ON NUMERICAL UPSCALING FOR STOKES AND STOKES-BRINKMAN FLOWS

Oleg Iliev, **Z.Lakdawala, J.Willems,**
Fraunhofer Institute for Industrial Mathematics, Kaiserslautern, Germany

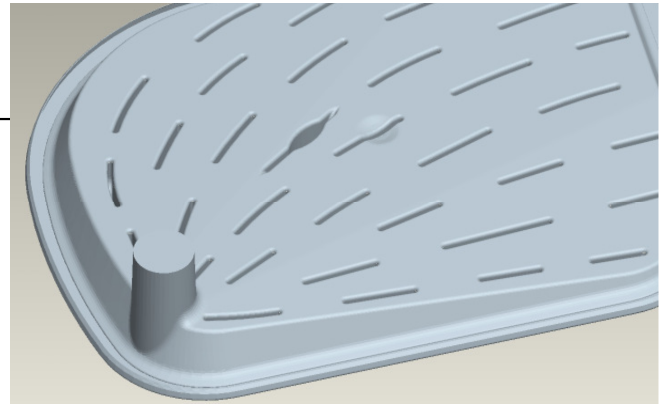
V.Starikovicius,
Vilnius Gediminas Technical University, Lithuania

P.Popov,
Inst. Scientific Computation, Texas A&M University, USA

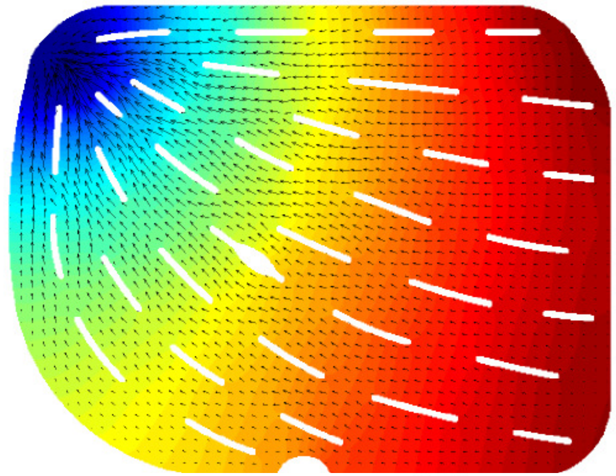
Fraunhofer Institut Techno- und Wirtschaftsmathematik

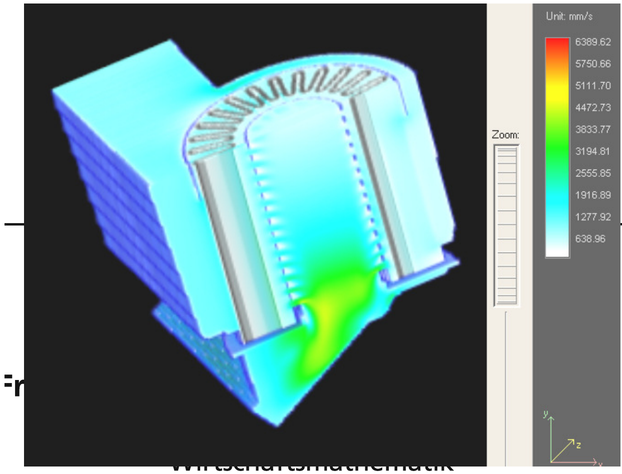
October 14, 2008

Content



1. Motivation and aims
2. Basic solver
3. Multiple scales. Subgrid approach
4. Computer simulations
5. Perspectives





Motivation and aims

CFD simulations for filtration

Main criteria determining the performance of a filter element:

1) **Pressure drop – flow rate ratio;**

1) **Dirt storage capacity;**

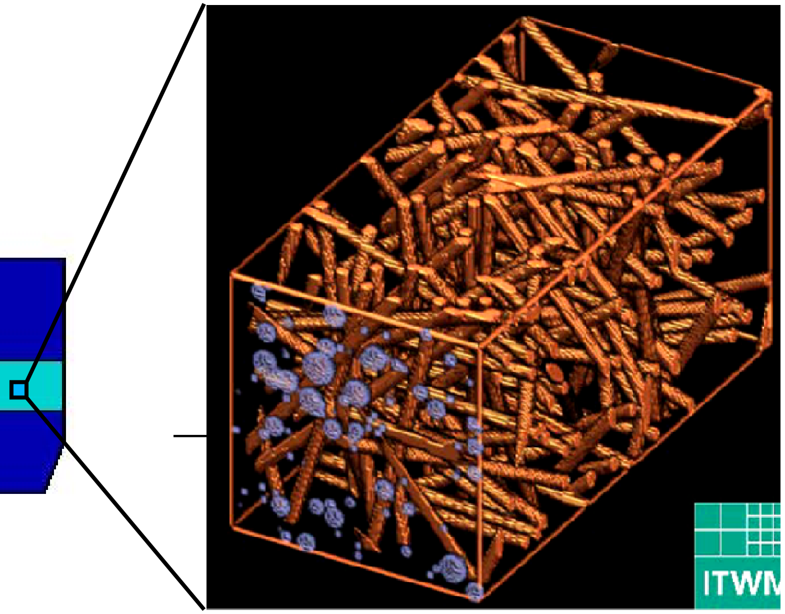
1) **Size of penetrating particles.**

depend on:

microscale (*e.g. fibrous geometry, local deposition of particles, etc*),

and

macroscale (*e.g., filter element geometry, pressure, velocity distribution, etc.*)



Challenges to CFD simulations

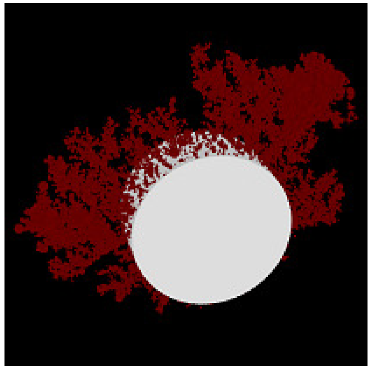
- Multiple scales (particles, fibres, pleats, ribs, housing,…);
- Time-dependent performance;
- Shortening the design time and Needs for new design ideas;
- Virtual filter element design;
- Extensive computational time;
- Parameters measurement or calculation (permeability, deposition rate,..)
- Validation of the numerical simulation results;
- …

Multiple scales in filtration

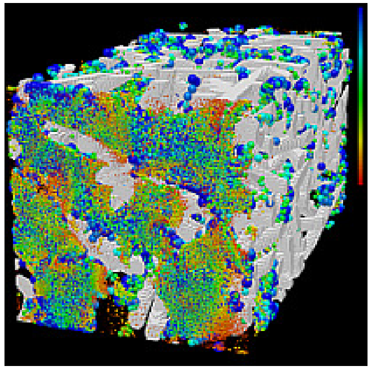
Particles level		Filter components		Filter element	Complete system
Nano scale	Micro scale		Millimeter	Centimeter	Meter

(Navier-)Stokes in pore space coupled with stochastic ODE for particles, ….

