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LDPC CODES FROM DEZA DIGRAPHS

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Abstract. In this paper, we present a construction of LDPC codes obtained from Deza digraphs with parameters $(n, k, 0, 1)$. The obtained LDPC codes have no cycles of length four in the Tanner graphs corresponding to the adjacency matrices of the Deza digraphs as parity-check matrices. We describe how LDPC codes can be obtained by applying the Cartesian product of certain Deza digraphs, and the Kronecker product of an adjacency matrix of a Deza digraph with parameters $(n, k, 0, 1)$ or $(n, k, 1, 1)$ and a permutation matrix. We also use several other combinatorial objects in the construction of LDPC codes.

1. Introduction

Binary low-density parity-check codes, commonly known as LDPC codes, were first described by Robert G. Gallager in the early 1960's (see [6]), with the purpose to obtain high data rates and insignificant error probabilities on noisy channels without the need for demanding equipment. MacKay and Neal examined them as well, demonstrating in the late 1990s that they have significantly better performance on Gaussian channels than conventional convolutional or concatenated codes ([13]). They showed that LDPC codes perform very close to the Shannon limit, almost as close as turbo codes. This performance close to the Shannon limit is one of the main motivations for investigating LDPC codes.

An LDPC code is a binary linear code defined by a sparse parity-check matrix H, which means that H contains a very small number of non-zero elements. More precisely, a matrix is sparse if more than half of its elements are equal to zero, i.e., if its sparsity (the ratio between the number of zero-valued elements and the total number of elements) is greater than 0.5. An

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LDPC code is called (s,t)-regular if each column of the parity-check matrix H has weight s and each row has weight t. The Tanner graph of the LDPC code defined by the parity-check matrix H, is the incidence graph corresponding to the incidence structure with the incidence matrix H. It is a bipartite graph consisting of two sets of vertices, called bit nodes and check nodes, where a bit node is adjacent to a check node if that bit is included in the corresponding parity check equation. The Tanner graph $\mathcal{G}$ associated with the parity-check matrix H of an LDPC code C has an adjacency matrix of the form $\begin{bmatrix} 0 & H \\ H^T & 0 \end{bmatrix}$. The Tanner graph can help us to examine the properties of the LDPC code.

The girth of the LDPC code defined by H is the girth of the corresponding Tanner graph. Short cycles in the Tanner graph of a code, especially cycles of length four, negatively impact the bit error rate (BER) performance of the code. The shorter the cycles, the greater the negative effect. Since the Tanner graph is bipartite, the length of a cycle must be even and at least four. Therefore, when constructing LDPC codes, the aim is to construct LDPC codes with girth at least six.

LDPC codes without cycles of length four were constructed from different combinatorial structures, for example, in [3, 10, 11, 17].

It is known that the error floor performance of LDPC codes depends on the existence of certain substructures of the Tanner graph. For an additive white gaussian noise (AWGN) channel, structures called trapping sets have a harmful effect on the iterative decoding performance of an LDPC code. Especially, their subclass called absorbing sets contributes significantly to the error floor. Therefore, examining the existence of absorbing sets is important for LDPC codes. More information on error floors of LDPC codes can be found in [15] and [16].

In [4], the authors investigated LDPC codes without cycles of length four from Deza graphs. A Deza graph with parameters (n,k,b,c) is a k-regular graph with n vertices such that any two of its vertices have b or c common neighbours.

In this paper, we investigate LDPC codes without cycles of length four obtained from adjacency matrices of Deza digraphs. We follow the definition of a Deza digraph, the directed version of a Deza graph, introduced by Wang and Feng in 2006 ([19]), and also used in [20] by Wang and Li.

A digraph Γ is said to be regular of valency k if there exists a constant number k such that $|u^+| = |u^-| = k$ for each vertex u in Γ, where u^+ denotes the set of all out-neighbours of u and u^- denotes the set of all in-neighbours of u.

Definition 1.1. *A regular digraph Γ with n vertices with adjacency matrix A is an (n,k,b,c)-Deza digraph if*

$$AA^T = kI + bB + cC,\ (b \leq c)$$

where B and C are symmetric non-zero binary matrices such that $B+C+I = J$. Here J is the all-one matrix of order n, and I is the identity matrix of order n.

We will assume that Γ has at least two vertices. Also, we will assume that the digraphs in this paper do not have more than one arc from one vertex to another, and no arcs from a vertex to itself.

Remark 1.2. As noted in [19], the definition of a Deza digraph can be stated in an equivalent form. An (n,k,b,c)-Deza digraph is a k-regular digraph with n vertices such that any two of its vertices have b or c common out-neighbours. In other words, for any two distinct vertices u and v, it holds that $|N_{u,v}| \in \{b,c\}$, where $N_{u,v} = u^+ \cap v^+$.

Note that this definition of Deza digraph is different than the one by Zhang and Wang from 2003 ([21]).

In [1], the authors examined another variant of directed Deza graphs, which they referred to as directed Deza graphs of type II. A digraph Γ with adjacency matrix A is said to be normal if $AA^T = A^TA$. A directed Deza graph of type II, as defined in [1], is a normal Deza digraph when regarding the definition of Wang and Feng from [19]. In the remainder of this paper, we will use the definition from [19].

For more information on concepts from graph theory and coding theory, we refer the reader to [5], [9] and [14].

The paper provides a description of the construction of LDPC codes using Deza digraphs and an analysis of the codes. It is organized as follows. In Sect. 2, we present a construction of LDPC codes using Deza digraphs with parameters $(n,k,0,1)$, and we describe the properties of the constructed codes. In Sect. 3, we describe how LDPC codes can be obtained by applying the Cartesian product of certain Deza digraphs, and the Kronecker product of an adjacency matrix of a Deza digraph with parameters $(n,k,0,1)$ or $(n,k,1,1)$ and a permutation matrix. We also use some other combinatorial objects in the construction of LDPC codes.

2. LDPC codes from Deza digraphs with parameters $(n,k,0,1)$

Let A be an adjacency matrix of an (n,k,b,c)-Deza digraph Γ. A is a $(0,1)$-matrix that has a constant row sum and a constant column sum, both equal to k. Consequently, the obtained binary code C with parity-check matrix A is then a (k,k)-regular code of length n and dimension equal to $n - \mathrm{rank}_2(A)$.

The sparsity of the adjacency matrix A of an (n,k,b,c)-Deza digraph, as well as for any other k-regular digraph with n vertices, is equal to $1-\frac{k}{n}$. The obtained code C will be an LDPC code under the condition that $1-\frac{k}{n}>\frac{1}{2}$, i.e., if $n>2k$.

