treatment of multiple rock types is currently studied.