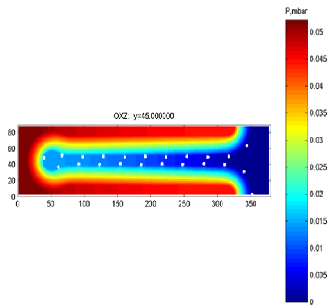
(Navier-)Stokes-Brinkmann in fluid and in porous media coupled with concentration of particles



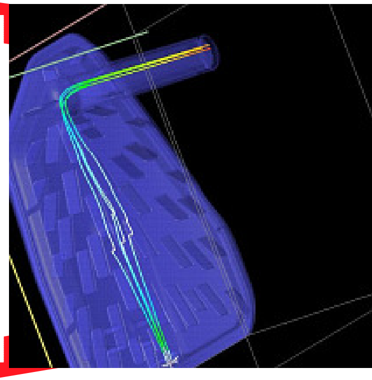
Particles-Fiber interaction



Dirt loading of the filtering medium



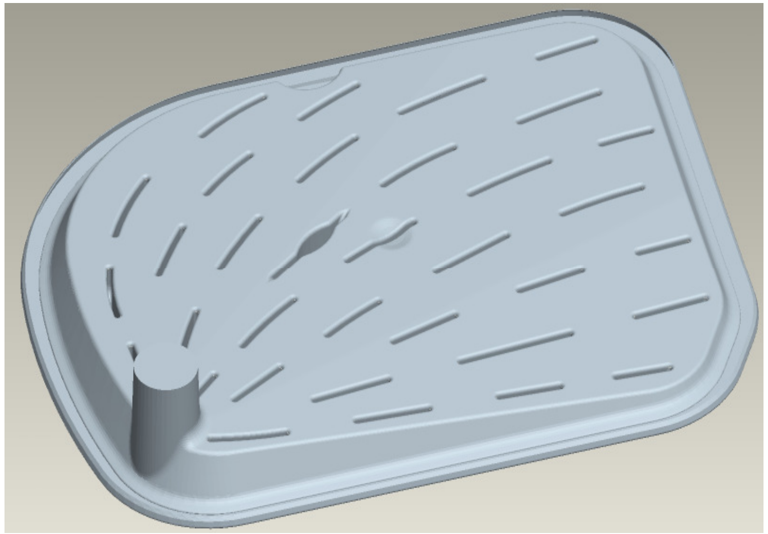
Pleats in cartrige filters

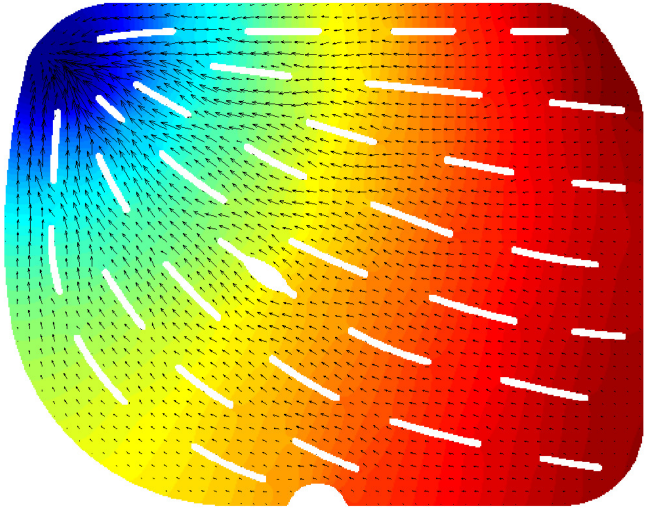


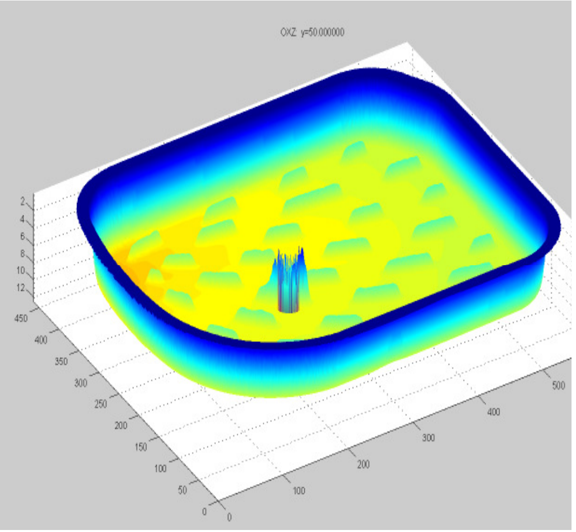
Flow within Filter element

Filter installation

Basic solver







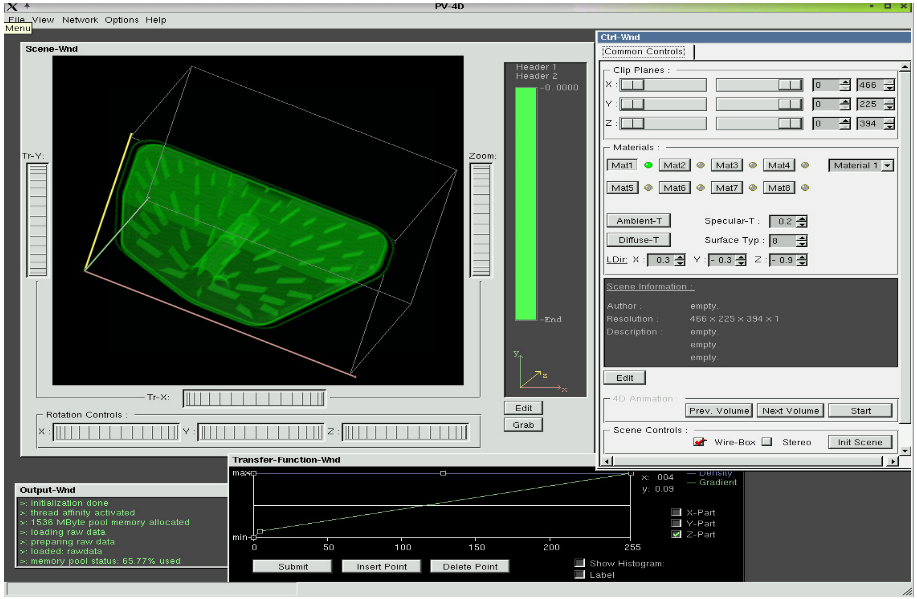
SUFIS

ITWM

Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

Basic CFD solver: SuFiS

- Grids: Cartesian grid
- Finite volume discretization on cell-centred collocated grid
- Chorin projection method with implicit treatment of Darcy term
- Proper treatment of discontinuous coefficients in pressure-correction equation
- Subgrid approach incorporated
- Specialized for filtration applications
- Paralleization



