GLASNIK MAT. - FIZ. I ASTR.
Tom 16. — No. 1—2 — 1961.

ON THE DEVIATIONS FROM THE ALLOWED SHAPE IN THE ALLOWED β-DECAY SPECTRA

B. Eman and D. Tadić, Zagreb

Abstract

The deviations from the allowed shape are discussed. The correction factors for the spectrum and longitudinal polarization are calculated including possible high order correction.

The allowed β-decays can be separeted into three groups: with small ft-values ($\log ft \approx 3$), with somewhat larger ft-values ($3 < \log ft < 5$) and with large ft-values ($\log ft \approx 6$). The decays with small and large ft-values were the most interesting ones in searching for possible deviations from the allowed shape. To explain the deviations in β-decays with small ft-values (if the deviations really exist) one must search for some modification of the β-decay interaction Hamiltonian. We tried to do so with a simple modification of the interaction Hamiltonian. We used the nonrelativistic Hamiltonian constructed with all possible combinations of available operators, which are invariant under the 3-dimensional rotation group. Each combination is multiplied by the form factor. Only the terms linear in the nucleon momentum and V-A-P combination of β-decay interaction are taken. In such a modification the standard β-decay theory is included as a particular case. It seems that such a simple modification is not capable of describe all the observed effects.

Of course, the best way of searching for effects which are connected with the structure of interaction Hamiltonian is to make measurements of the spectra and longitudinal polarization in mirror and $O^+ \to O^+$ β-transitions. We calculated the nuclear matrix elements for such transitions on the basis of the shell model (*Table* 3).

For some transitions with large ft-values it is possible to describe the deviation from the allowed shape with the standard β-decay theory. One of such transitions is l-forbidden transition, which is discussed in details and some experiments are suggested. A list of transitions which can be classified as l-forbidden, is given in *Table* 4. The most characteristic cases are those with large ft-value.

1. *Introduction*

High order corrections to the allowed β spectrum are of great interest at the present stage of development of the β decay theory. Systematic deviations from the allowed shape in some spectra have been reported [1], [2], [3], [4], [5], [6]. These deviations can be produced by some intrinsic properties of the β decay interaction (of course, combined with the influence of nuclear structure), or by the influence of nuclear structure only. The influence of nuclear structure occurs in nuclear matrix elements, and the intrinsic properties of the β decay interaction are connected with the form of the interaction Hamiltonian.

To obtain some conclusions we started from a slightly generalized nuclear non-relativistic Hamiltonian and calculated all possible high order corrections. The reasons for choosing such a Hamiltonian are the following:

a) The succes of non-relativistic models of atomic nuclei indicates that β decay may bee described by a non-relativistic Hamiltonian too.

b) Nuclear matrix elements can be calculated only by non-relativistic nuclear models. Thus it seems more natural to start with the non-relativistic Hamiltonian than to make the non-relativistic approximation of nuclear matrix elements [7], [8].

c) The guiding principle in constructing our Hamiltonian was that it had to be obtained from the relativistic Hamiltonian. (For example, by the Foldy-Wouthuysen transformation [9].) The investigations of the nonrelativistic approximations [10], [11], [12], [13] indicate that almost all possible combinations of available operators (nuclear moment $\mathbf{p}$, nuclear radius $\mathbf{r}$, $\vec{\sigma}$ matrix, and lepton covariants $\mathbf{L}$ and L_P) should be obtained. They are multiplied by different form factors, whose structure depends on the assumption about the structure of the relativistic Hamiltonian.

d) The investigation of mesonic effects in β decay [14], [15], [16] indicates that some new terms can be induced. The structure of one of them is the same as our term with B_4. The second, »induced pseudoscalar« is not of the same structure, as the phenomenological pseudoscalar term introduced in Hamiltonian (1). Some preliminary investigations show that such a term cannot significantly change our conclusions [41].

e) It seems that the β interaction is of the V-A type. We chose the lepton covariants in agreement with the propositions given by Feynman et al [14], and in agreement with the two-component neutrino theory.

f) in our calculations we assumed that all terms except the common ones (i. e. all terms with the form factors B_i) could give only small corrections (up to 10%). We, therefore, calculated only the cross terms with the leading terms.

It is possible to construct quite a general phenomenological theory. In such a theory all terms must be treated as equally important.

Our non-relativistic Hamiltonian is then:

$$H_{int} = g_V L + \frac{g_V}{2M} B_1 \mathbf{p}_L \mathbf{L} + \frac{g_V}{M} B_2 \mathbf{L}\mathbf{p} + 2i g_V B_3 \mathbf{L}\mathbf{r} +$$

$$+ \frac{i g_V}{2M} B_4 (\mathbf{p}_L \times \mathbf{L}) - g_V B_5 (\vec{\sigma} \times \mathbf{r}) \mathbf{L} + g_A \vec{\sigma} \mathbf{L} + \frac{g_A}{2M} B_6 \vec{\sigma} \mathbf{p}_L L +$$

$$+ \frac{g_A}{M} B_7 L \vec{\sigma} \mathbf{p} + 2i g_A B_8 \vec{\sigma} \mathbf{r} L - \frac{g_p}{2M} B_9 \vec{\sigma} \mathbf{p}_L L_P - i g_P B_{10} \vec{\sigma} \mathbf{r} L_P \quad (1)$$

$\mathbf{p}_L$ means that $\mathbf{p}$ operates on the lepton wave functions only. $\mathbf{L}$, L and L_P are the lepton covariants for the axial vector, vector and pseudoscalar, respectively. g_K are the β decay coupling constants. B_i are unknown form factors, containing all possible influences.

With $B_5 = B_8 = B_{10} = 0$ and with all other B_i equal to one, the approximation of Case I in ref. [13] is obtained. This corresponds to the usual form of the β decay theory. According to weak magnetism hypothesis [15] we can estimate $B_4 \approx (\mu_p - \mu_n) / 2M$.

2. *Correction Factors for Fermi and Gamow-Teller Transitions*

The correction factor for Fermi transitions is given by

$$C_F^\beta = g_V^2 \left(|\langle 1 \rangle|^2 C_{1F} + \langle 1 \rangle \langle r^2 \rangle^* C_{2F} + \langle 1 \rangle \langle B_1 \rangle^* C_{3F} + \right.$$
$$\left. + \langle 1 \rangle \langle i B_2 \mathbf{r}\mathbf{p} \rangle^* C_{4F} + \langle 1 \rangle \langle r^2 B_3 \rangle^* C_{5F} \right) \quad (2)$$

with

$$C_{1F} = L_0 \quad (3)$$

$$C_{2F} = 2 \left(\frac{q}{3} N_0 - \frac{q^2}{6} L_0 \right) \quad (4)$$

$$C_{2F} = \frac{1}{M} [(U + 2q) L_0 - P_0] \quad (5)$$

$$C_{4F} = \frac{2}{M} \left(\frac{2}{3} L_0 - N_0 \right) \quad (6)$$

$$C_{5F} = 4 \left(\frac{q}{3} L_0 - N_0 \right) \quad (7)$$

$$U = W - V - q \quad (8)$$

$$V = - \frac{\alpha Z}{r_0} \quad (9)$$

All notations have the usual meaning.

If the influence of the term C_{2F} is important (i. e. if the ratio of matrix elements $<r^2>$ and $<1>$ is not very small) it is necessary to take the variation of C_{1F} with r into account with higher accuracy. The actual form of L_0 is

$$L_0 = A(Z, W) + B(Z, W)\, r + C(Z, W)\, r^2 + \ldots \tag{10}$$

It is, therefore, necessary to replace (3) and (4) by

$$C_{1F} = A(Z, W) \tag{11}$$

$$C_{2F} = 2\left(\frac{q}{3} N_0 - \frac{q^2}{6} L_0\right) + C(Z, W) \tag{12}$$

and add

$$\bar{C}_{1F} = g_V^2 <1><|r|>^* B(Z, W) \tag{13}$$

to (2).

If the replacement of (11), (12) and (13) is not made it is necessary to make the estimation $<r^2> \approx <1>\, r_0^2$ to be consequent. That was overlooked in previous calculations [7,8], but the validity of such a treatment can easily be cheked by making a comparison with calculations in the plane wave approximation.

Longitudinal polarization is defined by

$$\mathbf{P} = \pm \frac{\mathbf{p}_e}{W} \frac{P^\beta}{C^\beta} \tag{14}$$

$\mathbf{p}_e$ is the electron momentum. The upper and lower signs refer to the electron and positron emission respectively.

$P_F{}^\beta$ ist of the form:

$$P_F{}^\beta = g_V^2 (|<1>|^2 P_{1F} + <1><r^2>^* P_{2F} + <1><B_1>^* P_{3F} + \\ + <1><i\, B_2\, \mathbf{r}\, \mathbf{p}>^* P_{4F} + <1><r^2\, B_3>^* P_{5F}) \tag{15}$$

with:

$$P_{1F} = -A_0\, n \tag{16}$$

$$P_{2F} = \left(\frac{1}{3} q^2 A_0 - \frac{2}{3} q\, C_0\right) n \tag{17}$$

$$P_{3F} = -\frac{1}{M} (W + q - V)\, A_0\, n \tag{18}$$

$$P_{4F} = \frac{2}{M}\left(C_0 - \frac{q}{3} A_0\right) n \tag{19}$$

$$P_{5F} = 4\left(C_0 - \frac{q}{3} A_0\right) n \tag{20}$$

$$n = \cos(\Delta_1 - \Delta_{-1}) \tag{21}$$

The form of A_0 is

$$A_0 = E(Z, W) + F(Z, W)\, r + G(Z, W)\, r^2 + \ldots \tag{22}$$

If the variation of r is important it is necessary to make the replacement:

$$P_{1F} = -E(Z, W)\, n \tag{23}$$

$$P_{2F} = \left(\frac{1}{3} q^2 A_0 - \frac{2}{3} q C_0 - G(Z, W) \right) n \tag{24}$$

and add

$$\bar{P}_{1F} = -g_V^2 \langle 1 \rangle \langle |r| \rangle^* F(Z, W)\, n \tag{25}$$

The definitions of $A_0, B_0, C_0, D_0, A(Z, W), B(Z, W), C(Z, W), E(Z, W), F(Z, W), G(Z, W)$ and n_k are given in Appendix I.