As mentioned earlier, when constructing LDPC codes, we aim to avoid cycles of length four in the Tanner graph corresponding to the given parity-check matrix. A cycle of length four will exist in the Tanner graph of the code defined by the parity check-matrix A, if and only if there exist two vertices of the digraph Γ with more than one common out-neighbour. When constructing an LDPC code without cycles of length four from the adjacency matrix A of an (n,k,b,c)-Deza digraph, it follows that $b,c\in\{0,1\}$. Note that for an $(n,k,0,0)$-Deza digraph, A will be an incidence matrix of a symmetric 2-$(n,k,0)$ design, where $k\in\{0,1\}$. Consequently, either the design has empty blocks or it is a symmetric 2-$(n,1,0)$ design. Similarly, for an $(n,k,1,1)$-Deza digraph, A will be an incidence matrix of a symmetric 2-$(n,k,1)$ design, i.e., of a projective plane. LDPC codes constructed from projective planes have been extensively studied in the literature (see, e.g., [12]). Therefore, in the rest of the paper, we will focus on LDPC codes constructed from $(n,k,0,1)$-Deza digraphs.

Remark 2.1. For an (n,k,b,c)-Deza digraph Γ, and a vertex $x\in V(\Gamma)$, let α denote the number of vertices from $V(\Gamma)$ that have b common out-neighbours with x, and let β denote the number of vertices from $V(\Gamma)$ that have c common out-neighbours with x. Then by [19, Proposition 1.2] the numbers α and β do not depend on the choice of the vertex x, and specifically for $b=0$ and $c=1$ we obtain:

$$\alpha=n-1-k^2+k,\quad \beta=k^2-k.$$

So, for an $(n,k,0,1)$-Deza digraph Γ, each vertex of Γ has exactly one common out-neighbour with k^2-k vertices, and zero common out-neighbours with all the remaining vertices.

Remark 2.2. When Γ is an $(n,k,0,1)$-Deza digraph, it follows that $\beta=k^2-k>0$, and therefore, $k\geq 2$. For $k=2$, the obtained code C will be an LDPC code if $n>4$. The following proposition states that for $k\geq 3$, C will always be an LDPC code.

Proposition 2.3. *Let Γ be an $(n,k,0,1)$-Deza digraph with adjacency matrix A, where $k\geq 3$. Then the code C defined by A as its parity-check matrix is an LDPC code.*

Proof. Let x be a vertex of Γ. By Remark 2.1, there exist k^2-k distinct vertices that have one common out-neighbour with x, and at least one vertex that has zero common out-neighbours with x. Consequently, for the number n of vertices in Γ, it holds that $n\geq k^2-k+2$. Since $k^2-k+2>2k$ for $k\geq 3$, it follows that $n>2k$. □

The conditions stated in Remark 2.2 exclude trivial cases, and therefore, in the remainder of the paper we will suppose that these conditions hold, that is, that the codes constructed from Deza digraphs are LDPC codes.

2.1. *Parameters of LDPC codes from $(n, k, 0, 1)$-Deza digraphs.* In this subsection, we present the results obtained regarding the parameters of LDPC codes constructed from $(n, k, 0, 1)$-Deza digraphs.

Remark 2.4. The minimum distance of an LDPC code of girth at least six, with column weight k is at least $k + 1$ (see [7, Theorem 1.8]).

The immediate consequence of Remark 2.4 is the following lemma.

Lemma 2.5. *Let Γ be an $(n, k, 0, 1)$-Deza digraph with adjacency matrix A, where $n > 2k$, and C the LDPC code defined by A as its parity-check matrix. Then the minimum distance of the code C is at least $k + 1$.*

Corollary 2.6. *Let Γ be an $(n, k, 0, 1)$-Deza digraph with adjacency matrix A, where $n > 2k$, and C the LDPC code defined by A as its parity-check matrix. Then the dimension of the code C is at most $n - k$.*

Proof. Let d be the minimum distance of C. By the Singleton bound it follows that $d \leq n - \dim(C) + 1$. Since Lemma 2.5 asserts us that $d \geq k + 1$, it follows that $\dim(C) \leq n - d + 1 \leq n - k$. □

2.2. *Absorbing sets.* As previously stated, absorbing sets are a type of trapping sets that significantly contribute to the error floor ([18]), and therefore have a negative effect on the decoding performance of an LDPC code.

Let C be an LDPC code obtained from a parity-check matrix A and let $\mathcal{G}$ be the corresponding Tanner graph. We say that a set T of a bit nodes from $\mathcal{G}$ is an (a, b) trapping set in $\mathcal{G}$, if there are exactly b check nodes of odd degree in the subgraph $\mathcal{G}[T]$ of $\mathcal{G}$ induced by $T \cup N(T)$, where $N(T)$ is the neighbourhood of T. The most harmful trapping sets are those with small sizes and small ratios $\frac{b}{a}$. An absorbing set is a trapping set T such that every bit node in $\mathcal{G}[T]$ is adjacent to fewer check nodes of odd degree than of even degree.

Remark 2.7. Let A be an adjacency matrix of an $(n, k, 0, 1)$-Deza digraph. Let T be an (a, b) absorbing set in the Tanner graph $\mathcal{G}$ of C corresponding to the parity-check matrix A. Then it can be easily shown that b is even if and only if a is even or k is even (see proof of [3, Remark 4.1]).

Theorem 2.8. *Let Γ be an $(n, k, 0, 1)$-Deza digraph with adjacency matrix A, where $n > 2k$. Furthermore, let C be the LDPC code defined by A as the parity check matrix, and let T be an (a, b) absorbing set in the corresponding Tanner graph $\mathcal{G}$. Then it holds:*

$$a \geq \left\lfloor \frac{k}{2} \right\rfloor + 2.$$

Proof. Let x be a bit node from T. It is of degree k, so it is adjacent to k check nodes in $\mathcal{G}[T]$. Since T is an absorbing set, at least $\left\lfloor \frac{k}{2} \right\rfloor + 1$ of those check nodes adjacent to x must be of even degree. Each check node of an even degree adjacent to x in $\mathcal{G}[T]$ must be adjacent to at least one other bit node from T. Since these check nodes can have at most one common neighbour, no two of them can be adjacent to the same bit node other than x. It follows that the size a of the absorbing set T is at least $\left\lfloor \frac{k}{2} \right\rfloor + 2$. ∎

3. Constructions of LDPC codes

3.1. *LDPC codes from the Kronecker product of an adjacency matrix of a Deza digraph and a permutation matrix.* An infinite family of LDPC codes whose Tanner graphs do not have cycles of length four can be constructed by applying the Kronecker product of the adjacency matrix of an $(n, k, 0, 1)$-Deza digraph and a suitable permutation matrix. An analogous construction starting from Deza graphs instead of Deza digraphs was described in [4], using the protograph "copy and permute" operation.

