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Tian Yue

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ON UNIFORM INSTABILITY IN MEAN OF STOCHASTIC SKEW-EVOLUTION SEMIFLOWS

Tian Yue
Hubei University of Automotive Technology, China

Abstract. In this paper we study three concepts of uniform instability in mean for stochastic skew-evolution semiflows: uniform exponential instability in mean, uniform polynomial instability in mean and uniform h-instability in mean. These concepts are natural generalizations from the deterministic case. Connections between these concepts are presented. Additionally, some expansion properties, logarithmic criteria and majorization criteria of these concepts are given, respectively.

1. Introduction

The topic of asymptotic behaviors of dynamical systems on Banach spaces has been intensively studied for many years. In the last decades, the results concerning this subject have witnessed significant progress (see [1, 2, 3, 4, 5, 8, 9, 10, 11, 17, 28, 30, 31] and the references therein). As a generalization of classical concepts such as C_0-semigroup, evolution family and linear skew-product semiflow, the notion of linear skew-evolution semiflow (or linear skew-product three-parameter semiflow) plays an important role in the qualitative theory of dynamical systems, especially in the deterministic case. The notion of linear skew-evolution semiflow was first introduced by Megan et al. in [22]. Starting with the work [22], several important papers were published regarding the asymptotic properties of skew-evolution semiflows (see [12, 14, 15, 16, 18, 19, 21, 24, 25, 26, 27]).

In recent years, many researchers focused on the exponential/polynomial stability and dichotomy of stochastic skew-evolution semiflows. For instance, a Datko type characterization for nonuniform exponential dichotomy in mean

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square of stochastic skew-evolution semiflows were studied by Stoica and Megan in [29]. In [20], through the usage of Banach function spaces, some discrete and continuous versions of Datko type theorem for uniform polynomial stability in mean and respectively uniform polynomial instability in mean of stochastic skew-evolution semiflows in Banach spaces were obtained by Hai. It is worth to mention that in [13] the authors investigated a general concept of uniform asymptotic stability in mean, the so-called uniform h-stability in mean. This concept includes the concepts of uniform exponential stability in mean and uniform polynomial stability in mean as particular cases.

It is well known that the instability problem has become one of the research hotspots in the field of the asymptotic behavior of dynamical systems. Motivated by the recent work of Fülöp, Megan and Borlea [13], in this paper, we introduce the concept of uniform h-instability in mean for stochastic skew-evolution semiflows which is an extension of concepts of uniform exponential stability in mean and uniform polynomial instability in mean. Our main objective is to give some characterizations for uniform exponential instability in mean, uniform polynomial instability in mean and uniform h-instability in mean of stochastic skew-evolution semiflows in Banach spaces.

This paper is organized as follows. In Section 2 we introduce some basic notions and the concept of unform h-instability in mean. Then, connections between uniform h-instability in mean, uniform exponential instability in mean and uniform polynomial instability in mean are established. In the last section we present the main results of our paper. At first we give some expansion properties for the concepts of uniform exponential instability in mean, uniform polynomial instability in mean and uniform h-instability in mean. Then some logarithmic criteria and respectively majorization criteria are presented for the concepts mentioned above, by extending the techniques used in the stable case (see [13]) to the unstable case.

2. Notions and preliminaries

In this section, we give some notations, definitions and preliminary facts which will be used in the sequel. Let $(\Omega, \mathcal{B}, \mathbf{P})$ be a probability space, X a Banach space, $\mathcal{L}(X)$ the Banach algebra of all bounded linear operators from X into itself. The norm on X and $\mathcal{L}(X)$ will denoted by $\|\cdot\|$. We denote by $\mathbb{N}$ the set of natural numbers, by $\mathbb{N}_+$ the set of positive integers, by $\mathbb{R}_+ = [0, \infty)$ and by $\Delta = \left\{(t, s) \in \mathbb{R}_+^2 : t \geq s\right\}$. Let us consider

$$\begin{aligned}
&\mathcal{L}^1(\Omega, X, \mathbf{P}) \\
&\quad = \left\{ f : \Omega \to X : f \text{ is Bochner measurable and } \int_\Omega \|f(\omega)\| \, d\mathbf{P}(\omega) < \infty \right\},
\end{aligned}$$

which is a Banach space endowed with the norm

$$\|f\|_1 = \int_\Omega \|f(\omega)\| \, d\mathbf{P}(\omega).$$

We denote $Y = \mathbb{R}_+ \times \mathcal{L}^1(\Omega, X, \mathbf{P})$ and $Z = \Delta \times \mathcal{L}^1(\Omega, X, \mathbf{P})$.