SUFiS

ITWM

Fraunhofer Institut Techno- und Wirtschaftsmathematik

IBS FILTRAN

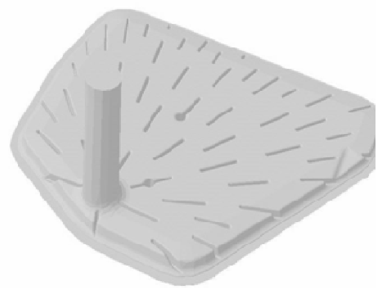
Macro scale: Flow through fluid and porous regions

$$\underbrace{\frac{\partial \vec{u}}{\partial t} - \nabla \cdot (\tilde{\mu} \nabla \vec{u}) + (\rho \vec{u}, \nabla)\vec{u}}_{Navier-Stokes} + \overbrace{K^{-1} \mu \vec{u} + \nabla p = f}^{Darcy}$$

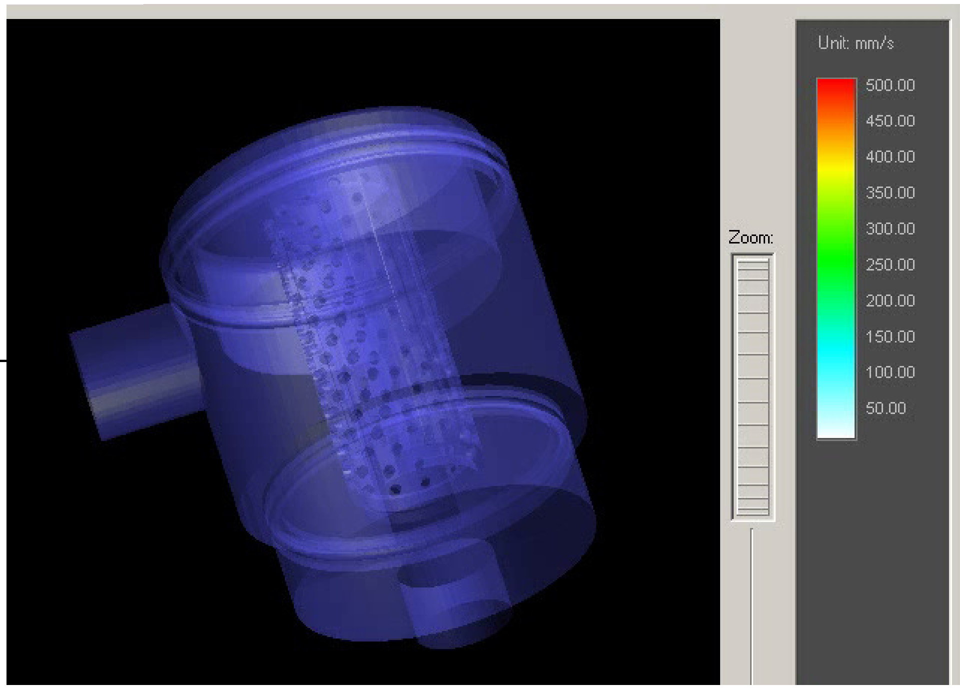
Momentum Equations

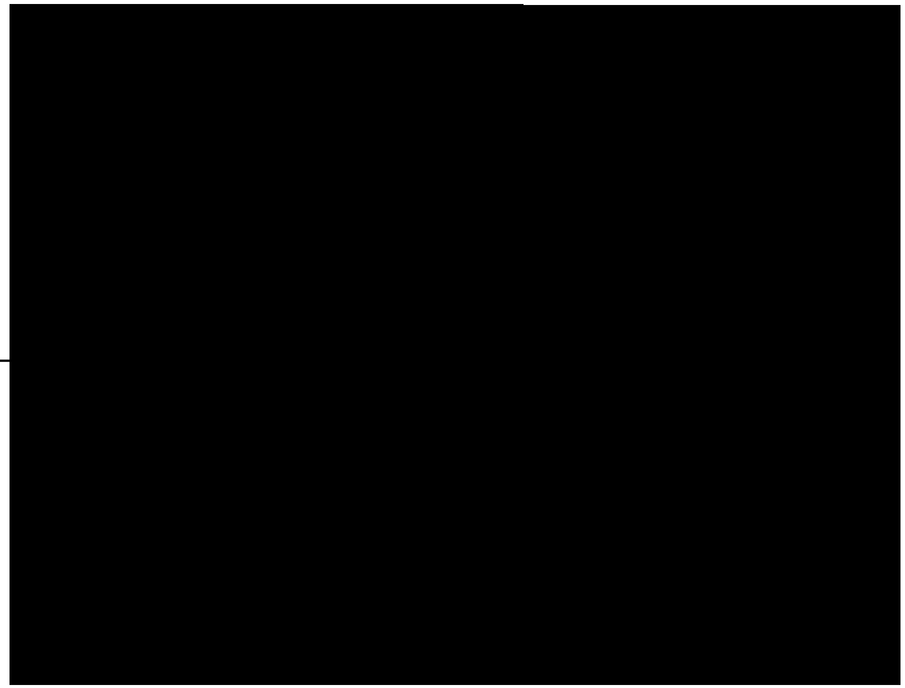
$$\nabla \cdot \vec{u} = 0$$

Continuity Equation

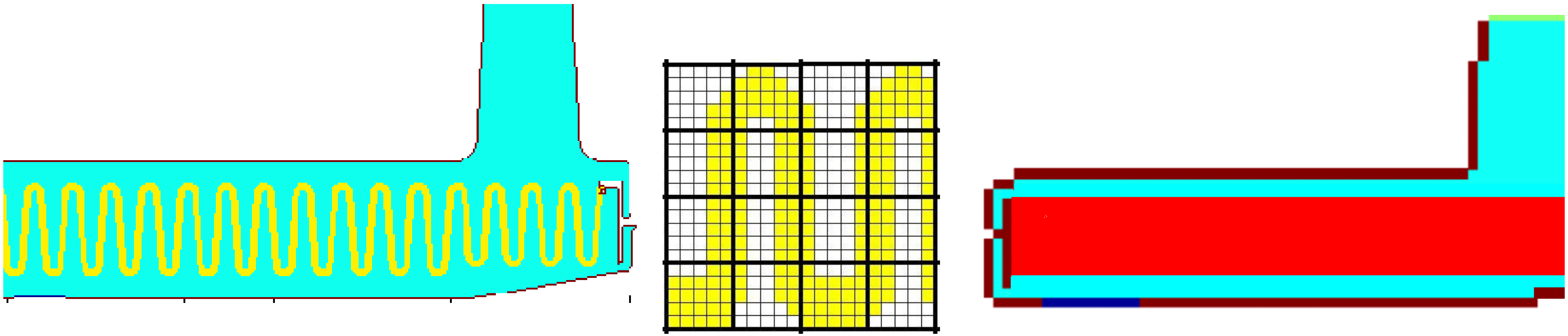