Theorem 3.1. *Let* Γ *be an* $(n, k, 0, 1)$*-Deza digraph with adjacency matrix* A*, where* $n > 2k$*, and let* C *be the* $[n, k', d]$ *binary LDPC code defined by* A *as its parity-check matrix. Furthermore, let* m *be a positive integer and* P *a permutation matrix of order* m*. Then* $A_m = A \otimes P$*, where* $\otimes$ *is the Kronecker product, is an adjacency matrix of an* $(nm, k, 0, 1)$*-Deza digraph* Γ_m *and the binary code* C_m *with the parity-check matrix* A_m *is an* $[nm, k'm, d]$ *LDPC code.*

Proof. The digraph Γ_m is a regular digraph of valency k with nm vertices. The matrix A_m is obtained from A by replacing every occurrence of one with a permutation matrix P, and every zero element with a null-matrix of order m. It can be easily seen that any two rows of A_m have either one or zero common ones, i.e., any two vertices of Γ_m have either one or zero common out-neighbours. Furthermore, both zero and one appear as the number of common out-neighbours for some vertices, and therefore, Γ_m is an $(nm, k, 0, 1)$-Deza digraph.
The dimension of C_m equals

$$nm - \operatorname{rank}_2(A_m) = nm - m \cdot \operatorname{rank}_2(A) = mk'.$$

A minimal set of linearly dependent columns from A_m can only contain columns from different blocks of columns of size m, but with the same indices within the blocks and is, therefore, equal to the minimal number d of linearly dependent columns of A. Consequently, the minimum distance of C_m is equal to d. ∎

Similarly, we can also use an adjacency matrix of an $(n, k, 1, 1)$-Deza digraph, instead of an $(n, k, 0, 1)$-Deza digraph, and apply an analogous construction. We again obtain an infinite family of larger Deza digraphs with

$b = 0$ and $c = 1$, and consequently an infinite family of LDPC codes with corresponding Tanner graps of girth at least 6. The adjacency matrix of an $(n, k, 1, 1)$-Deza digraph is an incidence matrix of a symmetric 2-$(n, k, 1)$ design, i.e., of a projective plane $PG(2, q)$, $q = p^s$, where p is prime and s a positive integer. For $q = 2^s$, we can determine the parameters of the obtained larger LDPC codes more precisely.

Remark 3.2. Let A be an incidence matrix of the projective plane - $PG(2, q)$, where $q = 2^s$. Let C be the LDPC code defined by the parity check matrix A. As stated in [12], the LDPC code C is $(q + 1, q + 1)$-regular and has parameters $[n, k', d]$, where:

$$n = q^2 + q + 1,\ k' = n - 3^s - 1 = q^2 + q - 3^s,\ d = q + 2.$$

Theorem 3.3. *Let Γ be an $(n, k, 1, 1)$-Deza digraph with adjacency matrix A which is an incidence matrix of the projective plane $PG(2, q)$, $q = 2^s$, and let C be the binary LDPC code defined by A as its parity-check matrix. Furthermore, let $m \geq 2$ be a positive integer and P a permutation matrix of order m. Then $A_m = A \otimes P$, where $\otimes$ is the Kronecker product, is an adjacency matrix of a $((q^2+q+1)m, q+1, 0, 1)$-Deza digraph Γ_m and the binary code C_m with the parity-check matrix A_m is a $[(q^2+q+1)m, (q^2+q-3^s)m, q+2]$ LDPC code.*

We omit the proof of Theorem 3.3, since it is analogous to the proof of Theorem 3.1.

Remark 3.4. We can make these constructions from $(n, k, 0, 1)$ and $(n, k, 1, 1)$-Deza digraphs even more general by putting different permutation matrices of the same order on the positions of ones in the adjacency matrix A of a Deza digraph. This generalization gives a greater diversity of parameters for the obtained LDPC codes.

3.2. *LDPC codes from the Cartesian product od Deza digraphs.* In the sequel, we describe how LDPC codes corresponding to $(n, k, 0, 1)$-Deza digraphs can be obtained by applying the Cartesian product of Deza digraphs that satisfy certain conditions.

For two digraphs Γ_1 and Γ_2, the Cartesian product, or the directed product as it is called in [19], $\Gamma_1 \times \Gamma_2$ from Γ_1 to Γ_2 is a digraph with the vertex set $V\Gamma_1 \times V\Gamma_2$ and the arc set:

$$\{((u_1, u_2), (v_1, v_2)) \mid (u_1 = v_1 \wedge (u_2, v_2) \in A\Gamma_2) \vee ((u_1, v_1) \in A\Gamma_1 \wedge u_2 = v_2)\}.$$

Remark 3.5. Let A_1 be the adjacency matrix of the digraph Γ_1, and A_2 the adjacency matrix of the digraph Γ_2. Then an adjacency matrix A of the Cartesian product $\Gamma_1 \times \Gamma_2$ is given by:

$$A = A_1 \otimes I_{n_2} + I_{n_1} \otimes A_2,$$

where $\otimes$ is the Kronecker product of matrices, $n_1 = |V\Gamma_1|$ and $n_2 = |V\Gamma_2|$. In other words, the matrix A can be obtained by replacing every occurrence of one in A_1 with the identity matrix of order n_2, every main diagonal entry with the matrix A_2, and every other zero element with the null-matrix of order n_2.

A digraph is called asymmetric if it has at most one arc between a pair of vertices, i.e. it does not contain oppositely oriented arcs between the same vertices. This implies that there cannot be two ones on the symmetric positions in the adjacency matrix of an asymmetric digraph. The Cartesian product $\Gamma_1 \times \Gamma_2$ is asymmetric if and only if both Γ_1 and Γ_2 are asymmetric.