The correction factor for Gamow-Teller transitions is

$$C_{GT^\beta} = 4\pi g_A^2 [|\langle T_{10} \rangle|^2 C_{1\,GT} + \langle T_{10} \rangle \langle r^2 T_{10} \rangle^* C_{2\,GT} +$$

$$+ \langle T_{10} \rangle \langle r^2 T_{12} \rangle^* \sqrt{2}\, C_{3\,GT} \pm \frac{g_V}{g_A} \langle T_{10} \rangle \langle B_4 T_{10} \rangle^* C_{4\,GT} \pm$$

$$\pm \frac{g_V}{g_A} \langle T_{10} \rangle \langle B_4 T_{12} \rangle^* \sqrt{2}\, C_{5\,GT} \pm$$

$$\pm \frac{g_V}{g_A} \langle T_{10} \rangle \langle i B_2 r T_{11}(\mathbf{p}) \rangle^* C_{6\,GT} + \langle T_{10} \rangle \langle B_6 T_{10} \rangle^* C_{7\,GT} +$$

$$\langle T_{10} \rangle \langle B_6 T_{12} \rangle^* \sqrt{2}\, C_{8\,GT} + \langle T_{10} \rangle \langle i B_7 r Y_1 \vec{\sigma}\, \mathbf{p} \rangle^* \frac{1}{\sqrt{3}} C_{9\,GT} +$$

$$+ \frac{g_P}{g_A} \langle T_{10} \rangle \langle B_9 T_{10} \rangle^* C_{10\,GT} +$$

$$+ \frac{g_P}{g_A} \langle T_{10} \rangle \langle B_9 T_{12} \rangle^* \sqrt{2}\, C_{11\,GT} \pm \frac{g_V}{g_A} \langle T_{10} \rangle \left(\langle r^2 B_5 T_{10} \rangle^* + \right.$$

$$\left. + \frac{\sqrt{2}}{2} \langle r^2 B_5 T_{12} \rangle^* \right) C_{12\,GT} + \langle T_{10} \rangle \left(\frac{1}{3} \langle r^2 B_8 T_{10} \rangle^* - \right.$$

$$\left. - \frac{\sqrt{2}}{3} \langle r^2 B_8 T_{12} \rangle^* \right) C_{13\,GT} + \frac{g_P}{g_A} \langle T_{10} \rangle \left(\frac{1}{3} \langle r^2 B_{10} T_{10} \rangle^* - \right.$$

$$\left. - \frac{\sqrt{2}}{3} \langle r^2 B_{10} T_{12} \rangle^* \right) C_{14\,GT}] \tag{26}$$

with:

$$C_{1\,GT} = L_0 \tag{27}$$

$$C_{2\,GT} = -\frac{q^2}{3} L_0 - \frac{2q}{9} N_0 \tag{28}$$

$$C_{3\,GT} = -\frac{4q}{9} N_0 \tag{29}$$

$$C_{4\,GT} = \frac{2}{3M} (P_0 - U L_0) \tag{30}$$

$$C_{5\,GT} = \frac{1}{3M} [P_0 - 3 N_0 - (W - V) L_0] \tag{31}$$

$$C_{6\,GT} = \frac{2}{M} \left(\frac{q}{3} L_0 + N_0 \right) \tag{32}$$

$$C_{7\,GT} = \frac{1}{3M} [(U + 2q) L_0 - P_0] \tag{33}$$

$$C_{8\,GT} = C_{5\,GT} \tag{34}$$

$$C_{9\,GT} = \frac{2}{M} \left(\frac{q}{3} L_0 - N_0 \right) \tag{35}$$

$$C_{10\,GT} = \frac{1}{3M} (L_0 - U P_0) \tag{36}$$

$$C_{11\,GT} = \frac{1}{3M} [(W - V) P_0 - L_0 + 3 R_0] \tag{37}$$

$$C_{12\,GT} = -\frac{2M}{3} C_{6\,GT} \tag{38}$$

$$C_{13\,GT} = 2M C_{9\,GT} \tag{39}$$

$$C_{14\,GT} = 2 \left(\frac{q}{3} P_0 + R_0 \right) \tag{40}$$

The upper and lower signs refer to the electron and positron emission, respectively.

If the variation of r is important it is necessary to make a change in the expression (26)

$$C_{1\,GT} = A (Z_0 W) \tag{41}$$

$$C_{2\,GT} = \frac{q^2}{3} L_0 + \frac{2q}{9} N_0 + C (Z, W) \tag{42}$$

$$\bar{C}_{1\,GT} = g_A^2 \langle T_{10} \rangle \langle |r| T_{10} \rangle^* B (Z, W) \tag{43}$$

The definition of the tensor operator is

$$T^M_{J\Lambda} (\mathbf{v}) = \sum_{\nu, m} (1\, \nu\, \Lambda\, m \,|\, 1\, \Lambda\, J\, M)\, v_1{}^{\nu}\, Y^m_{\Lambda} \tag{44}$$

where $\mathbf{v}$ is the vector operator. (The operator $\vec{\sigma}$ is omitted in our notation for nuclear matrix elements in (26) and (45).) The relevant

tensor operators are given in Cartesian representation in *Table* 1, with the corresponding selection rules.

Table 1.

Tensor operator	*Cartesian components*	*Selection rules*		
$T_{10}(\vec{\sigma})$	$\frac{\vec{\sigma}}{\sqrt{4\pi}}$	$\Delta J = 0;1$	$\Delta l = 0$	$\Delta\Pi = 0$
$r^2 T_{12}(\vec{\sigma})$	$\frac{1}{\sqrt{4\pi}} \frac{\sqrt{2}}{2} [\vec{\sigma} r^2 - 3(\vec{\sigma}\,\mathbf{r})\,\mathbf{r}]$	$\Delta J = 0;1$	$\Delta l = 0;2$	$\Delta\Pi = 0$
$r\,T_{11}(\mathbf{p})$	$\frac{1}{i\sqrt{4\pi}} \sqrt{\frac{3}{2}} (\mathbf{r} \times \mathbf{p})$	$\Delta J = 0;1$	$\Delta l = 0$	$\Delta\Pi = 0$
$r\,Y_1 \vec{\sigma}\,\mathbf{p}$	$\frac{1}{\sqrt{4\pi}} \sqrt{3}\,\mathbf{r}((\vec{\sigma})\,\mathbf{p})$	$\Delta J = 0;1$	$\Delta l = 0;2$	$\Delta\Pi = 0$

Δl means the possibility of changing the orbital momenta. This selection rule can be formulated only according to the shell model. The operator $T_{12}(\vec{\sigma})$ (see the terms $C_{5\,GT}$, $C_{8\,GT}$, $C_{11\,GT}$) aots only on the angular part and final state must be the same. (That is not valid for $r^2 T_{12}(\vec{\sigma})$.)

Except the common terms, some parts of the expressions (2) and (26) were derived in ref. [18]. Morita's treatment [7,8] will give the same result as the present one in the case $Z = 0$, if the relations of Feenberg et al [19] and Yamada [20] between the nuclear matrix elements of Morita are used. (Of course, with an appropriate choice of the nuclear form factors B_i.)

Longitudinal polarizaton can be calculated by the expression (14), where P_{GT^β} is given by:

$$P_{GT^\beta} = 4\pi g_A^2 \Big[|\langle T_{10}\rangle|^2 P_{1\,GT} + \langle T_{10}\rangle\langle r^2 T_{10}\rangle^* P_{2\,GT} +$$

$$+ \langle T_{10}\rangle\langle r^2 T_{12}\rangle^* \sqrt{2}\, P_{3\,GT} \pm \frac{g_V}{g_A} \langle T_{10}\rangle\langle B_4 T_{10}\rangle^* P_{4\,GT} \pm$$

$$\pm \frac{g_V}{g_A} \langle T_{10}\rangle\langle B_4 T_{12}\rangle^* \sqrt{2}\, P_{5\,GT} \pm$$

$$\pm \frac{g_V}{g_A} \langle T_{10}\rangle\langle i B_2\, r T_{11}(\mathbf{p})\rangle^* P_{6\,GT} + \langle T_{10}\rangle\langle B_8 T_{10}\rangle^* P_{7\,GT} +$$

$$+ \langle T_{10}\rangle\langle B_6 T_{12}\rangle^* \sqrt{2}\, P_{8\,GT} + \langle T_{10}\rangle\langle i B_5\, r\, Y_1 \vec{\sigma}\,\mathbf{p}\rangle^* \frac{1}{\sqrt{3}} P_{9\,GT} +$$

$$+ \frac{g_P}{g_A} \langle T_{10}\rangle\langle B_9 T_{10}\rangle^* P_{10\,GT} + \frac{g_P}{g_A} \langle T_{10}\rangle\langle B_9 T_{12}\rangle^* \sqrt{2}\, P_{11\,GT} \pm$$

$$\pm \frac{g_V}{g_A} <T_{10}> \left(<r^2 B_5 T_{10}>^* + \frac{\sqrt{2}}{2} <r^2 B_5 T_{12}>^* \right) P_{12\,GT} +$$

$$+ <T_{10}> \left(\frac{1}{3} <r^2 B_8 T_{10}>^* - \frac{\sqrt{2}}{3} <r^2 B_8 T_{12}>^* \right) P_{13\,GT} +$$

$$+ \frac{g_P}{g_A} <T_{10}> \left(\frac{1}{3} <r^2 B_{10} T_{10}>^* - \frac{\sqrt{2}}{3} <r^2 B_{10} T_{12}>^* \right) P_{14\,GT} \Big] \quad (45)$$

with

$$P_{1\,GT} = - A_0\, n \quad (46)$$

$$P_{2\,GT} = \left(\frac{2}{9} q\, C_0 + \frac{q^2}{3} A_0 \right) n \quad (47)$$

$$P_{3\,GT} = \frac{4}{9} q\, C_0\, n \quad (48)$$

$$P_{4\,GT} = \frac{2}{3M} U\, A_0\, n \quad (49)$$

$$P_{5\,GT} = \frac{1}{3M} [3\, C_0 + (W - V)\, A_0]\, n \quad (50)$$

$$P_{6\,GT} = - \frac{2}{M} \left(C_0 + \frac{q}{3} A_0 \right) n \quad (51)$$

$$P_{7\,GT} = - \frac{1}{3M} (E_0 - V)\, A_0\, n \quad (52)$$

$$P_{8\,TG} = P_{5\,GT} \quad (53)$$

$$P_{9\,GT} = \frac{2}{M} \left(C_0 - \frac{q}{3} A_0 \right) n \quad (54)$$

$$P_{10\,GT} = - \frac{1}{3M} A_0\, n \quad (55)$$

$$P_{11\,GT} = \frac{1}{3M} (A_0 - 3\, D_0)\, n \quad (56)$$

$$P_{12\,GT} = - \frac{2M}{3} P_{6\,GT} \quad (57)$$

$$P_{13\,GT} = 2M\, P_{9\,GT} \quad (58)$$

$$P_{14\,GT} = -2\, D_0\, n \quad (59)$$

If the variation of r is taken into account one has to replace (46) and (49) hy

$$P_{1\,GT} = - E(Z, W)\, n \quad (60)$$

$$P_{2\,GT} = \left(\frac{2}{9} q\, C_0 + \frac{q^2}{3} A_0 - G(Z, W) \right) n \quad (61)$$

and add the expression

$$\bar{P}_{1\,GT} = - g_A^2 <T_{10}> <|r|> T_{10}>^* F(Z, W) \quad (62)$$

to (45).

