Integral mean estimates for polynomials whose zeros are within a circle

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Abstract: Let $P(z)$ be a polynomial of degree n having all its zeros in $|z| \leq K$. For $K = 1$, it is known that for any $r > 0$,

$$n\left\{\int_0^{2\pi} \left|P(e^{i\theta})\right|^r d\theta\right\}^{\frac{1}{r}} \leq \left\{\int_0^{2\pi} \left|1 + e^{i\theta}\right|^r d\theta\right\}^{\frac{1}{r}} \underset{|z|=1}{\operatorname{Max}} |P'(z)|.$$

In this paper, we shall present several generalizations of this result in this sense that maximum $|P'(z)|$ on $|z| = 1$ is replaced by a factor involving integral mean of $|P'(z)|$ on $|z| = 1$, by considering the two cases $K \leq 1$ and $K > 1$.