In the following theorem, we state the conditions under which the Cartesian product of Deza digraphs is a Deza digraph with parameters $b = 0, c = 1$. In the proof, we use the result from [19, Theorem 3.6], which states that for any two distinct vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ from the Cartesian product $\Gamma_1 \times \Gamma_2$ of digraphs it holds: $|N_{u,v}| =$

$$\begin{cases} |N_{u_2,v_2}|, & \text{if } u_1 = v_1, \\ |N_{u_1,v_1}|, & \text{if } u_2 = v_2, \\ 2, & \text{if } \partial(u_1,v_1) = \partial(v_1,u_1) = \partial(u_2,v_2) = \partial(v_2,u_2) = 1, \\ 1, & \text{if } \partial(u_1,v_1) = \partial(v_2,u_2) = 1 \text{ and } \partial(v_1,u_1) \neq 1 \text{ or } \partial(u_2,v_2) \neq 1, \\ 1, & \text{if } \partial(v_1,u_1) = \partial(u_2,v_2) = 1 \text{ and } \partial(u_1,v_1) \neq 1 \text{ or } \partial(v_2,u_2) \neq 1, \\ 0, & \text{otherwise.} \end{cases}$$

THEOREM 3.6. *Let Γ_1 be an (n_1, k_1, b_1, c_1)-Deza digraph and let Γ_2 be an (n_2, k_2, b_2, c_2)-Deza digraph, where $(b_1, c_1), (b_2, c_2) \in \{(0,0), (0,1), (1,1)\}$, and where at least one of the digraphs Γ_1, Γ_2 is asymmetric and both are non-empty (i.e., $k_1, k_2 > 0$). Additionaly, if $(b_1, c_1) = (b_2, c_2) = (1,1)$, then suppose that either Γ_1 and Γ_2 are both distinct from the $(3,2,1,1)$-Deza digraph, or one of them is the $(3,2,1,1)$-Deza digraph and the other is an asymmetric Deza digraph whose adjacency matrix contains two zeros on symmetric positions. It follows that the Cartesian product $\Gamma = \Gamma_1 \times \Gamma_2$ is an $(n_1 n_2, k_1 + k_2, 0, 1)$-Deza digraph.*

PROOF. It is easy to see that $\Gamma_1 \times \Gamma_2$ is a regular digraph of valency $k_1 + k_2$ with $n_1 n_2$ vertices. Since at least one of the digraphs Γ_1 and Γ_2 does not contain oppositely oriented arcs between the same vertices, the case $|N_{u,v}| = 2$ is not possible, and therefore, $|N_{u,v}| \in \{0, 1\}$ for any two vertices u and v from Γ.

When at least one of the pairs (b_1, c_1), (b_2, c_2) is equal to $(0, 1)$, it is obvious that $|N_{u,v}|$ takes both values, 0 and 1. We will show that for $(b_1, c_1) = (b_2, c_2) = (0, 0)$ or $(b_1, c_1) = (b_2, c_2) = (1, 1)$, $|N_{u,v}|$ also takes both possible values. Let A be an adjacency matrix of $\Gamma_1 \times \Gamma_2$ given by $A = A_1 \otimes I_{n_2} + I_{n_1} \otimes A_2$, whose block structure is described in Remark 3.5.

1. Suppose that $(b_1, c_1) = (b_2, c_2) = (0, 0)$. Then the scalar product of any two rows of A from the same block of rows is equal to zero.

Furthermore, the scalar product of any two rows from different blocks of rows, that are on the same position within the blocks, is equal to zero. Since $k_1, k_2 > 0$, A contains a submatrix of the following form:

$$M = \begin{bmatrix} A_2 & \cdots & I \\ \vdots & \ddots & \vdots \\ B & \cdots & A_2 \end{bmatrix},$$

where B is either the null-matrix or the identity matrix.

i) If A_1 is asymmetric, then B must be the null-matrix. In that case, let r be the row of A that corresponds to the first row of M. Further, let s be the row of A that corresponds to the row from the last block of rows in M, in which the part within the block A_2 starts with a one (such a row exists, since $k_2 > 0$). Then the scalar product of the rows r and s is equal to one.

ii) If A_2 is asymmetric, regardless of whether B is the identity matrix or the null-matrix, we can again choose the same rows r and s as in the previous case and obtain the scalar product equal to one.

2. Suppose that $(b_1, c_1) = (b_2, c_2) = (1, 1)$. Then the scalar product of any two rows of A from the same block of rows is equal to one. Furthermore, the scalar product of any two rows from different blocks of rows, that are on the same position within the blocks, is equal to one.

   i) Assume that Γ_1 and Γ_2 are both distinct from the $(3, 2, 1, 1)$-Deza digraph. In that case, both A_1 and A_2 contain at least one element equal to zero outside the main diagonal in each of its rows and columns. Then A contains a submatrix of the form:

$$M = \begin{bmatrix} A_2 & \cdots & O \\ \vdots & \ddots & \vdots \\ B & \cdots & A_2 \end{bmatrix},$$

   where B is either the null-matrix or the identity matrix. Let B be the null-matrix and let r be the row of A corresponding to any row from the first block of rows of M. Further, let s be the row of A corresponding to a row from the last block of rows of M, such that r and s are on distinct positions within the blocks. Then the scalar product of these two rows is equal to zero. Now, suppose that B is the identity matrix. Let s be the row of A that corresponds to the first row of the last block of rows of M. Further, let r be the row of A that corresponds to the row from the first block of rows in M, that is not the first row in this block of rows, in which the part within the block A_2 starts with a zero. Then the scalar product of the rows r and s is equal to zero.

ii) Assume that one of Γ_1, Γ_2 is the $(3,2,1,1)$-Deza digraph and the other is an asymmetric Deza digraph whose adjacency matrix contains two zeros on symmetric positions.

If Γ_1 is the $(3,2,1,1)$-Deza digraph, then all the off-diagonal blocks of A are equal to the identity matrix, so A contains a submatrix of the form:

$$M = \begin{bmatrix} A_2 & \cdots & I \\ \vdots & \ddots & \vdots \\ I & \cdots & A_2 \end{bmatrix}.$$

Let (i,j) and (j,i), with $i \neq j$, be the symmetric positions on which A_2 has elements equal to zero. Take r to be the row of A that corresponds to the i-th row of the first block of rows of M, and take s to be the row of A that corresponds to the j-th row of the last block of rows of M. Then the scalar product of r and s is zero.

Finally, suppose that Γ_2 is the $(3,2,1,1)$-Deza digraph and that Γ_1 is an asymmetric Deza digraph whose adjacency matrix contains two zeros on symmetric positions. In this case A contains a submatrix of the form:

$$M = \begin{bmatrix} A_2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_2 \end{bmatrix}.$$

The scalar product of the row r of A that corresponds to any row from the first block of rows in M and the row s of A that corresponds to any row from the last block of rows in M, such that r and s are on distinct positions within the blocks of rows, is then equal to zero.

□

REMARK 3.7. Note that the only asymmetric $(n,k,1,1)$-Deza digraph whose adjacency matrix does not contain two zeros in symmetric positions is the asymmetric $(7,3,1,1)$-Deza digraph, i.e, its adjacency matrix is an asymmetric incidence matrix of a symmetric 2-$(7,3,1)$ design. For an asymmetric $(n,k,1,1)$-Deza digraph whose adjacency matrix does not contain two zeros in symmetric positions, the equality $n = 2k+1$ must hold. Using the necessary condition $k(k-1) = \lambda(n-1)$ for the existence of a symmetric block design, it is easy to prove the fact mentioned above.