DEFINITION 2.1. *(see [20, 29]) A pair (φ, Φ) is called a stochastic skew-evolution semiflow on $\Omega \times X$, if the measurable random field $\varphi : \Delta \times \Omega \to \Omega$ (often called as a stochastic evolution semiflow) satisfies*

(i) $\varphi(t, t, \omega) = \omega$, $\forall (t, \omega) \in \mathbb{R}_+ \times \Omega$;

(ii) $\varphi(t, s, \omega) = \varphi(t, r, \varphi(r, s, \omega))$, $\forall t \geq r \geq s \geq 0$, $\forall \omega \in \Omega$;

and the map $\Phi : \Delta \times \Omega \to \mathcal{L}(X)$ (often called as a stochastic evolution cocycle) satisfies

(iii) $\Phi(t, t, \omega) = \mathrm{Id}$ (where Id is the identity operator on X), $\forall (t, \omega) \in \mathbb{R}_+ \times \Omega$;

(iv) $\Phi(t, s, \omega) = \Phi(t, r, \varphi(r, s, \omega))\Phi(r, s, \omega)$, $\forall t \geq r \geq s \geq 0$, $\forall \omega \in \Omega$;

(v) $(t, s, \omega) \mapsto \Phi(t, s, \omega)x$ is Bochner measurable for every $x \in X$.

REMARK 2.2. We note that stochastic cocycles introduced in [2, 3, 6, 10, 28] are particular cases of stochastic skew-evolution semiflows. Some illustrative examples of stochastic skew-evolution semiflows are given by Hai in [20].

DEFINITION 2.3. *(see [23]) A function $h : \mathbb{R}_+ \to [1, \infty)$ is said to be a growth rate if it is nondecreasing and bijective.*

In what follows, we suppose that $h : \mathbb{R}_+ \to [1, \infty)$ is a growth rate.

DEFINITION 2.4. *A stochastic skew-evolution semiflow (φ, Φ) has uniform h-decay in mean (u.h.d.m.) if there exist $K > 1$ and $\alpha > 0$ such that*

(2.1)
$$Kh(t)^\alpha \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq h(s)^\alpha \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t, s, x) \in Z.$$

An example of a stochastic skew-evolution semiflow with uniform h-decay in mean is given in the following example.

EXAMPLE 2.5. Let X be a a real separable Hilbert space. Ω is the space of all continuous applications $\omega : \mathbb{R}_+ \to X$ such that $\omega(0) = 0$ with the compact open topology. Let $\mathcal{B}_t$, for $t \geq 0$, be the σ-algebra generated by the set $\{\omega \to \omega(s) \in X : s \leq t\}$ and let $\mathcal{B}$ be the associated Borel σ-algebra to Ω. Thus, $(\Omega, \mathcal{B}, \mathcal{B}_t, \mathbf{P})$ is a filtered probability space for a Wiener measure $\mathbf{P}$ on Ω (see [28, Example 2.1]). Then

$$\varphi : \Delta \times \Omega \to \Omega, \quad \varphi(t, s, \omega)(\tau) = \frac{h(t)}{h(s)}\omega(\tau)$$

is a stochastic evolution semiflow and

$$\Phi : \Delta \times \Omega \to \mathcal{L}(X), \quad \Phi(t,s,\omega)x = \frac{h(s)}{h(t)}x$$

is a stochastic evolution cocycle associated to the stochastic evolution semiflow φ. For any $K > 1$ and $(t,s,x) \in Z$, we have

$$\begin{aligned} Kh(t)\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) &= Kh(s)\int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega) \\ &\geq h(s)\int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega), \end{aligned}$$

which means that the stochastic skew-evolution semiflow (φ, Φ) satisfies Definition 2.4 for $\alpha = 1$ and for all $K > 1$. Hence, (φ, Φ) has uniform h-decay in mean.

Remark 2.6. As particular cases of Definition 2.4, we have the following.