K can be fixed, or can change due to loading of the filtering medium





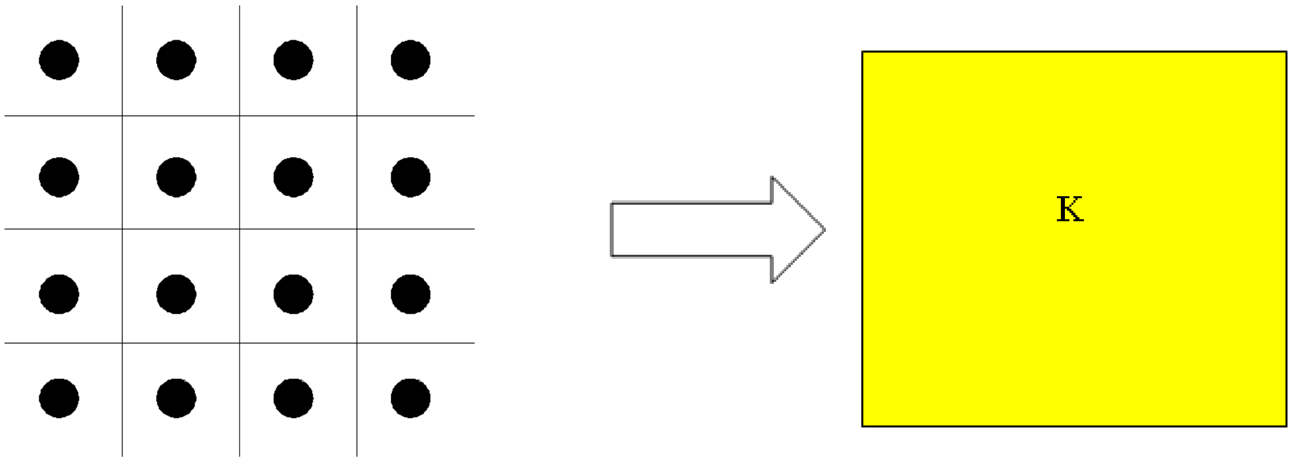
Multiple scales. Subgrid approach

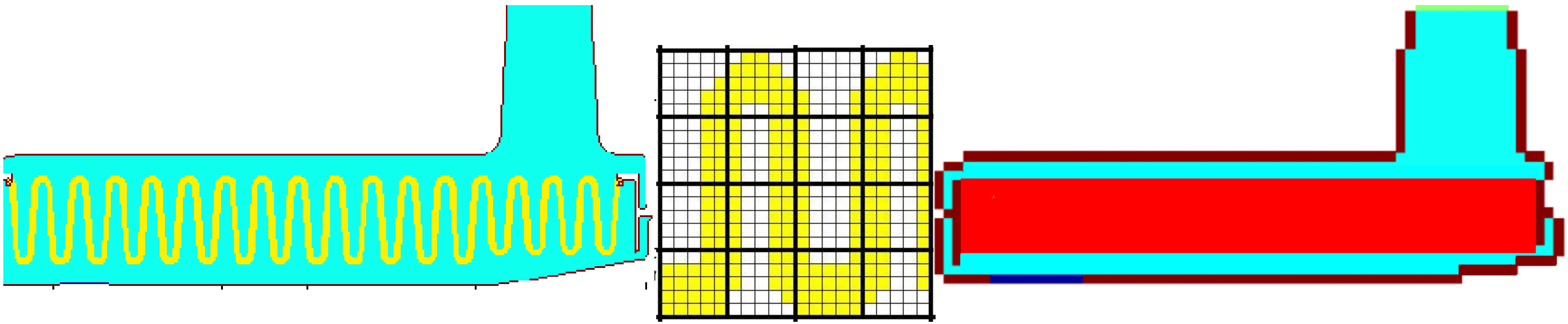


Fraunhofer Institut Techno- und Wirtschaftsmathematik

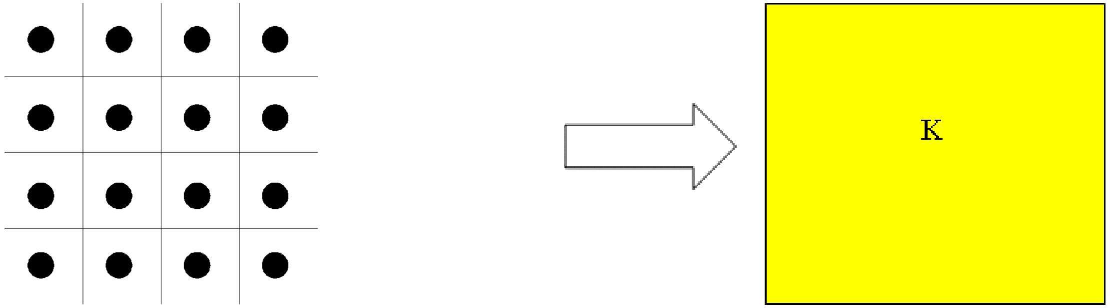
3. Multiple scales. Subgrid approach

- State of the art (Stokes to Darcy; Darcy to Darcy; two-level DD for multiscale)
- Microscale to mesoscale upscaling (Stokes to Darcy or to Brinkman
- Mesoscale to macroscale upscaling (Brinkman to Brinkman)