Theorem 3.8. *Let A be the adjacency matrix of the Cartesian product $\Gamma_1 \times \Gamma_2$, where Γ_1 and Γ_2 are Deza digraphs that satisfy the conditions of Theorem 3.6, such that $n_1n_2 > 2(k_1+k_2)$. Then the binary code C defined by the parity-check matrix A is a (k_1+k_2, k_1+k_2)-regular LDPC code of length n_1n_2. The dimension of the code C is at most $n_1n_2 - max\{n_2, k_1+k_2\}$ and the minimum distance is at least k_1+k_2+1. Furthermore, the Tanner graph of the code C corresponding to A has girth at least six.*

Proof. The result follows from Theorem 3.6, Remark 3.5, Lemma 2.5 and Corollary 2.6. □

Remark 3.9. As a special case of the previous theorem we can take $\Gamma_1 = \Gamma_2$. Let A_1 be an adjacency matrix of an asymmetric $(n,k,0,1)$-Deza digraph Γ_1, where $n > 2k$, so that the code C_1 defined by A_1 as parity-check matrix is an LDPC code. Then for arbitrary nonnegative integer m, the Cartesian product $\Gamma = \times_m \Gamma_1$ is an $(n^m, km, 0, 1)$-Deza digraph whose adjacency matrix A defines a parity-check matrix of a (km, km)-regular LDPC code of length n^m.

3.3. *LDPC codes from distance-regular graphs.* In [20], Wang and Li describe several constructions of Deza digraphs from incidence structures associated with distance regular graphs. The cases in which they produce $(n,k,0,1)$-Deza digraphs can be used to obtain LDPC codes.

Definition 3.10. *A connected graph $\Delta = (X, R)$ with diameter d is a distance-regular graph (DRG for short) if for all integers $h, i, j \in \{0, 1, \dots, d\}$, and for all $x, y \in X$ such that $\partial(x,y) = h$, the number*

$$p^h_{i,j} = |\{z \in X \mid \partial(x,z) = i,\ \partial(z,y) = j\}|$$

is independent of the choice of x and y. The numbers $p^h_{i,j}$ are the intersection numbers of the DRG Δ. We will denote $c_i = p^i_{i-1,1}$, for $1 \le i \le d$.

As an immediate consequence of [20, Theorem 4.1.], for $c_2 = 1$ and $d \ge 4$ we obtain the following result.

Theorem 3.11. *Let Δ be a bipartite DRG with a bipartition $X_1 \cup X_2$ with $2n$ vertices and with diameter $d \ge 4$ and valency k, for which $c_2 = 1$. Let A be the incidence matrix with zero main diagonal entries of the incidence structure (X_1, X_2), defined by*

$$x \in X_1 \text{ is incident with } y \in X_2 \text{ if and only if } \partial(x,y) = 1.$$

Then the digraph Γ with the adjacency matrix A is an $(n,k,0,1)$-Deza digraph, and for $n > 2k$ the binary code with the parity-check matrix A is a (k,k)-regular LDPC code of length n. The dimension of the code C is at most $n-k$ and the minimum distance is at least $k+1$. Furthermore, the Tanner graph of the code C has girth at least six.

The second construction of Deza digraphs from distance regular graphs by Wang and Li, which uses maximal cliques, directly gives $(n, k, 0, 1)$-Deza digraphs and is described in [20, Theorem 4.2.]. The constructed Deza digraphs from this construction can also be used to obtain LDPC codes.

3.4. *LDPC codes from divisible design digraphs.* As noted in [1], a divisible design digraph with parameters $(v, k, \lambda_1, \lambda_2, m, n)$ is a directed $(v, k, \lambda_1, \lambda_2)$-Deza graph of type II, i.e., a normal $(v, k, \lambda_1, \lambda_2)$-Deza digraph in the sense of our definition.

In [2, Theorem 2.17], the authors have proven the existence of divisible design digraphs with parameters $(n^2 - 1, n, 0, 1, n + 1, n - 1)$, for each n an odd prime power. The digraphs in question were constructed from certain skew balanced generalized weighing (BGW) matrices. As an immediate consequence, we obtain the following result.

Theorem 3.12. *Let n be an odd prime power. Then there exists a normal $(n^2 - 1, n, 0, 1)$-Deza digraph, and therefore there also exists an (n, n)-regular LDPC code of length $n^2 - 1$, dimension at most $n^2 - n - 1$ and minimum distance at least $n + 1$, whose corresponding Tanner graph has girth at least 6.*

3.5. *LDPC codes from Cayley digraphs.* In the sequel, we consider the $(n, k, 0, 1)$-Deza digraphs that are constructed from Cayley digraphs.

Definition 3.13. *Let G be a group, and let S be a subset of G such that the identity element e of G is not in S. The Cayley digraph $Cay(G, S)$ is a directed graph with the vertex set G and the set of arcs $\{(g, gs) \mid g \in G, s \in S\}$.*

Remark 3.14. Let $\mathbb{Z}[G]$ be the set of all formal sums of elements of the group G with coefficients from the set $\mathbb{Z}$. For any subset S of G, we will denote by $\overline{S}$ the sum of all elements of S in the group ring $\mathbb{Z}[G]$.

In [19, Theorem 2.2], the authors provided the necessary and sufficient condition for a Cayley digraph to be a Deza digraph. The question we have raised in this work is whether there exists a Cayley digraph that is an $(n, k, 0, 1)$-Deza digraph. The results we have obtained are summarized in the following theorem.

Theorem 3.15. *Let G be a group with identity element e, such that $|G| = n$. Let $k \geq 2$ and let $S = \{a_1, \ldots, a_k\}$ be a k-subset of G that does not contain e. The Cayley digraph $Cay(G, S)$ is an $(n, k, 0, 1)$-Deza digraph if and only if for*

$$C = \{a_i a_j^{-1} \mid i, j \in \{1, 2, \ldots, k\}, i \neq j\},$$

it holds that $|C| = k(k - 1)$, where $e + \overline{B} + \overline{C} = \overline{G}$, for a nonempty subset B of G.

Proof. Let $S^{-1} = \{a_1^{-1}, \dots, a_k^{-1}\}$ be the set of inverses of the elements of S. Now it follows

$$\begin{aligned}\overline{S}\cdot\overline{S^{-1}} =&(a_1+\cdots+a_k)(a_1^{-1}+\cdots+a_k^{-1})\\ =&ke+a_1(a_2^{-1}+\cdots+a_k^{-1})+a_2(a_1^{-1}+a_3^{-1}+\cdots+a_k^{-1})+\cdots\\ &+\ a_k(a_1^{-1}+\cdots+a_{k-1}^{-1}).\end{aligned}$$

For any two vertices u and v of $\Gamma = \mathrm{Cay}(G,S)$, $|N_{u,v}|$ is equal to the coefficient of $v^{-1}u$ in $\overline{S}\cdot\overline{S^{-1}}$. The proof now follows from [19, Theorem 2.2]. □

Remark 3.16. The sets B, C and $\{e\}$ from Theorem 3.15 form a partition of G.