(i) If $h(t) = e^t$, then we say that a stochastic skew-evolution semiflow (φ, Φ) has uniform exponential decay in mean (u.e.d.m.).

(ii) If $h(t) = t + 1$, then we say that a stochastic skew-evolution semiflow (φ, Φ) has uniform polynomial decay in mean (u.p.d.m.).

Definition 2.7. *A stochastic skew-evolution semiflow (φ, Φ) is said to be uniformly h-unstable in mean (u.h.us.m.) if there are $N > 1$ and $v > 0$ such that*

(2.2)
$$Nh(s)^v \int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) \geq h(t)^v \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega),\ \forall (t,s,x) \in Z.$$

Example 2.8. Let $\varphi : \Delta \times \Omega \to \Omega$ be the stochastic evolution semiflow in Example 2.5. It is easy to check that the map $\Phi : \Delta \times \Omega \to \mathcal{L}(X)$ defined by $\Phi(t,s,\omega)x = \frac{h(t)}{h(s)}x$ is a stochastic evolution cocycle associated to the stochastic evolution semiflow φ. For any $N > 1$ and $(t,s,x) \in Z$, we have

$$\begin{aligned} Nh(s)\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) &= Nh(t)\int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega) \\ &\geq h(t)\int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega), \end{aligned}$$

which gives that (φ, Φ) is uniformly h-unstable in mean.

Remark 2.9. As particular cases of Definition 2.7, we give the following.

(i) If $h(t) = e^t$, then we say that a stochastic skew-evolution semiflow (φ, Φ) is uniformly exponentially unstable in mean (u.e.us.m.).

(ii) If $h(t) = t + 1$, then we say that a stochastic skew-evolution semiflow (φ, Φ) is uniformly polynomially unstable in mean (u.p.us.m.).

REMARK 2.10. One can easily see that if a stochastic skew-evolution semiflow (φ, Φ) is u.h.us.m., then it has u.h.d.m. The converse implication is not necessarily valid. In addition, the relationships between the concepts of instability in mean and the concepts of decay in mean are given by the following diagram:

$$\begin{array}{ccc} u.e.us.m. & \Rightarrow & u.p.us.m. \\ \Downarrow & & \Downarrow \\ u.e.d.m. & \Leftarrow & u.p.d.m. \end{array}$$

DEFINITION 2.11. *A stochastic skew-evolution semiflow (φ, Φ) is said to be uniformly h-unstable (in the classical sense) if there are $N > 1$ and $v > 0$ such that*

$$Nh(s)^v \left\| \Phi(t, s, \omega)x \right\| \geq h(t)^v \left\| x \right\|,$$

for all $(t, s) \in \Delta$ and all $(\omega, x) \in \Omega \times X$.

It is obvious that each uniformly h-unstable stochastic skew-evolution semiflow admits a uniform h-instability in mean with respect to any probability measure $\mathbf{P}$ on Ω, but the converse is not valid. To show this, we consider the following example.

EXAMPLE 2.12. Consider a partition $\Omega = \bigcup_{i=0}^{\ell} \Omega_i$ of Ω ($\Omega_0 \neq \emptyset$, ℓ may be finite or infinite) with $\mathbf{P}(\Omega_0) = 0$ and numbers $N > 1$ and $v_0 = 0$ and $v_i > 0$ for $i \in \mathbb{N}_+$ with $\inf_{i \in \mathbb{N}_+} v_i > 0$. We assume that

$$N \int_{\Omega_i} \left\| \Phi(t, s, \omega)x(\omega) \right\| d\mathbf{P}(\omega) \geq \left(\frac{h(t)}{h(s)} \right)^{v_i} \int_{\Omega_i} \left\| x(\omega) \right\| d\mathbf{P}(\omega)$$

for all $(t, s, x) \in Z$ and $i \in \mathbb{N} \cap [0, \ell]$. Based on this assumption we have that there exists $v = \inf_{i \in \mathbb{N}_+} v_i$ such that the relation (2.2) holds for all $(t, s, x) \in Z$. Therefore, (φ, Φ) is uniformly h-unstable in mean. Since $v_0 = 0$, $\Omega_0 \neq \emptyset$, and $\mathbf{P}(\Omega_0) = 0$, (φ, Φ) is not uniformly h-unstable.

The next proposition gives a connection between the concept of uniform h-instability in mean and the concept of uniform exponential instability.