3. Multiple scales. Known: Upscaling Stokes to Darcy



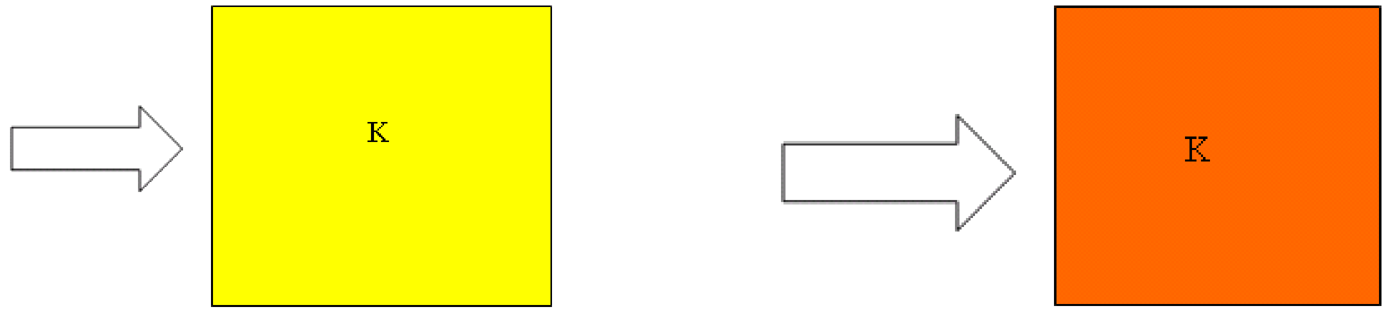
$$-\nu \Delta u = \nabla p \qquad\qquad -\mu K^{-1} u = \nabla p$$

$$\nabla \cdot u = 0 \qquad\qquad \nabla \cdot u = 0$$

+boundary conditions:

- **periodic (Sanchez Palencia)**
- **const. velocity (Allaire)**
- **engineering approach**

3. Multiple scales. Known: Darcy to Darcy



$$-\mu K^{-1} u = \nabla p$$

$$\nabla \bullet u = 0$$

$$-\mu \tilde{K}^{-1} u = \nabla p$$

$$\nabla \bullet u = 0$$

+boundary conditions:

- **periodic**
- **linear**
- **presure drop+oscilatory**
- **presure drop+Neumann**

Note:
Some results available for Macroheterogeneous case (block permeability, e.g., Wu, Efendiev,Hou)

3. Multiple scales. Brinkman to Darcy or Brinkman

Solve the local problem

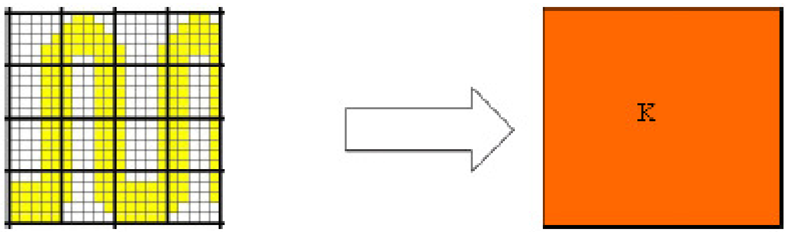
- Solve auxiliary cell problem in each direction for each quasi porous cell

$$-\nabla \cdot (\mu \nabla \vec{u}) + \hat{K}^{-1} \mu \vec{u} + \nabla p = f \qquad \text{momentum eqn}$$
$$\nabla \cdot \vec{u} = 0 \qquad \text{continuity eqn}$$

where

$$\hat{K}^{-1} = \begin{cases} K^{-1}, \ x \in \Omega_p \\ 0, \ x \in \Omega_f \end{cases} \quad \text{Fictitious Region Method - type continuation of coefficients}$$

- Get an upscaled block permeability tensor



  - Darcy
  - Sanchez Palencia
  - Allaire

Multiple scales. Subgrid approach

- Choose a basic grid on which the simulations are possible;

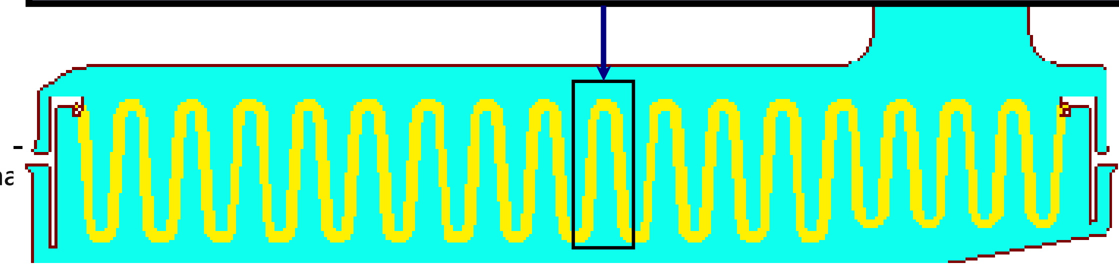
- Provide information about the fine geometrical details;

- For each grid cell check if it overlaps unresolved fine geometry details

- In marked cells (or their agglomeration) solve auxiliary problems on fine grid, and calculate effective permeability tensor;

- Solve the modified equations on the chosen grid (the fine details are accounted via the effective permeability).

Example of selected location for which effective permeability is calculated



Multiple scales. Subgrid approach

Usage of the subgrid approach:

- **Upscale and solve upscaled equations;**

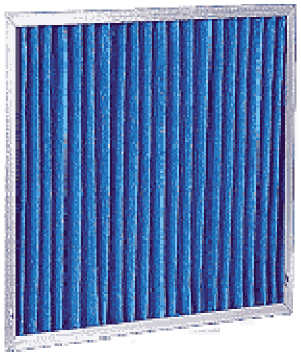
- **Upscale, solve upscaled equations and prolong the solution to the fine scale;**

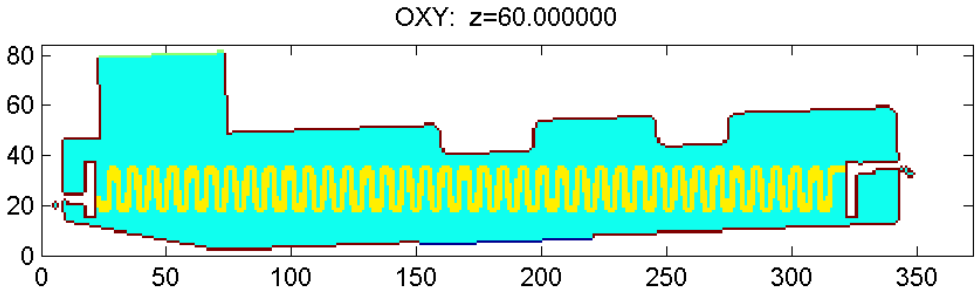
- **Iterate over scales (two-level DD with upscaling-based coarse scale operator).**