Remark 3.17. Suppose that there exists a Cayley digraph $\mathrm{Cay}(G,S)$ that is an $(n,k,0,1)$-Deza digraph. According to Theorem 3.15, the inequality $|G| \geq k(k-1)+2$ holds.
If $|C| = k(k-1)$ and B is the empty set, i.e., $|G| = k(k-1)+1$, then the corresponding Cayley digraph is an $(n,k,1,1)$-Deza digraph. Its adjacency matrix is an incidence matrix of a symmetric 2-$(n,k,1)$ design, i.e., of a projective plane.

Corollary 3.18. *Let Γ be a Cayley digraph that is an $(n,k,0,1)$-Deza digraph, where $n > 2k$, and let A be an adjacency matrix of Γ. Let C be an LDPC code C defined by A as its parity-check matrix. Then C is a (k,k)-regular LDPC code of length n. The dimension of the code C is at most $n-k$ and the minimum distance is at least $k+1$. Furthermore, the Tanner graph of the code C defined by the parity-check matrix A has girth at least six.*

Remark 3.19. If G is a cyclic group, then the Cayley digraph $\Gamma = \mathrm{Cay}(G,S)$ is circulant, i.e., the vertices of Γ can be ordered so that its adjacency matrix is a circulant matrix.

Proposition 3.20. *Let G be a cyclic group with identity element e and let S be a k-subset of G that does not contain e. Let $\Gamma = Cay(G,S)$ be a Cayley digraph that is an $(n,k,0,1)$-Deza digraph, where $n > 2k$, and let A be an adjacency matrix of Γ. Then the LDPC code C defined by the parity-check matrix A is a cyclic code. If A is a circulant matrix and if m is the largest number of consecutive zeros within a row of A cyclically, then the dimension of C is at most $n-m-1$.*

Proof. As a consequence of Remark 3.19, C is a cyclic code. If m is the largest number of consecutive zeros within a row of A cyclically and A is circulant, then any $m+1$ consecutive rows of A cyclically are linearly independent. The 2-rank of A is therefore at least $m+1$, so the dimension of the code C is at most $n-m-1$. □

Remark 3.21. Let G be a cyclic group of order n with generator σ and identity element e and let $S = \{a_1, \ldots, a_k\}$ be a k-subset of G that does not contain e. Let $a_j = \sigma^{i_j}$ for $j \in \{1, 2, \ldots, k\}$, where $1 \leq i_1 < i_2 < \ldots < i_k \leq n-1$. Furthermore, let $\Gamma = \mathrm{Cay}(G, S)$ be a Cayley digraph that is an $(n, k, 0, 1)$-Deza digraph, where $n > 2k$, with an adjacency matrix A and let C be the LDPC code defined by A as its parity-check matrix. Then the dimension of the code C is at most $n - m - 1$, where

$$m = \max\{i_2 - i_1 - 1, i_3 - i_2 - 1, \ldots, i_k - i_{k-1} - 1, n - i_k + i_1 - 1\}.$$

In other words, m is the maximum number of consecutive zeros cyclically, in the binary vector of length n that has ones exactly on the positions i_j, for $j \in \{1, \ldots, k\}$.

We have constructed Cayley digraphs of cyclic groups that are $(n, k, 0, 1)$-Deza digraphs for $k \in \{2, 3, 4, 5\}$, up to certain values of n in each case. Information about the non-trivial LDPC codes, that are non-isomorphic to repetition codes, whose parity-check matrices are adjacency matrices of these graphs, is presented in Tables 1 - 4. Trivial codes and repetition codes also appear, but they are excluded from the tables. Up to isomorphism, there exists exactly one code with each of the given parameters, for fixed values of n and k. Moreover, we have checked whether the codes with the same parameters corresponding to different values of k are isomorphic. The results we obtained show that LDPC codes with the same parameters, but listed in different tables, are isomorphic in all cases except for the $[24, 4, 8]$ LDPC codes constructed from $Cay(Z_{24}, \{1, 3, 17\})$ and $Cay(Z_{24}, \{1, 2, 4, 17\})$, and $[30, 8, 8]$ LDPC codes constructed from $Cay(Z_{30}, \{1, 3, 9\})$ and $Cay(Z_{30}, \{1, 2, 9, 13, 15\})$.

Some of the constructed codes, marked with *, achieve an upper bound for the minimum distance, for the given length and dimension, and are therefore optimal (see [8]). The codes marked with ** are near-optimal codes, i.e., their minimum distance is one less than the upper bound. All constructed codes are cyclic.

4. Conclusion

In this paper, we give a construction of LDPC codes using Deza digraphs with parameters $(n, k, 0, 1)$. By taking an adjacency matrix of a Deza digraph with parameters $(n, k, 0, 1)$, where $n > 2k$, as a parity-check matrix of an LDPC code, the corresponding Tanner graph does not contain cycles of length 4. Using the constructions described in the paper, infinite families of LDPC codes with various parameters can be obtained. Moreover, we have shown that we can obtain optimal and near-optimal LDPC codes from Deza digraphs.

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Table 1. Parameters of the non-trivial LDPC codes, other than repetition codes, constructed from the cyclic Cayley digraphs that are $(n, 2, 0, 1)$-Deza digraphs, $n \leq 20$

Parameters of Deza digraphs	Cayley digraph	LDPC	Girth
(6,2,0,1)	$Cay(\mathbb{Z}_6, \{1, 3\})$	$[6, 2, 3]^{**}$	6
(8,2,0,1)	$Cay(\mathbb{Z}_8, \{1, 3\})$	$[8, 2, 4]^{**}$	8
(9,2,0,1)	$Cay(\mathbb{Z}_9, \{1, 4\})$	$[9, 3, 3]^{**}$	6
(10,2,0,1)	$Cay(\mathbb{Z}_{10}, \{1, 3\})$	$[10, 2, 5]^{**}$	10
(12,2,0,1)	$Cay(\mathbb{Z}_{12}, \{1, 3\})$	[12,2,6]	12
	$Cay(\mathbb{Z}_{12}, \{1, 4\})$	[12,3,4]	8
	$Cay(\mathbb{Z}_{12}, \{1, 5\})$	[12,4,3]	6
(14,2,0,1)	$Cay(\mathbb{Z}_{14}, \{1, 3\})$	[14,2,7]	14
(15,2,0,1)	$Cay(\mathbb{Z}_{15}, \{1, 4\})$	[15,3,5]	10
	$Cay(\mathbb{Z}_{15}, \{1, 6\})$	[15,5,3]	6
(16,2,0,1)	$Cay(\mathbb{Z}_{16}, \{1, 3\})$	[16,2,8]	16
	$Cay(\mathbb{Z}_{16}, \{1, 5\})$	[16,4,4]	8
(18,2,0,1)	$Cay(\mathbb{Z}_{18}, \{1, 3\})$	[18,2,9]	18
	$Cay(\mathbb{Z}_{18}, \{1, 4\})$	[18,3,6]	12
	$Cay(\mathbb{Z}_{18}, \{1, 7\})$	[18,6,3]	6
(20,2,0,1)	$Cay(\mathbb{Z}_{20}, \{1, 3\})$	[20,2,10]	20
	$Cay(\mathbb{Z}_{20}, \{1, 5\})$	[20,4,5]	10
	$Cay(\mathbb{Z}_{20}, \{1, 6\})$	[20,5,4]	8