PROPOSITION 2.13. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it is uniformly h-unstable in mean if and only if the stochastic skew-evolution semiflow (φ_h, Φ_h) (see [13, Theorem 1]) is uniformly exponentially unstable in mean, where*

$$\varphi_h : \Delta \times \Omega \to \Omega, \ \varphi_h(t, s, \omega) := \varphi(h^{-1}(e^t), h^{-1}(e^s), \omega)$$

and

$$\Phi_h : \Delta \times \Omega \to \mathcal{L}(X), \ \Phi_h(t, s, \omega) := \Phi(h^{-1}(e^t), h^{-1}(e^s), \omega).$$

Proof. Necessity. If (φ, Φ) is u.h.us.m., then by Definition 2.7, there are two constants $N > 1$ and $v > 0$ such that relation (2.2) holds. It follows from (2.2) that

$$\begin{aligned}
Ne^{vs} \int_\Omega \|\Phi_h(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) &= Ne^{vs} \int_\Omega \left\|\Phi(h^{-1}(e^t), h^{-1}(e^s), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\
&\geq e^{vs} \left(\frac{h(h^{-1}(e^t))}{h(h^{-1}(e^s))}\right)^v \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega) \\
&= e^{vt} \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega),
\end{aligned}$$

which means that (φ_h, Φ_h) is u.e.us.m.

Sufficiency. If (φ_h, Φ_h) is u.e.us.m., then it follows that there are $N > 1$ and $v > 0$ such that for all $(t, s, x) \in Z$ we have

$$\begin{aligned}
&Nh(s)^v \int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) \\
&= Nh(s)^v \int_\Omega \left\|\Phi(h^{-1}(h(t)), h^{-1}(h(s)), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\
&= Nh(s)^v \int_\Omega \left\|\Phi(h^{-1}(e^{\ln h(t)}), h^{-1}(e^{\ln h(s)}), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\
&= Nh(s)^v \int_\Omega \|\Phi_h(\ln h(t), \ln h(s), \omega)x(\omega)\|\, d\mathbf{P}(\omega) \\
&\geq h(s)^v e^{v(\ln h(t) - \ln h(s))} \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega) \\
&= h(t)^v \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega).
\end{aligned}$$

Hence (φ, Φ) is u.h.us.m. □

As a direct consequence of Proposition 2.13, we obtain the following result.

Corollary 2.14. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it is uniformly polynomially unstable in mean if and only if the stochastic skew-evolution semiflow (φ_1, Φ_1) is uniformly exponentially unstable in mean, where*

$$\varphi_1 : \Delta \times \Omega \to \Omega,\ \ \varphi_1(t,s,\omega) := \varphi(e^t - 1, e^s - 1, \omega)$$

and

$$\Phi_1 : \Delta \times \Omega \to \mathcal{L}(X),\ \ \Phi_1(t,s,\omega) := \Phi(e^t - 1, e^s - 1, \omega).$$

In a similar manner to Proposition 2.13, below we give a connection between the concept of uniform h-decay in mean and the concept of uniform exponential decay.

PROPOSITION 2.15. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it has uniform h-decay in mean if and only if the stochastic skew-evolution semiflow (φ_h, Φ_h) (see [13, Theorem 1]) has uniform exponential decay in mean, where*

$$\varphi_h : \Delta \times \Omega \to \Omega,\ \varphi_h(t,s,\omega) := \varphi(h^{-1}(e^t), h^{-1}(e^s), \omega)$$

and

$$\Phi_h : \Delta \times \Omega \to \mathcal{L}(X),\ \Phi_h(t,s,\omega) := \Phi(h^{-1}(e^t), h^{-1}(e^s), \omega).$$

PROOF. It is similar to the proof of Proposition 2.13. □

COROLLARY 2.16. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it has uniform polynomial decay in mean if and only if the stochastic skew-evolution semiflow (φ_1, Φ_1) has uniform exponential decay in mean, where (φ_1, Φ_1) is defined as in Corollary 2.14.*

PROOF. It follows immediately from Proposition 2.15 for $h(t) = t + 1$. □

We now discuss the relation between uniform h-instability in mean and uniform polynomial instability in mean.