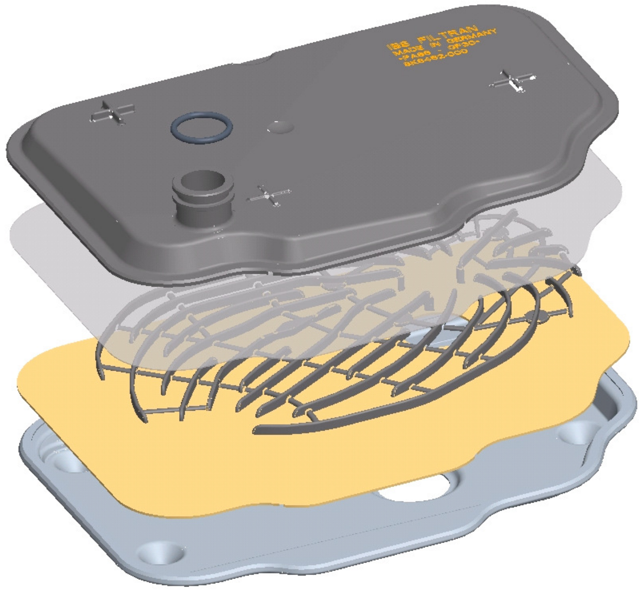
Open problems:

- **No theory for upscaling blocks containing solid, porous and fluid;**
- **No theory for macroheterogeneous case;**
- **.....**