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References

[1] D. Crnković, H. Kharaghani, S. Suda and A. Švob, *New constructions of Deza digraphs,* Int. J. Group Theory **13** (2024), 225–240.

[2] D. Crnković, H. Kharaghani and A. Švob, *Divisible design Cayley digraphs,* Discrete Math. **343** (2020), 111784.

[3] D. Crnković, N. Mostarac and A. Švob, *LDPC LCD codes from odd graphs,* Adv. Math. Commun. **20** (2026), 68–83.

[4] D. Crnković, S. Rukavina and M. Šimac, *LDPC codes from Deza graphs,* Probl. Inf. Transm., to appear.

[5] R. Diestel, Graph Theory, Springer-Verlag, Berlin Heidelberg, 2017.

[6] R. G. Gallager, Low-Density Parity-Check Codes, MIT Press, Cambridge, MA, 1963.

[7] M. Greferath, C. Rößing and L. Storme, *Galois geometries and low-density parity-check codes,* in: Current Research Topics in Galois Geometry, Nova Science Publishers, New York, 2014, 245–270.

[8] M. Grassl, Bounds on the minimum distance of linear codes and quantum codes, Online available at http://www.codetables.de. Accessed on 2025-11-13.

[9] W. C. Huffman and V. Pless, Fundamentals of Error-Correcting Codes, Cambridge University Press, Cambridge, 2003.

TABLE 2. Parameters of the non-trivial LDPC codes, other than repetition codes, constructed from the cyclic Cayley digraphs that are $(n,3,0,1)$-Deza digraphs, $n \leq 30$

Parameters of Deza digraphs	Cayley digraph	LDPC	Girth
(12,3,0,1)	$Cay(\mathbb{Z}_{12},\{1,2,6\})$	$[12,2,8]^*$	6
(14,3,0,1)	$Cay(\mathbb{Z}_{14},\{1,2,4\})$	$[14,3,8]^*$	6
	$Cay(\mathbb{Z}_{14},\{1,3,7\})$	$[14,6,4]^{**}$	6
(15,3,0,1)	$Cay(\mathbb{Z}_{15},\{1,2,6\})$	$[15,2,10]^*$	6
	$Cay(\mathbb{Z}_{15},\{1,2,5\})$	$[15,4,8]^*$	6
(18,3,0,1)	$Cay(\mathbb{Z}_{18},\{1,2,6\})$	$[18,2,12]^*$	6
(21,3,0,1)	$Cay(\mathbb{Z}_{21},\{1,2,9\})$	$[21,2,14]^*$	6
	$Cay(\mathbb{Z}_{21},\{1,2,4\})$	$[21,3,12]^*$	6
	$Cay(\mathbb{Z}_{21},\{1,2,6\})$	$[21,5,10]^*$	6
	$Cay(\mathbb{Z}_{21},\{1,4,10\})$	[21,9,4]	6
(24,3,0,1)	$Cay(\mathbb{Z}_{24},\{1,2,6\})$	$[24,2,16]^*$	6
	$Cay(\mathbb{Z}_{24},\{1,3,17\})$	[24,4,8]	6
(27,3,0,1)	$Cay(\mathbb{Z}_{27},\{1,2,6\})$	$[27,2,18]^*$	6
(28,3,0,1)	$Cay(\mathbb{Z}_{28},\{1,2,4\})$	$[28,3,16]^*$	6
	$Cay(\mathbb{Z}_{28},\{1,3,7\})$	[28,6,8]	6
	$Cay(\mathbb{Z}_{28},\{1,5,21\})$	[28,12,4]	6
(30,3,0,1)	$Cay(\mathbb{Z}_{30},\{1,2,6\})$	$[30,2,20]^*$	6
	$Cay(\mathbb{Z}_{30},\{1,5,21\})$	[30,4,10]	6
	$Cay(\mathbb{Z}_{30},\{1,2,5\})$	$[30,4,16]^*$	6
	$Cay(\mathbb{Z}_{30},\{1,3,9\})$	[30,8,8]	6

[10] F. Ivanov and V. Zyablov, *LDPC codes based on steiner quadruple systems and permutation matrices,* in: Fourteenth International Workshop on Algebraic and Combinatorial Coding Theory, Nova Science Publishers/Novinka, New York, 2014, 175–180.

[11] J.-L. Kim, U. N. Peled, I. Perepelitsa, V. Pless and S. Friedland, *Explicit construction of families of LDPC codes with no 4-cycles,* IEEE Trans. Inform. Theory **50** (2004), 2378–2388.

[12] Y. Kou, S. Lin and M. P. C. Fossorier, *Low Density Parity Check Codes Based on Finite Geometries: A Rediscovery and New Results,* IEEE Trans. Inform. Theory **47** (2002), 2711–2736.

[13] D. J. C. MacKay and R. M. Neal, *Near Shannon limit performance of low density parity check codes,* Electron. Lett. **32** (1997), 457–458.

[14] F. J. MacWilliams and N. J. A. Sloane, The Theory of Error-Correcting Codes, North-Holland, Amsterdam, 1977.

[15] E. McMillon, A. Beemer and C. A. Kelley, *Extremal absorbing sets in low-density parity-check codes,* Adv. Math. Commun. **17** (2023), 465–483.

[16] T. Richardson, *Error floors of LDPC codes,* in: Proc. Allerton Conference on Communications, Control, and Computing, Monticello, 2003, 1426–1435.

[17] J. Rosenthal and P. O. Vontobel, *Constructions of LDPC codes using Ramanujan graphs and ideas from Margulis,* in: 38th Allerton Conference on Communication, Control and Computing, 2000, 248–257.