PROPOSITION 2.17. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it is uniformly h-unstable in mean if and only if the stochastic skew-evolution semiflow (ψ_h, Ψ_h) (see [13, Theorem 2]) is uniformly polynomially unstable in mean, where*

$$\psi_h : \Delta \times \Omega \to \Omega,\ \psi_h(t,s,\omega) := \varphi(h^{-1}(t+1), h^{-1}(s+1), \omega)$$

and

$$\Psi_h : \Delta \times \Omega \to \mathcal{L}(X),\ \Psi_h(t,s,\omega) := \Phi(h^{-1}(t+1), h^{-1}(s+1), \omega).$$

PROOF. Necessity. If (φ, Φ) is u.h.us.m., then by Definition 2.7, there are two constants $N > 1$ and $v > 0$ such that the relation (2.2) holds. It follows from (2.2) that

$$\begin{aligned}
&N(s+1)^v \int_\Omega \|\Psi_h(t,s,\omega)x(\omega)\|\, d\mathbf{P}(\omega) \\
&= N(s+1)^v \int_\Omega \left\|\Phi(h^{-1}(t+1), h^{-1}(s+1), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\
&\geq (s+1)^v \left(\frac{h(h^{-1}(t+1))}{h(h^{-1}(s+1))}\right)^v \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega) \\
&= (t+1)^v \int_\Omega \|x(\omega)\|\, d\mathbf{P}(\omega),
\end{aligned}$$

which means that (ψ_h, Ψ_h) is u.p.us.m.

Sufficiency. If (ψ_h, Ψ_h) is u.p.us.m., then it follows that there are $N > 1$ and $v > 0$ such that for all $(t, s, x) \in Z$ we have

$$\begin{aligned}
&Nh(s)^v \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&= Nh(s)^v \int_\Omega \left\|\Phi(h^{-1}(h(t)), h^{-1}(h(s)), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\
&= Nh(s)^v \int_\Omega \|\Psi_h(h(t) - 1, h(s) - 1, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq h(s)^v \left(\frac{h(t) - 1 + 1}{h(s) - 1 + 1}\right)^v \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= h(t)^v \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega).
\end{aligned}$$

Hence (φ, Φ) is u.h.us.m. □

The next proposition establishes a connection between the concept of uniform h-decay in mean and the concept of uniform polynomial decay.

Proposition 2.18. *Let (φ, Φ) be a stochastic skew-evolution semiflow. Then it has uniform h-decay in mean if and only if the stochastic skew-evolution semiflow (ψ_h, Ψ_h) has uniform polynomial decay in mean, where (ψ_h, Ψ_h) is defined as in Proposition 2.17.*

Proof. It is similar to the proof of Proposition 2.17. □

3. The main results

3.1. *Some expansion properties.* In this subsection we present some key expansion properties for the concepts of uniform exponential instability in mean, uniform polynomial instability in mean and uniform h-instability in mean for stochastic skew-evolution semiflows in Banach spaces.

Theorem 3.1. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform exponential decay in mean. Then it is uniformly exponentially unstable in mean if and only if there exist $r > 1$ and $c > 1$ such that*

$$\int_\Omega \|\Phi(r + s, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq c \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (s, x) \in Y. \tag{3.1}$$

Proof. Necessity. It is a simple verification for $r = \frac{\ln N}{v} + 1$ and $c = \frac{e^{vr}}{N}$, where $N > 1$ and $v > 0$ are given by Remark 2.9(i).

Sufficiency. Since (φ, Φ) has u.e.d.m., it follows that there are $K > 1$ and $\alpha > 0$ such that

$$Ke^{\alpha t} \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq e^{\alpha s} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t, s, x) \in Z. \tag{3.2}$$

Obviously, for every $(t, s) \in \Delta$ there are $n \in \mathbb{N}$ and $l \in [0, r)$ such that $t - s = rn + l$.