The above expressions for the correction factors in the low Z limit are given in Appendix IV.

3. *Applications*

For the elements Na^{22}, Na^{24}, P^{32}, Zr^{89}, In^{114} the deviations from the allowed shape result in the decreasing of the spectrum shape factor with the electron energy for electrons and positrons. The experimental results are given in *Table* 2.*

Table 2

Element	*Type of decay Max. energy in MeV log ft*	*Experimental shape factor*	*Reference*
Na^{22}	β^+	$1 - \frac{0.016}{W}$	[4]
	0.545		
	7.42	$1 + \frac{b}{W}$, $0.23 < b < 0.35$	[2]
Na^{24}	β^-	$1 + \frac{0.072}{W}$	[4]
	1.389		
	6.11	$1 - 0.015\ W$	[4]
P^{32}	β^-	$1 - 0.041\ W$	[4]
	1.707	$1 - 0.042\ W \pm 0.01\ W$	priv. com.
	7.9	$1 + \frac{b}{W}$, $0.2 < b < 0.4$	[1]
Zr^{89}	β^+	$1 + \frac{b}{W}$ $0.25 < b < 0.45$	[3]
	0.897		
	6.1	$1 - 0\,39\ W + 0.09\ W^2$	[3]
In^{114}	β^-	$1 + \frac{b}{W}$, $0.2 < b < 0.3$	[1]
	1.984		
	4.4		

* *Note added in proof*

* The β spectra of In^{114} and P^{32} have been measured recently by Nichols, McAdams and Jensen (Phys. Rev. 122 (1961) 172)

They obtained the next results

element	experimental shape factor
In^{114}	$1 + (0.0036 \pm 0.0021)$ W
P^{32}	$1 - (0.0133 \pm 0.0011)$ W

The result for P^{32} is in qualitative agreement with result of ref. [4], but for In^{114} is in disagreement with ref. [1]. Dr. McAdams informed us that their results need aditional justification. The authors are very thankfull to Dr. McAdams for this communications.

Case I — small ft-values

First we shall discuss the possibility of producing the deviations from the allowed shape by some intrinsic properties of the β decay interaction. The consequence of such hypothesis is that the deviation is present in all allowed decays.* The most suitable decays for studying this problem are β decays with low ft-values. We can assume that the contributions of the correction factors C_{2F} (4), C_{2GT} (28), C_{3GT} (29), P_{2F} (17), P_{2GT} (47), P_{3GT} (48) are small because their nuclear matrix elements are of an order of r_0^2. (There are some cases where the influence of nuclear structure can make these terms important, for example the decays with large ft values, $\log ft \approx 6$ or even larger. This will be discussed in Case II.) The other correction factors can become important because the form factors B_i by which they are multiplied may be perhaps larger than 1. From the present calculations it seems that the exact expression for L_0 etc. [46], [47], finite size corrections [44], [45], correction for screening [44], radiative correction [48] and correction due to the inner bremstrahlung [48] can change the results for only a few percents. Since we are trying to explain the measured deviations which are at least 10 times larger, we can neglect such fine corrections.

The correction factors for the allowed decays can then be written in the low Z approximation. (This corresponds to the ξ approximation of Kotant et al. [42])

$$C^\beta = g_V^2 \left|<1>\right|^2 C_F^\beta + g_A^2 \left|<\vec{\sigma}>\right|^2 C_{GT}^\beta \tag{63}$$

$$P^\beta = g_V^2 \left|<1>\right|^2 P_F^\beta + g_A^2 \left|<\vec{\sigma}>\right|^2 P_{GT}^\beta \tag{64}$$

with

$$C_F^\beta = 1 + \frac{2B_1}{M}(E_0 \pm \xi) + a\left(\frac{E_0}{3} \pm \xi\right) - \frac{2B_1}{M} W - \frac{1}{W}\left(\frac{B_1}{M} + \frac{a}{3}\right) \tag{65}$$

$$P_F^\beta = -\left[1 + \frac{B_1}{M}(E_0 \pm 2\xi) + a\left(\frac{E_0}{3} \pm \xi\right)\right] \tag{66}$$

$$a = \frac{2}{M} B_2 \varepsilon_1 + 4 B_3 \varepsilon_2 \tag{67}$$

$$C_{GT}^\beta = c_1 + c_2 \frac{1}{W} + c_3 W \tag{68}$$

$$P_{GT}^\beta = -(c_4 + c_3 W) \tag{69}$$

* That is also the consequence of the hypothesis about an admixture of the neutrino with the spin $^3/_2$ ([23]), except in $O^+ \to O^+$ transitions.

Here we use the following abbreviations:

$$\begin{pmatrix} c_1 \\ c_4 \end{pmatrix} = 1 - a_1(E_0 \mp 3\xi) - \frac{c_3}{2}(E_0 \mp 3\xi) \mp \frac{1}{3M} B_6(1-\varkappa_4)\xi - \\ - \frac{1}{3M} \Lambda B_4(2+\varkappa_4)\xi + \begin{pmatrix} - \\ + \end{pmatrix} a_2 \tag{70}$$

$$c_2 = a_1 + a_2(E_0 \mp 3\xi) \pm \frac{g_P}{g_A} \frac{1}{3M} B_9(1-\varkappa_4)\xi - \frac{c_3}{2} \tag{71}$$

$$c_3 = \pm \frac{2\Lambda}{3M}(2B_4 + 2B_2\varkappa_5 - B_5\varkappa_3) \tag{72}$$

$$a_1 = -(B_6 + 2B_7\varkappa_1 + 2B_8\varkappa_2)\frac{1}{3M} \tag{73}$$

$$a_2 = \frac{g_P}{g_A}(B_9 + B_{10}\varkappa_2)\frac{1}{3M} \tag{74}$$

The upper and lower signs refer to the electron and positron emission, respectively. $\begin{pmatrix} - \\ + \end{pmatrix}$ in the last term of (70) means that — must be taken for c_1 and + for c_4. E_0 is the maximal lepton energy. We assumed that the r dependence of B_i is not important. The other notations used are as follows:

$$\xi = \left| \frac{\alpha Z}{2r_0} \right| \tag{75}$$

$$\Lambda = \left| \frac{g_V}{g_A} \right| \tag{76}$$

$$\varepsilon_1 = \frac{<i\,\mathbf{r}\,\mathbf{p}>}{<1>} \tag{77}$$

$$\varepsilon_2 = \frac{<r^2>}{<1>} \tag{78}$$

$$\varkappa_1 = \frac{<i\,r\,Y_1\,\vec{\sigma}\,\mathbf{p}>}{<T_{10}>\sqrt{3}} \tag{79}$$

$$\varkappa_2 = \frac{2M}{3<T_{10}>}(<r^2\,T_{10}> - \sqrt{2}<r^2\,T_{12}>) \tag{80}$$

$$\varkappa_3 = \frac{4M}{3<T_{10}>}(<r^2\,T_{10}> + \frac{\sqrt{2}}{2}<r^2\,T_{12}>) \tag{81}$$

$$\varkappa_4 = \frac{<T_{12}>\sqrt{2}}{<T_{10}>} \tag{82}$$

$$\varkappa_5 = \frac{<i\,r\,T_{11}(\mathbf{p})>}{<T_{10}>} \tag{83}$$

If the reason of the deviations is really the intrinsic property of the β decay interaction it can be examined by measuring the spectrum shape and longitudinal polarization in the superallowed transitions [21], [22], [35]. For such decays satisfactory propositions about the relative sign and the order of magnitude of the nuclear matrix elements could be done on the basis of the shell model. Thus we can separate the effects produced by the influence of nuclear structure.

The examination of the correction factors (63) and (64) shows that if C^β is energy dependent, then longitudinal polarization for some combinations of parameters can vary with energy too. The direct information on the variation of P^β with the electron energy can be very useful too. According to this we defined the »relative longitudinal polarization«. (See Appendix I).

In *Table* 3 the nuclear matrix elements for mirror transitions calculated on the basis of the shell model are given.

Table 3.

J	l	$\lvert<\vec{\sigma}>\rvert^2$	$\varkappa_1$	$\frac{5}{2Mr_0^2}\varkappa_2$	$\frac{5}{4Mr_0^2}\varkappa_3$	$\varkappa_4$	$\varkappa_5$
$\frac{1}{2}$	0	3	$-\frac{1}{\sqrt{8}}$	1	1	0	$\frac{1}{\sqrt{6}}$
$\frac{1}{2}$	1	$\frac{1}{3}$	$-\frac{1}{\sqrt{24}}$	-3	3	4	$-\sqrt{2}$
$\frac{3}{2}$	1	$\frac{5}{3}$	$-\frac{1}{\sqrt{24}}$	$\frac{3}{5}$	$\frac{6}{5}$	$\frac{2}{5}$	$\frac{1}{\sqrt{2}}$
$\frac{3}{2}$	2	$\frac{3}{5}$	$-\frac{1}{\sqrt{40}}$	-1	2	2	$-\sqrt{\frac{27}{10}}$
$\frac{5}{2}$	2	$\frac{7}{5}$	$-\frac{1}{\sqrt{40}}$	$\frac{3}{7}$	$\frac{9}{7}$	$\frac{4}{7}$	$\sqrt{\frac{6}{35}}$
$\frac{5}{2}$	3	$\frac{5}{7}$	$-\frac{1}{\sqrt{56}}$	$-\frac{3}{5}$	$\frac{9}{5}$	$\frac{8}{5}$	$-\sqrt{\frac{24}{7}}$
$\frac{7}{2}$	3	$\frac{9}{7}$	$-\frac{1}{\sqrt{56}}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{2}{3}$	$\sqrt{\frac{27}{2}}$

For all cases holds: $|<1>|^2 = 1$, $\varepsilon_1 = -\frac{3}{2}$, and $\varepsilon_2 = \frac{3}{5} r_0^2 \cdot J$ is the spin of the initial and final nuclei, l is the orbital angular momentum of the last nucleon.

The radial part of these matrix elements is calculated with oscillator wave functions normalized in such a way that the diagonal matrix element of r^2 is always 3/5 r_0^2. This table covers all known mirror transitions. The corrected values of $|<\vec{\sigma}>|^2$ are given in ref. [24], [25]. It seems that these corrections would not influence the ratios of the nuclear matrix elements $\varkappa_i$ too much.