TABLE 3. Parameters of the non-trivial LDPC codes, other than repetition codes, constructed from the cyclic Cayley digraphs that are $(n, 4, 0, 1)$-Deza digraphs, $n \leq 25$

Parameters of Deza digraphs	Cayley digraph	LDPC	Girth
(14,4,0,1)	$Cay(\mathbb{Z}_{14}, \{1, 2, 5, 7\})$	$[14, 4, 6]^{**}$	6
(15,4,0,1)	$Cay(\mathbb{Z}_{15}, \{1, 2, 5, 7\})$	[15,3,5]	6
	$Cay(\mathbb{Z}_{15}, \{1, 2, 4, 8\})$	$[15, 7, 5]^{*}$	6
(16,4,0,1)	$Cay(\mathbb{Z}_{16}, \{1, 2, 4, 13\})$	[16,2,8]	6
(18,4,0,1)	$Cay(\mathbb{Z}_{18}, \{1, 2, 4, 9\})$	[18,2,9]	6
	$Cay(\mathbb{Z}_{18}, \{1, 2, 4, 8\})$	[18,3,6]	6
(20,4,0,1)	$Cay(\mathbb{Z}_{20}, \{1, 2, 4, 9\})$	[20,2,10]	6
	$Cay(\mathbb{Z}_{20}, \{1, 2, 4, 15\})$	$[20, 3, 10]^{**}$	6
	$Cay(\mathbb{Z}_{20}, \{1, 2, 5, 10\})$	[20,4,5]	6
(21,4,0,1)	$Cay(\mathbb{Z}_{21}, \{1, 2, 4, 8\})$	[21,3,7]	6
	$Cay(\mathbb{Z}_{21}, \{1, 2, 5, 17\})$	$[21, 4, 9]^{**}$	6
	$Cay(\mathbb{Z}_{21}, \{1, 2, 5, 7\})$	$[21, 6, 7]^{**}$	6
	$Cay(\mathbb{Z}_{21}, \{1, 2, 4, 9\})$	$[21, 7, 8]^{*}$	6
(22,4,0,1)	$Cay(\mathbb{Z}_{22}, \{1, 2, 4, 9\})$	[22,2,11]	6
(24,4,0,1)	$Cay(\mathbb{Z}_{24}, \{1, 2, 4, 9\})$	[24,2,12]	6
	$Cay(\mathbb{Z}_{24}, \{1, 2, 4, 8\})$	[24,3,8]	6
	$Cay(\mathbb{Z}_{24}, \{1, 2, 4, 19\})$	$[24, 3, 12]^{**}$	6
	$Cay(\mathbb{Z}_{24}, \{1, 2, 5, 18\})$	[24,4,6]	6
	$Cay(\mathbb{Z}_{24}, \{1, 2, 4, 17\})$	[24,4,8]	6
	$Cay(\mathbb{Z}_{24}, \{1, 3, 4, 18\})$	[24,5,8]	6
	$Cay(\mathbb{Z}_{24}, \{1, 2, 5, 10\})$	[24,6,6]	6
(25,4,0,1)	$Cay(\mathbb{Z}_{25}, \{1, 2, 6, 17\})$	[25,5,5]	6

[18] C. Schlegel and S. Zhang, *On the dynamics of the error floor behavior in (regular) LDPC codes,* IEEE Trans. Inf. Theory **56** (2010), 3248–3264.

[19] K. Wang and Y.-Q. Feng, *Deza digraphs,* Eur. J. Comb. **27** (2006), 995–1004.

[20] K. Wang and F. Li, *Deza digraphs II,* Eur. J. Comb. **29** (2008), 369–378.

[21] G. Zhang and K. Wang, *A directed version of Deza graphs - Deza digraphs,* Australas. J. Combin. **28** (2003), 239–244.

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Table 4. Parameters of the non-trivial LDPC codes, other than repetition codes, constructed from the cyclic Cayley digraphs that are $(n, 5, 0, 1)$-Deza digraphs, $n \leq 40$

Parameters of Deza digraphs	Cayley digraph	LDPC	Girth
(24,5,0,1)	$Cay(\mathbb{Z}_{24}, \{1, 2, 4, 10, 21\})$	$[24, 2, 16]^*$	6
(27,5,0,1)	$Cay(\mathbb{Z}_{27}, \{1, 2, 5, 18, 20\})$	$[27, 2, 18]^*$	6
(28,5,0,1)	$Cay(\mathbb{Z}_{28}, \{1, 2, 4, 14, 21\})$	$[28, 3, 16]^*$	6
(30,5,0,1)	$Cay(\mathbb{Z}_{30}, \{1, 2, 4, 10, 15\})$	$[30, 2, 20]^*$	6
	$Cay(\mathbb{Z}_{30}, \{1, 2, 5, 14, 21\})$	[30,4,10]	6
	$Cay(\mathbb{Z}_{30}, \{1, 2, 7, 10, 20\})$	$[30, 4, 16]^*$	6
	$Cay(\mathbb{Z}_{30}, \{1, 2, 9, 13, 15\})$	[30,8,8]	6
(31,5,0,1)	$Cay(\mathbb{Z}_{31}, \{1, 2, 4, 9, 20\})$	$[31, 5, 16]^*$	6
	$Cay(\mathbb{Z}_{31}, \{1, 2, 4, 8, 16\})$	[31,15,6]	6
(33,5,0,1)	$Cay(\mathbb{Z}_{33}, \{1, 2, 4, 10, 21\})$	$[33, 2, 22]^*$	6
(35,5,0,1)	$Cay(\mathbb{Z}_{35}, \{1, 2, 4, 10, 17\})$	$[35, 3, 20]^*$	6
	$Cay(\mathbb{Z}_{35}, \{1, 2, 4, 18, 30\})$	[35,4,14]	6
(36,5,0,1)	$Cay(\mathbb{Z}_{36}, \{1, 2, 4, 10, 15\})$	$[36, 2, 24]^*$	6
	$Cay(\mathbb{Z}_{36}, \{1, 2, 4, 13, 18\})$	[36,4,12]	6
	$Cay(\mathbb{Z}_{36}, \{1, 2, 4, 16, 29\})$	[36,6,8]	6
	$Cay(\mathbb{Z}_{36}, \{1, 2, 4, 18, 31\})$	[36,6,12]	6
(39,5,0,1)	$Cay(\mathbb{Z}_{39}, \{1, 2, 4, 9, 31\})$	$[39, 2, 26]^*$	6
	$Cay(\mathbb{Z}_{39}, \{1, 2, 7, 30, 32\})$	[39,12,12]	6
(40,5,0,1)	$Cay(\mathbb{Z}_{40}, \{1, 2, 4, 8, 20\})$	$[40, 4, 16]$	6

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