Computer simulations using subgrid approach

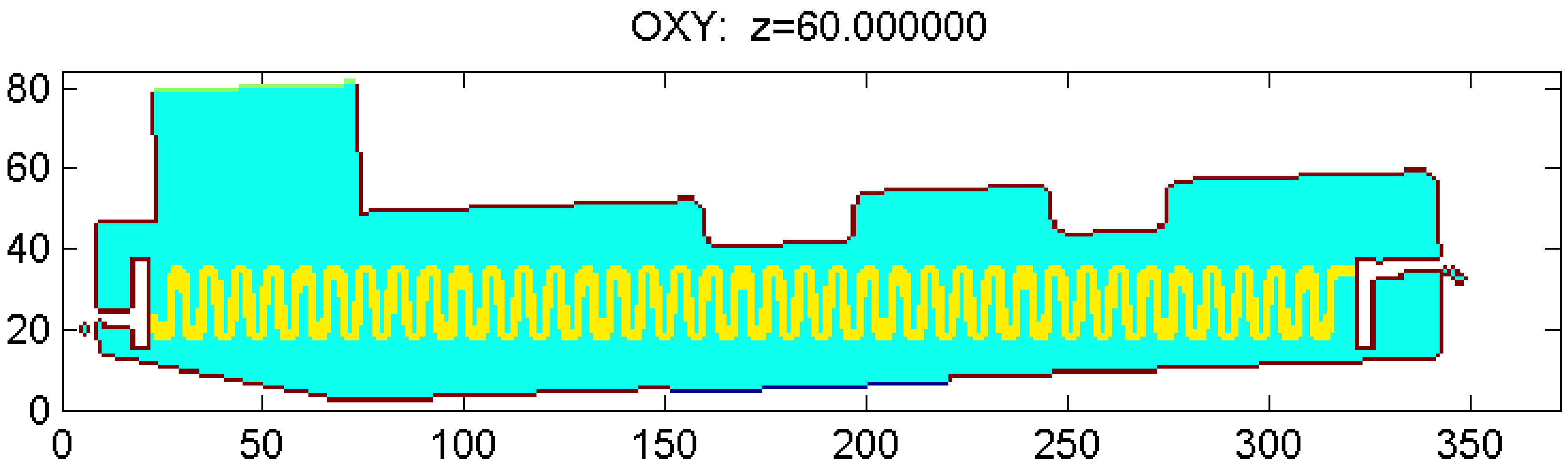


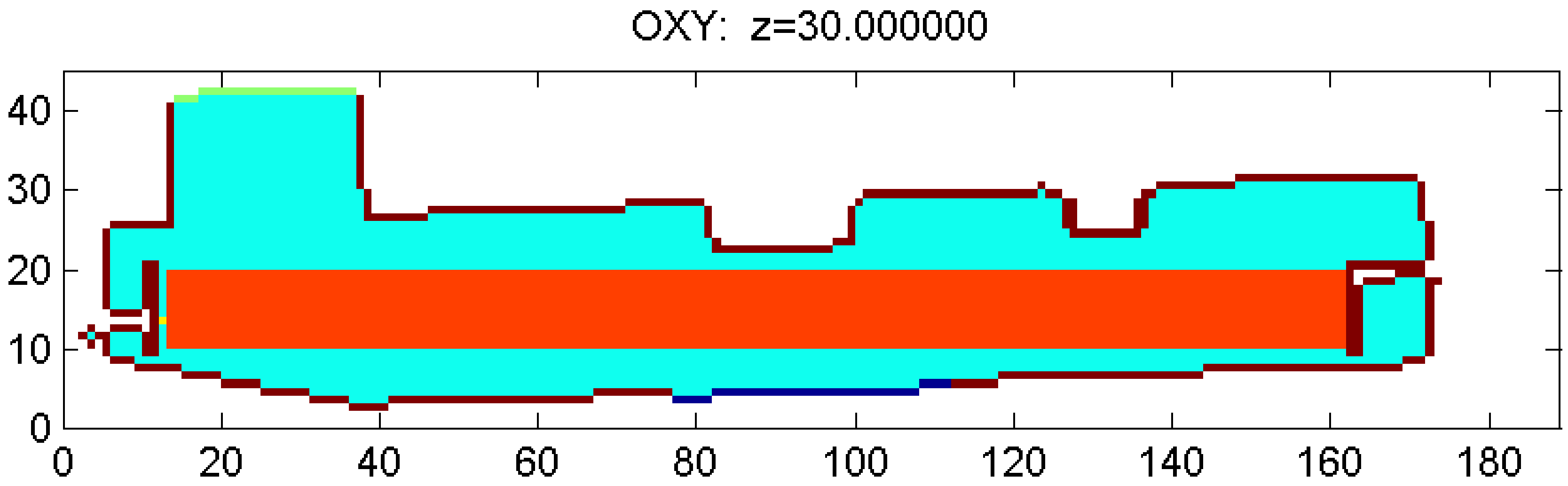




Fraunhofer Institut
Techno- und
Wirtschaftsmathematik

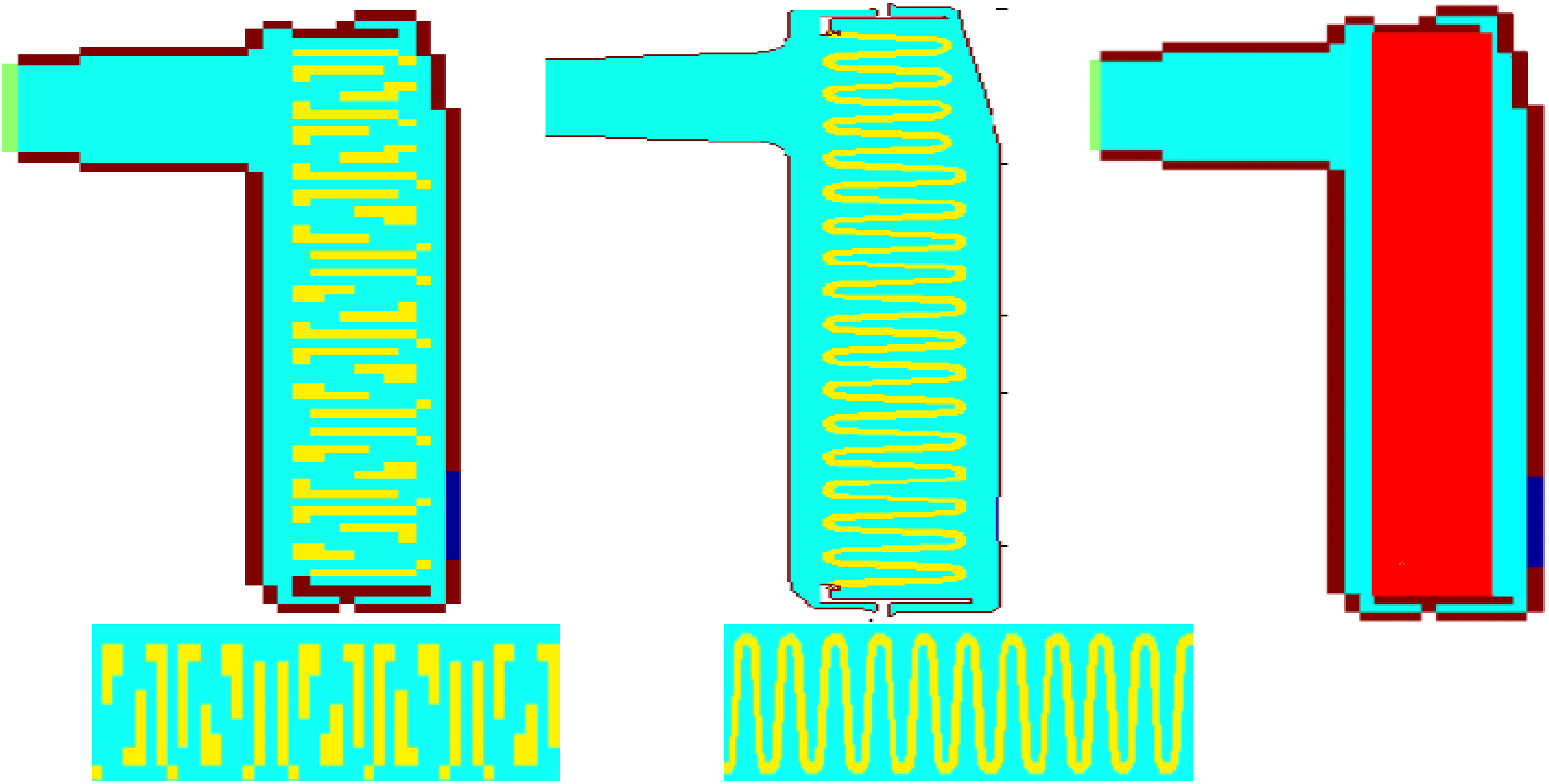
Pleated filter, simulations with subgrid approach