In the standard β decay theory, the correction factors for β decay and longitudinal polarization for mirror transitions are given with the formulas (63) and (64), where we have

$$C_F^{\beta} = 1 + \frac{1}{M}(E_0 - \xi) + \bar{a}\left(\frac{E_0}{3} - \xi\right) - \frac{2}{M} W - \frac{1}{W}\left(\frac{1}{M} + \frac{\bar{a}}{3}\right) \tag{65a}$$

$$P_F^{\beta} = -\left[1 + \frac{1}{M}(E_0 - 2\xi) + \bar{a}\left(\frac{E_0}{3} - \xi\right)\right] \tag{66a}$$

$$\bar{a} = \frac{2}{M}\varepsilon_1 + 4\varepsilon_2 \tag{67a}$$

$$C_{GT}^{\beta} = \bar{c}_1 + \bar{c}_2 \frac{1}{W} + \bar{c}_3 W \tag{68a}$$

$$P_{GT}^{3} = -(\bar{c}_1 + \bar{c}_3 W) \tag{69a}$$

$$\bar{c}_1 = 1 - \bar{a}_1(E_0 + 3\xi) - \frac{\bar{c}_3}{2}(E_0 + 3\xi) + \frac{1}{3M}(1 - \varkappa_4)\xi -$$

$$- \frac{1}{3M} A B_4 (2 + \varkappa_4)\xi \tag{70a}$$

$$\bar{c}_2 = \bar{a}_1 - \frac{\bar{c}_3}{2} \tag{71a}$$

$$\bar{c}_3 = -\frac{2A}{3M}(2B_4 + 2\varkappa_5) \tag{72a}$$

$$\bar{a}_1 = -(1 + 2\varkappa_1)\frac{1}{3M} \tag{73a}$$

B_4 is one, or according to the weak magnetism hypothesis $B_4 = \mu_p - \mu_n$.

For example, the correction factors C^{β} and P^{β} for the transitions $Al^{25} \to Mg^{25}$ are drawn as a function of c_2 (71). The other terms are given as in the standard β decay theory.

$O^+ \to O^+$ transitions can give some information about the pure Fermi interaction. The nuclear matrix elements in such a case are: $|<1>|^2 = 2$; $\varepsilon_1 = -\frac{3}{2}$; $\varepsilon_2 = \frac{3}{2} r_0^2$.

For pure G.-T. transitions we can make the following conclusions from the correction factors (68) and (69):

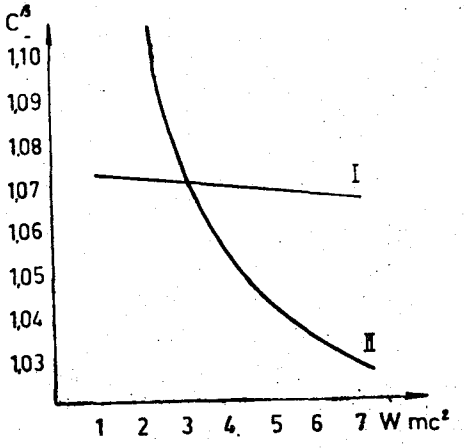

Fig. 1. The correction factors C^β for the decay of Al^{25} is plotted as a function of energy in arbitrary units. Curve I corresponds to C_{F^β} (65a) and C_{GT^β} (68a). Curve II is drawn for $c_2 = 0.3$. B_4 is taken 1.

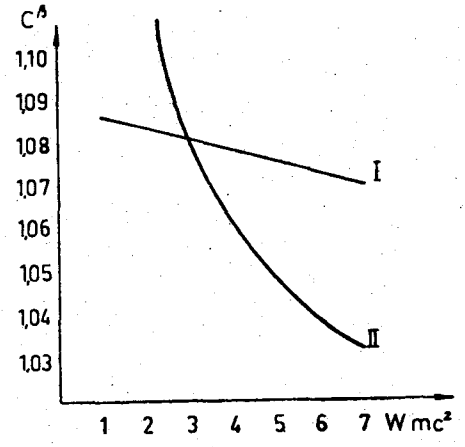

Fig. 2. The same as in *Fig.* 1, only $B_4 = 4.705$.

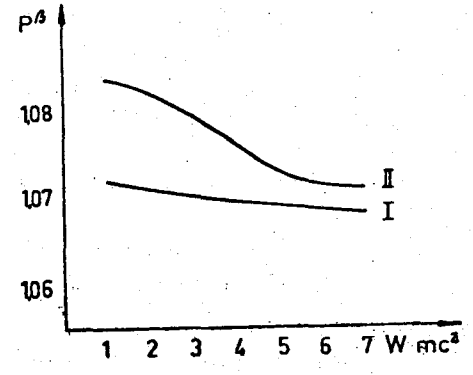

Fig. 3. The correction factors for the relative longitudinal polarization for the decay of Al^{25} are plotted as a function of energy in arbitrary units. Curve I corresponds to $B_4 = 1$ and Curve II to $B_4 = 4.705$. P^β is given with the formulas (66a) and (69a).

a) It is possible to combine the factors c_2 (71) and c_3 (72) in such a way as to obtain the correction factors which decrease with

energy for electrons and positrons. But the shape of the correction factors will then not be the same for electrons and positrons.

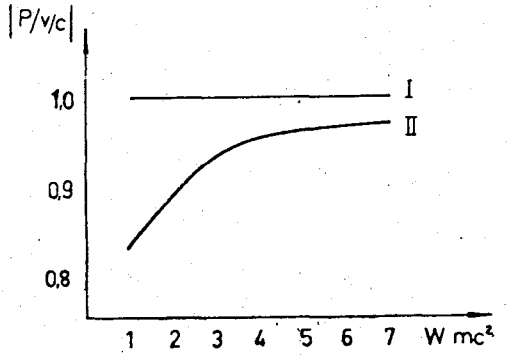

Fig. 4. The absolute value of longitudinal polarization of positrons divided by v/c for the decay of Al^{25} is given as a function of the electron energy. Curve I corresponds to c_2 (71a) and Curve II corresponds to $c_2 = 0.3$. The results are insensitive to the changes of B_4.

b) The relative sign of the nuclear matrix elements may be rather important. In our version we have some terms linear in nuclear matrix elements, for example $<T_{10}><T_{12}>^* f(Z, W)$. The relative sign of such combinations can in principle vary from nuclei to nuclei, so it is not possible to determine uniquely the sign of such terms for positrons or electrons respectively. This caused great difficulties in making some general conclusions.

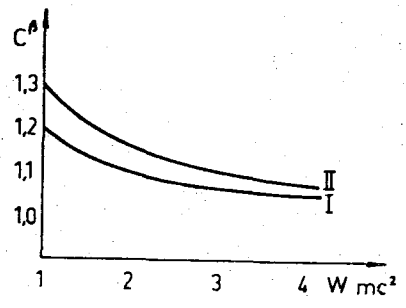

Fig. 5. The correction factors C^β for the decay of In^{114} are plotted as a function of energy. Curve I refers to the correction factor $1 + \frac{0.2}{W}$ and Curve II to the correction factor $1 + \frac{0.3}{W}$.

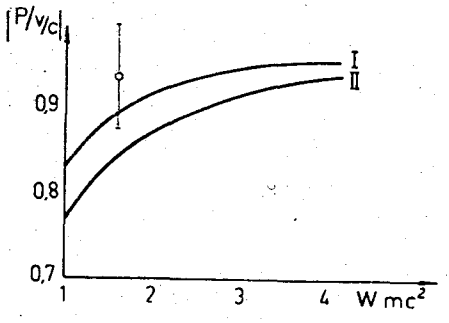

Fig. 6. The absolute value of longitudinal polarization of electrons divided by $\frac{v}{c}$ in the decay of In^{114} is given as a function of electron energy. Curves I and II correspond to the Curves I and II in Fig. 6 respectively. The experimental point measured by Spivak et al [43], is indicated.

c) The variation of longitudinal polarization with energy can be rather large in some cases. For example, in the case of In^{114}, although

it has $ft = 4.4$, it seems that it is possible to make a supposition that its spectrum shape is not influenced by nuclear structure. It is taken $c_3 \approx 0$ and c_1 and c_2 are adjusted to reproduce the spectrum shape [1]. Then the calculated longitudinal polarization will have a variation of about 20% with energy. (See *Fig.* 5 and *Fig.* 6)

Spivak et al. measured longitudinal polarization at 300 kev. Their value, $(-0.93 \pm 0.06)\frac{v}{c}$, is in poor agreement with the calculated one. The measurement of the variation of longitudinal polarization with energy, can serve as a good test.

d) P^{β} connected with measurements of the relative longitudinal polarization (see Appendix I) is independent of energy, or varies linearly with energy.

e) It is impossible to make any definite conclusions about the presence of the pseudoscalar type of interaction. (a_2 in the expressions (70) and (71).)

f) It seems that with a reasonable choice of the form factor B_i it is impossible to reproduce the same deviation from the allowed shape for β^+ and β^- decay. The form factors B_i must be small enough so that it is not necessary to take into account the terms with the squares of the form factors. Of course this conclusion is not sure enough, because it is difficult to take into account the influence of nuclear structure. W_R can obtain some definite conclusions only in an extremely simplified case.

We shall investigate two hypothetical nuclei, one decaying by β^- and the other by β^+. Let them have the same values of E_0, ξ and $\varkappa_i$. We are seeking for such a B_i as to obtain the same spectrum shape for both nuclei. The first conclusion is that c_3 must be small, since it changes the sign for β^- and β^+, and we shall neglect it. The conditions on c_1 and c_2 are: $c_1 > 0$ and $0.2 < c_2 / c_1 < 0.4$ for β^+ and β^- decay. The first condition is obvious, and the second follows from the experimental results.

The correction factors can then be written in the form:

$$c_1 = 1 - E_0 a_1 \mp \xi a_3 - a_2 - \xi a_6 \tag{84}$$

$$c_2 = a_1 + a_2 E_0 \mp \xi a_4 \tag{85}$$

$$a_3 = 3 a_1 + a_5 \tag{86}$$

$$a_4 = 3 a_2 - a_7 \tag{87}$$

a_1 and a_2 are given by (73) and (74) respectively.

$$a_5 = \frac{1}{3M} B_6 (1 - \varkappa_4) \tag{88}$$

$$a_6 = \frac{1}{3M} \Lambda B_4 (2 + \varkappa_4) \tag{89}$$

$$a_7 = \frac{g_P}{g_A} \frac{1}{3M} B_9 (1 - \varkappa_4) \tag{90}$$

With conditions

$$\left(\frac{c_2}{c_1}\right)_{\beta+} = \left(\frac{c_2}{c_1}\right)_{\beta-} = x \tag{91}$$

$$c_{1\,\beta^-} = y \quad c_{1\,\beta^+} = z \tag{92}$$

we have six equations for seven parameters a_i. The solutions of the system are:

$$a_1 = \frac{1}{1 - E_0^2}\,(x\,z + z\,E_0 - x\,a_3\,\xi - E_0 - a_3\,\xi\,E_0 + a_6\,\xi\,E_0) \tag{93}$$

$$a_2 = \frac{1}{E_0^2 - 1}\,(x\,z\,E_0 + z - 1 - a_3\,\xi - \mathrm{x}\,a_3\,E_0 + a_6\,\xi) \tag{94}$$

$$a_3 = \frac{z - y}{2\,\xi} \tag{95}$$

$$a_4 = x\,a_3 \tag{96}$$

a_5 and a_7 are fixed with the equations (86) and (87). For reasonable choices of y and z we can fit these results with very large B_i. In such a way all the calculations are not valid because the quadratic terms with B_i must be taken into account. For example $x = 0.3$, $y \approx z$, $\varkappa_4 \approx 1$ we obtain $B_4 \approx -250$.

Since such a conclusion is obtained for a case that does not exist in reality, it can serve only as an indication, and not as a proof.

With the modification of the β decay theory proposed by Kuchowicz [26] it is difficult to reproduce the same deviations for β^- and β^+ decays. The theory of Kuchowicz is a particular case of our version if one takes into account the time derivative too. All B_i are small except B_6, which is connected with Kuchowicz's **B** as follows:

$$\vec{\sigma}\,B_6 = -\mathbf{B} \tag{97}$$

So we shall neglect all terms except the term with B_6 and add the time derivative term to the correction factor. With the notation

$$\mathbf{C} = C\,\vec{\sigma} \text{ and } \mathbf{B} = B\,\vec{\sigma} \tag{98}$$

the result is

$$C_{GT^\beta} = 1 - (C + \frac{2}{3}\,B)\,E_0 \mp \frac{2}{3}\,B\,(2 + \varkappa_4)\,\xi + \frac{2}{3}\,B\,\frac{1}{W} \tag{99}$$

The third term changing the sign for β^- and β^+ could not be small, Because B must be large enough to give the observed deviation. $\varkappa_4$, which represents the influence of nuclear structure, will vary from nuclei to nuclei.

Case II — large f t values

If the observed deviation results from the effect dependent on nuclear structure only, then it is impossible to make any general predictions. The effect can vary from nuclei to nuclei, and it may be dependent on the f t value. The deviation must not be present in all allowed decays. The correction factors which were neglected in Case I can now be very important. These factors are obtained as a result of further expansion of the lepton covariants in the powers of r. For all other terms, except the dominant one, we took only the terms with the lowest possible power of r. The choice of the form factors is given on page 93 and 114. (For possible modifications see ref. [13].)

l-forbidden Transitions

The influence of nuclear structure can very easily be taken into account in the case of P^{32}. With the assumed shell model configuration $(s\ 1/2)^p\ (d\ 3/2)^n$ for P^{32} and $(s\ 1/2)^{2p}$ for S^{32} this transition is classified as l-forbidden ([27]). In the first approximation the matrix element $<T_{10}>$ is exactly zero. The nonvanishing of this element can be explained by configuration mixing. The first attempt to explain the deviation on the basis of l-forbiddenness was performed by I b e n ([28]). We tried to obtain the experimental correction factor by the expression:

$$C_{GT^\beta} = C_{1\,GT} + \varkappa_6\, C_{3\,GT} + \varkappa_1\, C_{9\,GT} \tag{100}$$

$$\varkappa_6 = \frac{\sqrt{2} < r^2\, T_{12} >}{< T_{10} >} \tag{101}$$

The reported experimental results are not in very good agreement [1], [4], [5], [6]. Numerical analyses showed that the term $C_{9\,GT}$, is very small and rather unimportant. The experimental results can be obtained with

$$2 \cdot 10^{-3} < \varkappa_6 < 1 \cdot 10^{-2}$$

This means that the $<r^2\, T_{12}>$ matrix element is enhanced toward the $<T_{10}>$ matrix element, because the decay is l-forbidden. Some estimation of the absolute magnitude of $\varkappa_6$ can be obtained if $<r^2\, T_{12}>$ is calculated by the shell model functions and $<T_{10}>$ from the experimental $f t$ value.* The result

$$|\varkappa_6| = \left| \frac{< r^2\, T_{12} >}{< T_{10} >_{\exp}} \right| = 1.2 \cdot 10^{-2}$$

is in good agreement with the experiment. Some attempt to obtain the relative sign on the basis of some simple models is described in Appendix V. If the $\varkappa_6$ matrix element is so enlarged, all other terms in C_{GT^β} except those in (100) are practically quite unimportant.

* $\langle T_{10} \rangle_{\exp}$ is calculated by the approximation $C_{GT^\beta} = 1$, so this value can serve only for rough estimation.

In the low Z limit approximation the correction factor will be approximately

$$C_{GT^\beta} = 1 - \varkappa_6 \frac{4}{27} [W^2 + W(3\xi - E_0) + \frac{1}{W} E_0 - 3 E_0 \xi - 1] \qquad (102)$$

and for the relative longitudinal polarization (Appendix I)

$$P_{GT^\beta} = -\{1 - \varkappa_6 \frac{4}{27} [W^2 + W(3\xi - E_0) - 3 E_0 \xi]\} \qquad (103)$$

For positrons it is necessary to replace ξ with $-\xi$. It follows from (102) that for a not too high energy C_{GT^β} is roughly of the form $1 - a W$
where

$$a = \frac{4}{27} \varkappa_6 (3\xi - E_0) [1 + \varkappa_6 \frac{12}{27} E_0 \xi]^{-1}$$

The value of $a \approx 0.02$ is approximately in agreement with the results of Daniel [4]. The correction factor C^β, P^β and longitudinal polarization are plotted in *Fig.* 7, *Fig.* 8 and *Fig.* 9.

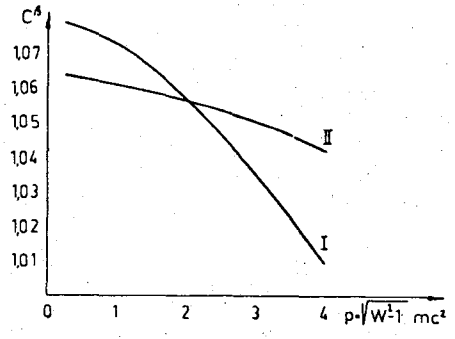

Fig. 7. The correction factors C^β (100) for the decay of P^{32} are plotted in arbitrary units as a function of the momentum. Curve I corresponds to $\varkappa_6 = 10^{-2}$ and Curve II to $\varkappa_6 = 3 \cdot 10^{-3}$.

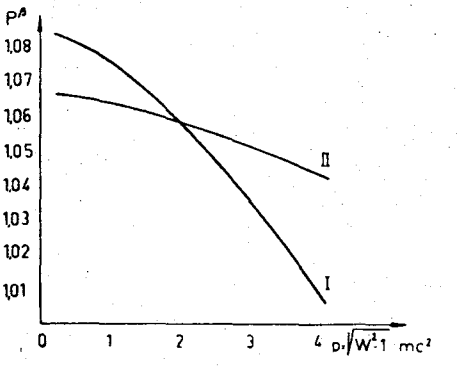

Fig. 8. The correction factors for the relative longitudinal polarization for the decay of P^{32} are plotted in arbitrary units as a function of the momentum. Curves I and II correspond to Curves I and II in Fig. 7, respectively.

Some consequence of such an explanation are:

a) The deviation for β^+ decay can be different and C_{GT^β} can be increased by the electron energy if $\varkappa_6$ has the same sign as for

P^{32}. (Of course, that depends on nuclear structure.). One can search for such effects in some β^+ l-forbidden decays with large ft values (perhaps Ge^{68} or Ge^{69}).

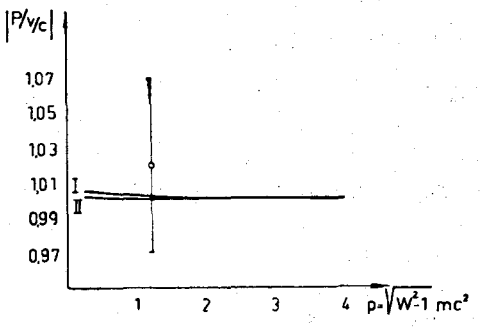

Fig. 9. The absolute value of longitudinal polarization of electrons divided by $\frac{v}{c}$ for the decay of P^{32} is plotted as function of the momentum. The experimental point measured by Spivak et al [43] is indicated.

b) If in the β decay interaction there is some intrinsic effect producing the deviation, it will combine with the effect caused by l-forbiddenness.

c) The $P_{GT}{}^{\beta}$ will be energy dependent with rather a large variation. The measurement of the relative longitudinal polarization (Appendix I) can, therefore, serve as a test for the above theory.

To make the problem clearer it will be important to make measurements in the case of some other l-forbidden transitions. They are listed in *Table* 4.

Table 4.

I. Even A Nuclei

Initial nucley and spin	*Final nucley and spin*	*Initial and final configuration*	*Decay and maximal kinetic energy MeV*	*log ft*
$_{15}P^{32}$	$_{16}S^{32}$	$d\ 3/2 \quad s\ 1/2$	β^-	7.9
1 +	0 +		1.707	
—	—			
$_{14}Si^{32}$	$_{15}P^{32}$	$d\ 3/2 \quad s\ 1/2$	β^-	≈ 7.8
0 +	1 +		0.1	
	—			
$_{15}P^{34}$	$_{16}S^{34}$	$d\ 3/2 \quad s\ 1/2$	β^-	5.2
1 +	0 +		5.1	
$_{29}Cu^{62}$	$_{28}Ni^{62}$	$p\ 3/2 \quad f\ 5/2$	β^+	> 5.1
(1 +, 0 +)	0 +		2.91	
$_{29}Cu^{64}$	$_{28}Ni^{64}$	$p\ 3/2 \quad f\ 5/2$ *or*	β^+	5.0
1 +	0 +	$p\ 3/2 \quad p\ 3/2$	0.656	

$_{29}Cu^{64}$	$_{30}Zn^{64}$	$f\ ^5/_2$	$p\ ^3/_2$	β^-	5.3
1+	0+		or		
		$p\ ^3/_2$	$p\ ^3/_2$	0.573	
$_{28}Ni^{66}$	$_{29}Cu^{66}$	$f\ ^5/_2$	$p\ ^3/_2$	β^-	$\gtrsim 4.1$
0+	(1+)		or		
		$p\ ^3/_2$	$p\ ^3/_2$	0.20	
$_{32}Ge^{68}$	$_{31}Ga^{68}$	$p\ ^3/_2$	$f\ ^5/_2$	β^+	6.7
0+	1+		or		
	—	$p\ ^3/_2$	$p\ ^3/_2$	0.7	
$_{31}Ga^{68}$	$_{30}Zn^{68}$	$p\ ^3/_2$	$f\ ^5/_2$	β^+	5.2
1+	0+		or		
—		$p\ ^3/_2$	$p\ ^3/_2$	1.894	