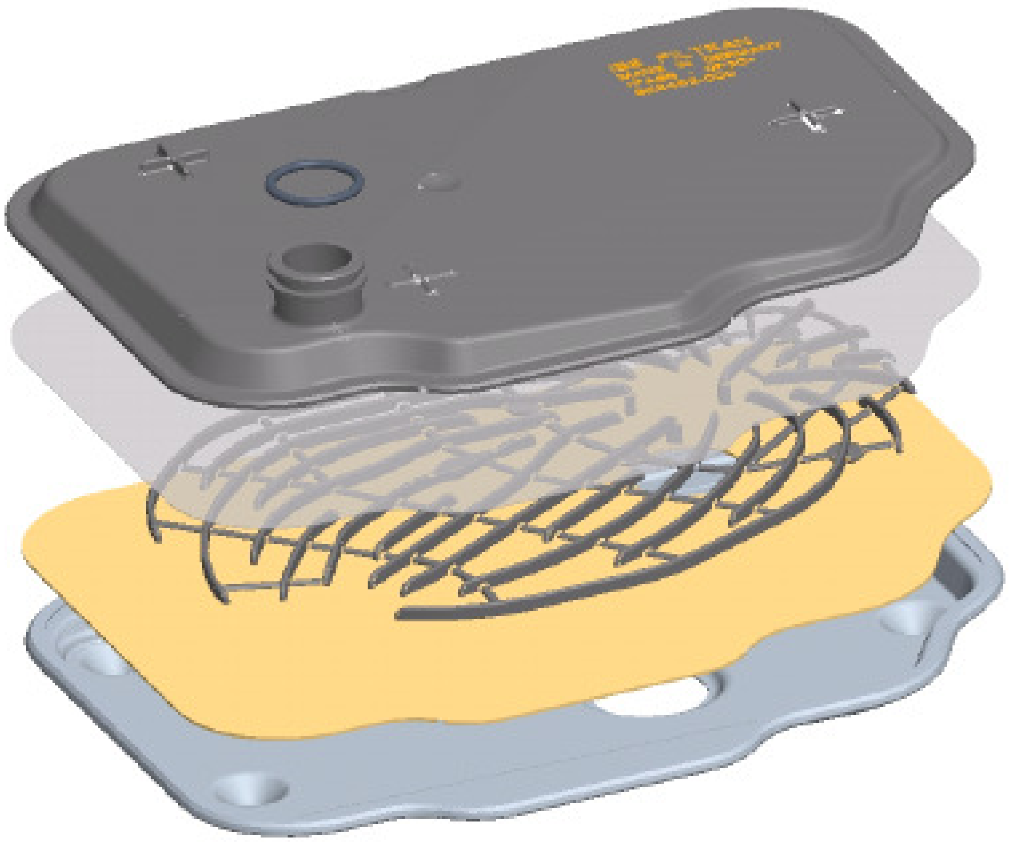
4. Computer simulations

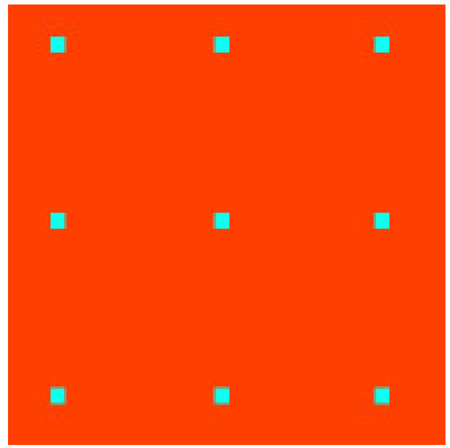
Pleated Filter

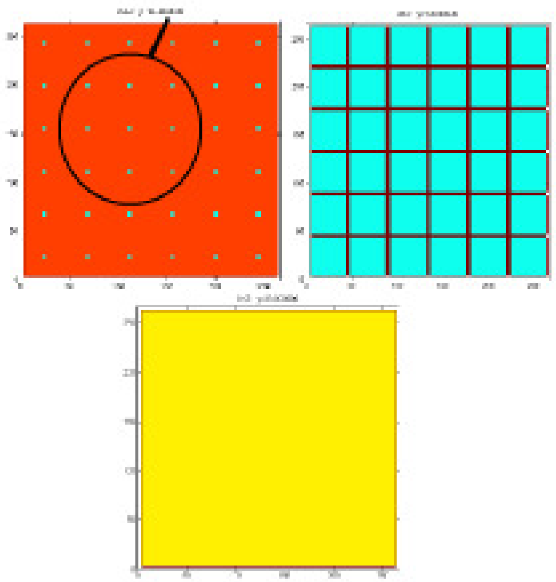


	dp (mbar)	time (s)
1mm	2700	1421
0.5mm	1874	149102
subgrid-1mm-0.5mm	1760	65000

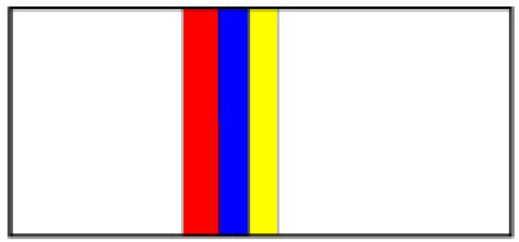
Combi Filter with multiple porous media separated by a mesh





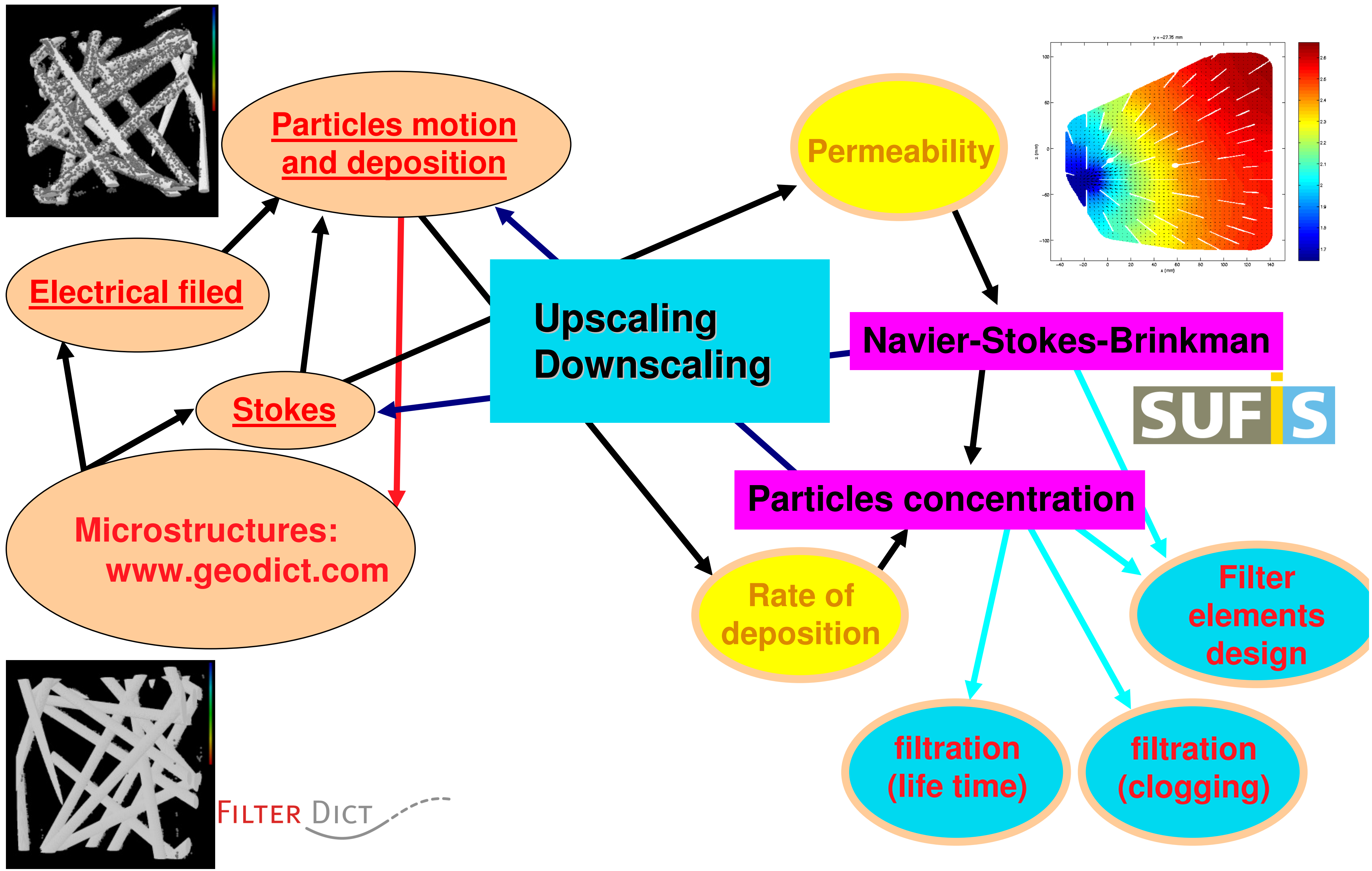


	dp (mbar)	time (s)
1mm	115	1248
0.25mm	16.8	73703
subgrid-1mm-0.25mm	17.0	5000



Perspectives

Microscale # Multiscale **Macroscale**



Thank you

www.itwm.fhg.de Fraunhofer ITWM

www.dasmod.de Dependable Adaptive Systems and Mathematical Modeling, TU Kaiserslautern