II. Odd A Nuclei

$_{14}Si^{31}$	$_{15}P^{31}$	$d\ ^3/_2$	$s\ ^1/_2$	β^-	> 5.5
$\left(\frac{3}{2}+\right)$	$\frac{1}{2}+$				
	—			1.48	
$_{15}P^{33}$	$_{16}S^{33}$	$d\ ^3/_2$	$s\ ^1/_2$	β^-	5.0
$\left(\frac{1}{2}+\right)$	$\frac{3}{2}+$				
	—			0.249	
$_{24}C^{55}$	$_{25}Mn^{55}$	$p\ ^3/_2$	$f\ ^7/_2$	β^-	5.2
$\frac{3}{2}-$	$\frac{5}{2}-$				
	—			2.85	$I=j-1$
$_{29}Cu^{61}$	$_{28}Ni^{61}$	$p\ ^3/_2$	$f\ ^3/_2$	β^+	≈ 5.1
$\frac{3}{2}-$	$\frac{3}{2}-$				
—	—			1.21	$I=j-1$
$_{28}Ni^{63}$	$_{29}Cu^{63}$	$f\ ^5/_2$	$p\ ^3/_2$	β^-	6.7
$\left(\frac{3}{2}-\right)$	$\frac{3}{2}-$				
	—			0.067	$I=j-1$
$_{28}Ni^{65}$	$_{29}Cu^{65}$	$f\ ^5/_2$	$p\ ^3/_2$	β^-	6.5
$\left(\frac{5}{2}-\right)$	$\frac{3}{2}-$				
	—			2.10	

$_{31}Ga^{65}$ $\left(\frac{3}{2}-\right)$	$_{30}Zn^{65}$ $\left(\frac{5}{2}-\right)$	$p\ ^3/_2$	$f\ ^5/_2$	β^+ 2.24	5.7
$_{30}Zn^{65}$ $\frac{5}{2}-$	$_{29}Cu^{65}$ $\frac{3}{2}-$ —	$p\ ^3/_2$	$f\ ^5/_2$	β^+ 0.326	7.4
$_{29}Cu^{67}$ $\left(-\frac{3}{2}\right)$	$_{30}Zn^{67}$ $\frac{5}{2}-$	$f\ ^5/_2$	$p\ ^3/_2$	β^- 0.557	6.3
$_{32}Ge^{69}$ $\left(\frac{5}{2}, \frac{3}{2}-\right)$	$_{31}Ga^{69}$ $\frac{3}{2}-$ —	$p\ ^3/_2$	$f\ ^5/_2$	β^+	<6.4

The spins of a state measured directly are underlined in the table. I is the total spin and j the spin of the last nucleon. Only the configurations of transferred particle are noted here. The data for the maximal kinetic energy and ft values are taken from Nuclear Data Sheets or from Stromings et al. [17].

The investigation of this table shows that some of the elements have rather a small ft value. In these cases we can expect that the classification will not be very good and that the effect will be small, if it will be present at all. The most characteristic elements are those with a large ft value. It will be useful to make a comparison of two β emitters from this table, one of them having a large ft value and the other a small ft value.

The reported values for longitudinal polarization are:

$-(1.00 \pm 0.10)\ v/c$[29]
$-(0.91 \pm 0.08)\ v/c$[30]
$-(0.76 \pm 0.17)\ v/c$[31]
$-(1.13 \pm 0.08)\ v/c$[32]
$-(0.97 \pm 0.06)\ v/c$[33]
$-(0.97 \pm 0.03)\ v/c$[34]
$-(1.02 \pm 0.05)\ v/c$[43], at 300 kev.

The measurements in almost all the cases are not undertaken at a well defined electron energy. It seems that the experiments indicate that the polarization is $< |v/c|$, which is in contradiction with theory. But the precision of the measurements is still not siffucient to draw any definite conclusions. The last measurements of Spivak et al. [43] indicate that the polarization is $> |v/c|$.

Longitudinal polarization (14) calculated by the expression (100) and by the analogous expression for P_{GT^β} is almost exactly $-v/c$ and practically energy independent ($P_{GT^\beta}/C_{GT^\beta}$ varies from 1.002 to 1.000.) With our explanation of the spectrum shape it is impossible to obtain the polarization smaller than $|v/c|$. These results seem to be in good agreement with the measurements of Spivak et al. [43].

4. *Conclusion*

The deviations from the allowed shape ($C^\beta \approx$ const) have been found in a rather small number of allowed decays. In order to get a complete knowledge of such phenomena it is necessary to make more experiments with a large number of atomic nuclei. From the present experimental results it is not possible to draw any definite conclusions about the nature of the observed deviations. The experiments of different groups of experimentalist are in poor agreement.

In analysing the longitudinal polarization we concluded that some important information can be obtained if one measures the variation with energy. If it is possible to measure the quantity which we called relative longitudinal polarization can serve as a test for some theoretical predictions.

β-γ angular correlation and β-γ circular correlation experiments are not analysed in the text, but can be a useful source of information.

It will be useful to investigate two groups of allowed transitions:

a) Transitions with very small f t values (superallowed), namely $O^+ \to O^+$ and mirror transitions. By investigating such transitions it is possible to investigate the effects caused by the structure of β decay interaction Hamiltonian. As a working hypothesis, we tried with modified β decay interaction Hamiltonian (the details are given in introduction) and we derive some consequences of such an approach. It seems that it does not work well. By comparing the results obtained for $O^+ \to O^+$ and mirror transitions it is possible to establish the presence of the spin 3/2 neutrino because $O^+ \to O^+$ transitions cannot be influenced by the spin 3/2 neutrino.

b) In transitions with rather a large ft value ($\log ft \approx 6$), the influence of nuclear structure results in some additional selection rules. The ratios of some nuclear matrix elements can become anomalously large, which results in the deviation from the allowed spectrum shape. Among such transitions we investigated l-forbidden transitions in detail. Only one of the measured transitions, P^{32}, can be classified as l-forbidden. To make the situation clearer, it is necessary to make measurements of a greater number of transitions which can be classified as l-forbidden. Our conclusion is that the most characteristic of them are those with the largest ft value

(log $ft \approx 6$). For some of them with a small ft value (for example Cu^{64}) we do not expect any remarkable deviation from the allowed shape. It will be useful to make measurements for some positron emitters from this class with a sufficiently large ft value (Ge^{68} and Ge^{69}).

According to theory the absolute longitudinal polarization for l-forbidden transitions is practically $\sim \left|\frac{v}{c}\right|$ for all energies.

Only the combination of the results obtained by measuring the transitions from group *a*) and *b*) can give the possibility of separating the effects produced by the structure of the interaction Hamiltonian from the effects produced by the influence of nuclear structure.

We are very grateful to Dr. G. Alaga for many discussions and for constructive criticism of the manuscript.

Appendix

I. *Longitudinal Polarization*

Longitudinal polarization is defined as

$$P = \frac{N_+ - N_-}{N_+ + N_-} \tag{A1}$$

where N_+ and N_- is number of electrons with the spin in the direction of motion and in the opposite direction, respectively. The relative polarization $N_+ - N_-$ is energy dependent and is proportional to

$$N_+ - N_- \sim p^2 (W - E_0)^2 F(Z, W) P^\beta \tag{A2}$$

where P^β is given by (15) and (45). The polarization correction factor P^β is slightly energy dependent for the allowed transitions, similarly as C^β. It will be worthwile to find an experiment for measuring the variation of $N_+ - N_-$ with energy.

II. *Some notations*

Combinations of the radial wave functions are as follows:

$$f_\nu g_{-\nu} = p^2 F \frac{p}{W} r_0^{2\nu-2} A_{\nu-1} \tag{A3}$$

$$f_{-\nu} g_\nu = p^2 F \frac{p}{W} r_0^{2\nu} B_{\nu-1} \tag{A4}$$

$$f_\nu f_{-\nu} - g_\nu g_{-\nu} = 2 p^2 F \frac{p}{W} r_0^{2\nu-1} C_{\nu-1} \tag{A5}$$

$$f_\nu f_{-\nu} + g_\nu g_{-\nu} = 2 p^2 F \frac{p}{W} r_0^{2\nu-1} D_{\nu-1} \tag{A6}$$

$$n_\nu = \cos(\Delta_\nu - \Delta_{-\nu}) = \frac{\nu S_\nu}{\left[\nu^2 + \left(\frac{\alpha Z}{p}\right)^2\right]^{1/2}} \qquad S_\nu = \frac{\nu}{|\nu|} \tag{A7}$$

$$L_0 = A(Z, W) + B(Z, W) r + C(Z, W) r^2 + \ldots \tag{A8}$$

where

$$A(Z, W) = \frac{1}{2}(\gamma + 1) \tag{A8a}$$

$$B(Z, W) = -\frac{y}{2\gamma + 1}\left(2\gamma + 3 + \frac{\gamma}{W^2}\right) p \tag{A8b}$$

$$C(Z, W) = -\frac{(\gamma + 1)(4\gamma^3 + \gamma^2 - \gamma - 1) - 2y^2(7\gamma + 6)}{(2\gamma + 1)^2(\gamma + 1)} p^2 \tag{A8c}$$

$$A_0 = E(Z, W) + F(Z, W) r + G(Z, W) r^2 + \ldots \tag{A9}$$

where

$$E(Z, W) = \frac{1}{2}(\gamma^2 + y^2)(\gamma + 1) \tag{A9a}$$

$$F(Z, W) = -(\gamma^2 + y^2)\frac{y(2\gamma + 3)}{(2\gamma + 1)} p \tag{A9b}$$

$$G(Z, W) = -\frac{(\gamma^2 + y^2)}{(2\gamma + 1)^2(\gamma + 1)}[\gamma(\gamma + 1)(\gamma + 2) - y^2(4\gamma + 3)(\gamma + 2)] p^2 \tag{A9c}$$

$$\gamma = +[1 - (\alpha Z)^2]^{1/2}, \quad y = \frac{\alpha Z W}{p} \tag{A10}$$

Other notations are well known in β decay theory.

III. *Mirror Transitions*

In the calculation of the matrix elements for mirror transitions we used the formulas

$$\langle j_1 || i r Y_1 \vec{\sigma} \mathbf{p} || j_2 \rangle = (-)^{j_1 - j_2 - 1} \frac{3}{2}\left[\frac{(2j_1 + 1)(2j_2 + 1)}{4\pi(2l_1 + 1)}\right]^{1/2} [(2l_1 + 3)(l_1 + 1) \cdot W(l_1 + 1\, l_1\, 1/2\, 1/2 \mid 1\, j_2)\, W(j_2\, j_1\, l_1 + 1\, l_1 \mid 1\, 1/2) - l_1(2l_1 - 1) \cdot W(l_1 - 1\, l_1\, 1/2\, 1/2 \mid 1\, j_2)\, W(j_2\, j_1\, l_1 - 1\, l_1 \mid 1\, 1/2)]\, \delta_{l_1 l_2} \tag{A11}$$

$$<j_1 \,||\, i\, r\, T_{11}(\mathbf{p}) \,||\, j_2> = \left[\frac{3\,(2j_1+1)\,(2j_2+1)\,(l_1+1)\,l_1}{8\pi}\right]^{1/2} \cdot$$

$$\cdot\, W(j_2\, 1/2\, 1\, l_1 \,|\, l_1\, j_1)\, \delta_{l_1 l_2} \tag{A12}$$

$$<j_1 \,||\, T^{n}_{k\lambda} \,||\, j_2>_w = (-)^{1+j_1+j_2+l_1+l_2} \left[\frac{(2n+1)\,(2j_1+1)\,(2j_2+1)}{4\pi}\right]^{1/2} \cdot$$

$$[(2k+1)\,(2\Lambda+1)\,(2l_2+1)]^{1/2}\,(l_2\,0\;\Lambda\,0 \,|\, l_2\,\Lambda\,l_1\,0) \begin{Bmatrix} j_2 & k & j_1 \\ 1/2\,n & 1/2 & \\ l_2 & \Lambda & l_1 \end{Bmatrix} \tag{A13}$$

IV. Low Z Approximation

The correction factors for Gamow-Teller transitions for the spectrum and longitudinal polarization in the low Z approximation are:

$$C_{1\,GT} = 1 \tag{A14}$$

$$C_{2\,GT} = -\frac{1}{27}[11\,W^2 + 2\,W\,(3\,\xi - 10\,E_0) + \frac{2}{W}E_0 + 3\,E_0\,(3\,E_0 - 2\,\xi) - 2] \tag{A15}$$

$$C_{3\,GT} = -\frac{4}{27}\,[W^2 + W\,(3\,\xi - E_0) + \frac{1}{W}\,E_0 - 3\,E_0\,\xi - 1] \tag{A16}$$

$$C_{4\,GT} = \frac{2}{3M}\left(\frac{1}{W} - 2\,W + E_0 - 2\,\xi\right) \tag{A17}$$

$$C_{5\,GT} = \frac{1}{3M}\,\xi \tag{A18}$$

$$C_{6\,GT} = \frac{2}{3M}\left(\frac{1}{W} - 2\,W + E_0 - 3\,\xi\right) \tag{A19}$$

$$C_{7\,GT} = \frac{1}{3M}\left(E_0 + 2\,\xi - \frac{1}{W}\right) \tag{A20}$$

$$C_{8\,GT} = C_{5\,GT} \tag{A21}$$

$$C_{9\,GT} = \frac{2}{3M}\left(E_0 + 3\,\xi - \frac{1}{W}\right) \tag{A22}$$

$$C_{10\,GT} = \frac{1}{3M}\left(\frac{E_0 - 2\,\xi}{W} - 1\right) \tag{A23}$$

$$C_{11\,GT} = -\frac{1}{3M}\,\frac{\xi}{W} \tag{A24}$$

$$C_{12\,GT} = \frac{4}{9}\left(2\,W - \frac{1}{W} - E_0 + 3\,\xi\right) \tag{A25}$$

$$C_{13\,GT} = \frac{4}{3}\left(E_0 + 3\,\xi - \frac{1}{W}\right) \tag{A26}$$

$$C_{14\,GT} = \frac{2}{3}\left(\frac{E_0 - 3\,\xi}{W} - 1\right) \tag{A27}$$

$$P_{1\,GT} = -1 \tag{A28}$$

$$P_{2\,GT} = \frac{1}{27}\left[11\,W^2 + 2\,W\,(3\,\xi - 10\,E_0) + 3\,E_0\,(3\,E_0 - 2\,\xi)\right] \tag{A29}$$

$$P_{3\,GT} = \frac{4}{27}\left[W^2 + W\,(3\,\xi - E_0) - 3\,E_0\,\xi\right] \tag{A30}$$

$$P_{4\,GT} = \frac{2}{3\,M}\,(2\,W + 2\,\xi - E_0) \tag{A31}$$

$$P_{5\,GT} = -\frac{1}{3\,M}\,\xi \tag{A32}$$

$$P_{6\,GT} = \frac{2}{3\,M}\,(2\,W + 3\,\xi - E_0) \tag{A33}$$

$$P_{7\,GT} = -\frac{1}{3\,M}\,(E_0 + 2\,\xi) \tag{A34}$$

$$P_{8\,GT} = P_{5\,GT} \tag{A35}$$

$$P_{9\,GT} = -\frac{2}{3\,M}\,(E_0 + 3\,\xi) \tag{A36}$$

$$P_{10\,GT} = -\frac{1}{3\,M} \tag{A37}$$

$$P_{11\,GT} = 0 \tag{A38}$$

$$P_{12\,GT} = -\frac{4}{9}\,(2\,W + 3\,\xi - E_0) \tag{A39}$$

$$P_{13\,GT} = -\frac{4}{3}\,(E_0 + 3\,\xi) \tag{A40}$$

$$P_{14\,GT} = -\frac{2}{3} \tag{A41}$$

$$\xi = -\frac{\alpha\,Z}{2\,r_0} \tag{A42}$$

Z is the atomic number, r_0 is the radius of atomic nuclei, α is the fine structure constant. For positrons it is necessary to replace Z by $-Z$.

V. *Calculation of Nuclear Matrix Elements for* P^{32}

In order to calculate the ratio of the matrix elements $<r^2 T_{12}>$ and $<T_{10}>$ we made several attempts.

In the single particle shell model of atomic nuclei the matrix element $<T_{10}>$ is exactly zero for the $P^{32} \to S^{32}$ transition. One must therefore take into account an admixture of nuclear states which is produced by residual interactions between nucleons.

a) First we tried to calculate the matrix elements by taking into account a short range residual interaction in the well-known form [22]

$$H_{int} = -V_0 [(1-\alpha) + \alpha \vec{\sigma}_1 \vec{\sigma}_2] \delta(\mathbf{r}_1 - \mathbf{r}_2) \tag{A43}$$

α and V_0 are some constants. In calculating the matrix elements of H_{int} we used the formulas:

$$<j_1 j_2 J M | \delta(\mathbf{r}_1 - \mathbf{r}_2) | j_3 j_4 J M> = F(r) \frac{1}{16\pi} (-)^{j_2+j_4+l_2+l_4+1} \cdot$$

$$\cdot [1 + (-)^{l_1+l_2+l_3+l_4}] [(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)]^{1/2} \cdot$$

$$\cdot \left\{ \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & J \\ 1/2 & -1/2 & 0 \end{pmatrix} - \right.$$

$$\left. - (-)^{j_2+j_4+l_2+l_4} \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & J \\ 1/2 & 1/2 & -1 \end{pmatrix} \right\} \tag{A44}$$

$$<j_1 j_2 J M | \vec{\sigma}_1 \vec{\sigma}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) | j_3 j_4 J M> = F(r) \frac{1}{8\pi} (-)^{j_2+j_3+l_2+l_3+1} \cdot$$

$$\cdot [1 + (-)^{l_1+l_2+l_3+l_4}] [(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)]^{1/2} \cdot$$

$$\left\{ [(-)^{J+j_3+j_4} + \frac{1}{2} (-)^{j_1+j_2+l_1+l_2}] \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & 0 \\ 1/2 & -1/2 & J \end{pmatrix} + \right.$$

$$\left. + [(-)^{j_2+j_3+l_2+l_3+1} + \frac{1}{2} (-)^{j_1+j_4+l_1+l_4}] \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & 1/2 & -1 \end{pmatrix} \begin{pmatrix} j_3 & j_4 & J \\ 1/2 & 1/2 & -1 \end{pmatrix} \right\} \tag{A45}$$

$$F(r) = \int R_{j_1}(r_1)^* R_{j_2}{}^*(r_2) \delta(r_1 - r_2) R_{j_3}(r_1) R_{j_4}(r_2) r_1 \, dr_1 \, r_2 \, dr_2 \tag{A46}$$

$$\sigma_n^\nu Y_\Lambda^\mu = \sum_{k\kappa} (n \nu \Lambda \mu | n \Lambda k \varkappa) T_{k\Lambda}^\kappa \; ; \quad n = 0, 1 \tag{A47}$$

The double bar matrix element used in β decay is

$$<j_1 || T_{k\Lambda} || j_2>_\beta = \frac{1}{\sqrt{2j_2+1}} <j_1 || T_{k\Lambda} || j_2>_w \tag{A48}$$

The wave function of the ground state of P^{32} was assumed to be

$$\overline{\Psi}_{1+} = \Psi_{1+} [(s\,1/2)^p (d\,3/2)^n] + \cdot$$

$$\frac{<(d\,3/2)^p (d\,3/2)^n | H_{int} | (s\,1/2)^p (d\,3/2)^n>}{E_{s1/2} - E_{d3/2}} \cdot \Psi_{1+} [(d\,3/2)^p (d\,3/2)^n] \tag{A49}$$

and for S^{32}

$$\overline{\Psi}_{0+} = \Psi_{0+}\,[(s\,1/2)^{2p}] + \frac{<(d\,3/2)^{2p}\,|\,H_{int}\,|\,(s\,1/2)^{2p}>}{2\,(E_{s1/2} - E_{d3/2})}\;\Psi_{0+}\,[(d\,3/2)^{2p}] \tag{A50}$$

$V_0\,a$ was fixed from the separation energy ($E_{2+} - E_{1+} = 0.077$ MeV) of 1^+ and 2^+ levels in P^{32}. The spin orbit coupling term was used in the form $C\,\mathbf{l}\,\mathbf{s}$ with $C = -1.29$ MeV.

Finally we get the result:

$$\varkappa_6 = \frac{\sqrt{2}<r^2\,T_{12}>}{<T_{10}>} = \left(\frac{343}{9-c^2} + 0.083\right)\cdot 10^{-3}, \quad c = \frac{1-a}{a}$$

Even for $c = 0$ the ratio is too large. For $c > y$ the result has the wrong sign.

b) We tried to fit the experimental data by assuming the paring force interaction between the last two protons in S^{32} [40]

$$<j\,j\,|\,H_{int}\,|\,j'\,j'> = -\frac{19}{2\,A}\,[(2\,j+1)\,(2\,j'+1)]^{1/2} \tag{A51}$$

For the ratio of the matrix elements $\varkappa_6$ we obtained

$$\varkappa_6 = -\left(\frac{27.8}{c+3} - 0.083\right)\cdot 10^{-3}$$

$c < -3$ gives the correct sign, but then the force between the s 1/2 proton and the d 3/2 neutron is repulsive.

c) The calculation with the model of strongly deformed nuclei (for details see ref. [36], [37], [38]) give the best result for the parameter of deformation $\eta = -2$

$$\varkappa_6 = 0.135\cdot 10^{-3}$$

This is for factor 10 too small. For deformations with $\eta > 0$ we obtain the wrong sign.

d) Finally we tried to calculate the ratio $\varkappa_6$ on the basis of long range residual forces. We assumed that the deformation of P^{32} and S^{32} is small and that the effect can be described with a small Y_2 coupling term to the nuclear surface. In the weak coupling approximation (for details and notation see Bohr and Mottelson [39]) the wave function of an individual nucleons is:

$$\Psi_{j^m} = \Psi_{j^m} + \eta \sum_{j'} \frac{<j\,|\,h\,|\,j'>}{\hbar\,\omega + E_{j'} - E_j}\;(j'\,m'\,R\,R_z\,|\,j'\,R\,j\,m)\;\Psi_{j'^{m'}}\,\varkappa_R{}^{R_z} \tag{A52}$$

where Ψ_{j^m} is the shell model function, $\varkappa_R{}^{R_z}$ describe surface oscillations and η is treated as an adjustable parameter. The results

are tabulated for several combinations of parameter η, and for $\hbar\omega = 3.3$ MeV.

η^2	$\varkappa_6 \cdot 10^3$	$4\pi \left\| < T_{10} >_\beta \right\|^2$
0.0118	2	$0.38 \cdot 10^{-3}$
0.00583	4	$0.11 \cdot 10^{-3}$
0.00231	10	$0.24 \cdot 10^{-4}$

The results are insensitive to the changing of $\hbar\omega$. But for $\hbar\omega \leq 3.3$ MeV the wrong sign for $\varkappa_6$ is obtained. In hydrodynamic approximation $\hbar\omega$ is 4.5 MeV. From the experimental ft value $4\pi \left\| < T_{10} >_\beta \right\|^2$ is calculated to be $0.6 \cdot 10^{-4}$.

Institute »Ruđer Bošković«, Zagreb

REFERENCES

[1] *O. E. Jonson, R. G. Jonson* and *L. M. Langer,* Phys. Rev. **112** (1958), 2004,
[2] *J. H. Hamilton, L. M. Langer* and *W. G. Smith,* Phys. Rev. **112** (1958), 2010,
[3] *J. H. Hamilton, L. M. Langer* and *W. G. Smith,* Phys. Rev. **119** (1960), 772,
[4] *H. Daniel,* Nuclear Physics **8** (1958), 191, and private communication,
[5] *R. L. Graham, J. S. Geiger* and *T. A. Eastwood,* Can. J. Phys. **36** (1958), 1084,
[6] *F. T. Porter, F. Wagner Jr.* and *M. S. Freedman,* Phys. Rev. **107** (1957), 135,
[7] *M. Morita,* Nuclear Physics **14** (1959), 106,
[8] *M. Morita,* Phys. Rev. **113** (1959), 1584,
[9] *L. L. Foldy* and *S. A. Wouthuysen,* Phys. Rev. **78** (1950), 20,
[10] *M. E. Rose* and *R. K. Osborne,* Phys. Rev. **93** (1954), 1315,
[11] *E. J. Konopinski,* Phys. Rev. **94** (1954), 492,
[12] *R. F. Herbst,* Phys. Rev. **96** (1954), 372,
[13] *D. Tadić,* Glasnik mat.-fiz. i astr. **13** (1958), 267; Nuclear Physics **18** (1960), 138,
[14] *R. P. Feyman* and *M. Gell-Mann,* Phys. Rev. **109** (1958), 193,
[15] *M. Gell-Mann,* Phys. Rev. **111** (1958), 362,
[16] *M. L. Goldberger* and *S. B. Treiman,* Phys. Rev. **111** (1958), 355,
[17] *D. Strominger, J. M. Hollander* and *G. T. Seaborg,* Rev. Mod. Phys. **30** (1958), 585,
[18] *G. Alaga, L. Šips* and *D. Tadić,* Nuclear Physics **6** (1958), 305,
[19] *T. Ahrens* and *E. Feenberg,* Phys. Rev. **86** (1952), 64,
[20] *M. Yamada,* Progr. Theort. Phys. **9** (1953), 268,
[21] *E. Feenberg,* Shell Theory of the Nucleus, Princeton University Press, 1955,
[22] *D. Elliott* and *A. Lain,* Structure of Atomic Nuclei, Handbuch der Physik, Band 39,
[23] *C. L. Hammer* and *R. H. Good, Jr.,* Phys. Rev. **117** (1960), 889,
[24] *L. Kofoed-Hansen* and *A. Winter,* Mat.-Fys. Medd. Dan. Vid. Selsk **27**, No 14 (1953) and **30** No 20 (1956),
[25] *G. L. Trigg,* Phys. Rev. **86** (1952), 506,
[26] *B. Kuchowicz,* Polish Acad. Sci. **7**, No **94** (1959),
[27] *L. W. Nordheim,* Rev. Mod. Phys. **23** (1951), 322,
[28] *I. Iben,* Jr., Phys. Rev. **109** (1958), 2059,
[29] *H. Frauenfelder, A. O. Hanson, N. Levine, A. Rossi* and *G. de Pasquali,* Phys. Rev. **107** (1957), 643,

[30] *I. S. Geiger, G. T. Ewan, R. L. Graham, D. R. MacKenzie,* Bull. Am. Phys. Soc. *3,* No 1, 51 SA10.
[31] *H. de Waard* and *O. J. Pappema,* Physics **23** (1957), 597,
[32] *C. R. Dulgeroff, E. D. Lambe* and *T. A. Pond.* Bull. Am. Phys. Soc. **2,** No 7, 348 M2 (1957),
[33] *F. Boehm* and *A. H. Wapstra,* Phys. Rev. **109** (1958), 456,
[34] *P. E. Spivak* and *L. A. Mihaelian,* Nuclear Physics **20** (1960), 475,
[35] *R. Wallace* and *J. A. Welch,* Jr., Phys. Rev. **117** (1960), 1297,
[36] *S. G. Nilsson,* Mat. Fys. Medd. Dan. Vid. Selsk **29,** No 16 (1955), 1,
[37] *G. Alaga,* Glasnik mat.-fiz. i astr. **12** (1957), 245,
[38] *C. J. Gallagher,* Nuclear Physics **16** (1960), 215,
[39] *A. Bohr* and *B. R. Mottelson,* Mat. Fys. Medd. Dan. Vid. Selsk **27,** No 16 (1953),
[40] *L. S. Kisslinger* and *R. A. Sorensen* — to be published,
[41] *D. Tadić* Nuclear Physics **23** (1961), 777,
[42] *T. Kotani* and *M. H. Ross,* Prog. Theoret. Phys. **20** (1958), 643,
T. Kotani and *M. H. Ross,* Phys. Rev. **113** (1959), 622,
[43] *P. E. Spivak, L. A. Mihaeljan, I. E. Kutikov* and *V. F. Analin,* JETF **39** (1960), 1477,
[44] *B. S. Djelepov* and *L. N. Zirjanova,* Vlijanje električeskogo polja atoma na β-raspad,
[45] *M. E. Rose* and *D. K. Holmes,* ORNL-1022,
[46] *M. E. Rose, C. L. Perry, N. M. Dismuke,* ORLN-1459,
[47] *M. E. Rose and C. L. Perry,* Phys. Rev. **90** (1953), 479,
[48] *S. M. Berman,* Phys. Rev. **112** (1958), 267,

O ODSTUPANJU OD DOZVOLJENOG OBLIKA SPEKTRA KOD DOZVOLJENIH β RASPADA

B. Eman i D. Tadić, Zagreb

Sadržaj

Mjerenjem je ustanovljeno da kod β^+ i β^- raspada oblik spektra odstupa od dozvoljenog i to u istom smjeru.

Ovo odstupanje može biti posljedica svojstava Hamiltoniana interakcije ili posljedica utjecaja matričnih elemenata. Konstruiran je generalizirani nerelativistički Hamiltonian (1) kojemu su matrični elementi množeni sa form faktorima. Izračunati su korekcioni faktori i longitudinalna polarizacija za Fermieve (2) (5) i Gamow-Tellerove (26) (45) prijelaze, koji pružaju mogućnost da se izvrši analiza eksperimentalnih podataka (tabela 2).

Pretpostavka, da je odstupanje od dozvoljenog oblika spektra uvjetovano strukturom Hamiltoniana, vodi na ispitivanje spektra dozvoljenih prijelaza sa malom ft vrijednosti, a to su zrcalne jezgre i $O^+ \rightarrow O^+$ prijelazi. Definitivni zaključci su otežani, jer se predznak nuklearnih matričnih elemenata mijenja od jezgre do jezgre, a točnija mjerenja polarizacije kod zrcalnih jezgri nisu vršena. Tabelirani su matrični elementi za zrcalne jezgre izračunati na osnovu modela ljusaka (Tabela 3). Moguće je naći takove kombinacije parametara (71) i (72) da korekcioni faktor pada sa energijom i za elektrone i za pozitrone, ali oblik korekcionog faktora nije isti

za oba raspada. Da bi se odredila vrijednost parametara s kojima su množeni matrični elementi Hamiltoniana (1), uzeta je hipotetska jezgra koja se raspada preko β^+ i β^- grane. Uz ovakovu pretpostavku B_i poprimaju jako velike vrijednosti.

U slučaju, kada je mjereno odstupanje uzrokovano utjecajem nuklearnih matričnih elemenata, ne može se dati nikakva definitivna prognoza o spektru i o polarizaciji. Efekt će varirati od jezgre do jezgre. Da bi se opisali ovakvi efekti moguće je pretpostaviti da jedan od matričnih elemenata Hamiltoniana (1) postaje bitan. To je vjerojatno slučaj kod l-zabranjenih prijelaza. Prijelazi koji se eventualno mogu smatrati l-zabranjenima tabelirani su u tabeli 4. Izračunata je polarizacija i korekcioni faktor za P^{32} i uspoređena sa mjerenim vrijednostima [43].

Za daljnji razvoj teorije nužno je da se izvrši više eksperimenata sa raznim jezgrama. Naročito bi bilo važno usporediti dozvoljene prijelaze sa velikom i malom ft vrijednosti. Korisna bi bila usporedba jednog β^- i β^+ l-zabranjenog prijelaza. Za sada se rezultati eksperimentalnih fizičara ne slažu međusobno.

U dodatku definirana je relativna polarizacija, unesene su formule za računanje reduciranih matričnih elemenata za zrcalne prijelaze i izvršeno je računanje matričnih elemenata za P^{32}. Priložene su vrijednosti korekcionih faktora u aproksimaciji malog Z.

(Primljeno 15. I 1961.)