Let $(t, s, x) \in Z$. According to (3.2) and (3.1) we have that

$$\begin{aligned}
&\int_\Omega \|\Phi(t,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \int_\Omega \|\Phi(s+nr+l, s+nr, \varphi(s+nr,s,\omega))\Phi(s+nr,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{K} e^{-\alpha l} \int_\Omega \|\Phi(s+nr,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{K} e^{-\alpha r} \int_\Omega \|\Phi(s+nr,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{K} e^{-\alpha r} c \int_\Omega \|\Phi(s+(n-1)r,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \cdots \geq \frac{1}{K} e^{-\alpha r} c^n \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{1}{K} e^{-\alpha r} e^{vrn} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{1}{K} e^{-\alpha r - vl} e^{v(t-s)} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{K} e^{-(\alpha+v)r} e^{v(t-s)} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{N} e^{v(t-s)} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega),
\end{aligned}$$

where $v = \frac{\ln c}{r}$ and $N = Ke^{(\alpha+v)r} + 1$. Thus, we can conclude that (φ, Φ) is u.e.us.m. ∎

Theorem 3.2. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform polynomial decay in mean. Then it is uniformly polynomially unstable in mean if and only if there exist $r > e - 1$ and $c > 1$ such that*

$$\int_\Omega \|\Phi(rs+r+s,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq c \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (s,x) \in Y. \tag{3.3}$$

Proof. Necessity. If (φ, Φ) is u.p.us.m., then from Corollary 2.14, we have that (φ_1, Φ_1) is u.e.us.m. In light of Theorem 3.1 we have that there exist $u > 1$ and $c > 1$ such that

(3.4)
$$\begin{aligned}
\int_\Omega \|\Phi_1(u+v,v,\omega)x(\omega)\| \, d\mathbf{P}(\omega) &= \int_\Omega \left\|\Phi(e^{u+v}-1, e^v - 1, \omega)x(\omega)\right\| \, d\mathbf{P}(\omega) \\
&\geq c \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)
\end{aligned}$$

for all $(v,x) \in Y$.

Let $r = e^u - 1$ and $v = \ln(s+1)(s \geq 0)$, which implies $r > e - 1$, $s = e^v - 1$ and $e^{u+v} - 1 = rs + r + s$. Thus, from (3.4) we deduce that

$$\int_\Omega \|\Phi(rs+r+s,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) = \int_\Omega \left\|\Phi(e^{u+v}-1, e^v-1,\omega)x(\omega)\right\| d\mathbf{P}(\omega) \geq c\int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)$$

for all $(s,x) \in Y$.

Sufficiency. Since (φ, Φ) has u.p.d.m, therefore (φ_1, Φ_1) has u.e.d.m. in view of Corollary 2.16.

Let $u = \ln(1+r)$ and $v \geq 0$, which implies $u > 1$, $r = e^u - 1$, $s = e^v - 1 \geq 0$ and $e^{u+v} - 1 = rs + r + s$. By (3.3) we have

$$\begin{aligned}\int_\Omega \|\Phi_1(u+v,v,\omega)x(\omega)\| \, d\mathbf{P}(\omega) &= \int_\Omega \left\|\Phi(e^{u+v}-1, e^v-1,\omega)x(\omega)\right\| d\mathbf{P}(\omega) \\ &= \int_\Omega \|\Phi(rs+r+s,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\ &\geq c\int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega).\end{aligned}$$

From Theorem 3.1 we obtain that (φ_1, Φ_1) is u.e.us.m. which implies from the Corollary 2.14 that (φ, Φ) is u.p.us.m. ∎

Theorem 3.3. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform h-decay in mean. Then it is uniformly h-unstable in mean if and only if there exist $r > e$ and $c > 1$ such that*

$$\int_\Omega \left\|\Phi(h^{-1}(re^s), h^{-1}(e^s),\omega)x(\omega)\right\| d\mathbf{P}(\omega) \geq c\int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \;\; \forall (s,x) \in Y. \tag{3.5}$$

Proof. It follows from Proposition 2.13, Proposition 2.15 and Theorem 3.1. ∎

3.2. *Logarithmic criteria.* In this subsection, we give some logarithmic type characterizations for the concepts considered in our study.

Theorem 3.4. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform polynomial decay in mean. Then it is uniformly polynomially unstable in mean if and only if there exists a constant $L > 1$ such that*

$$L\int_\Omega \|\Phi(t,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq \ln\frac{t+1}{s+1}\int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \;\; \forall (t,s,x) \in Z. \tag{3.6}$$

PROOF. Necessity. We suppose that (φ, Φ) is u.p.us.m. Then, via Remark 2.9(ii) there are $N > 1$ and $v > 0$ such that

(3.7)

$$\begin{aligned}
\ln\frac{t+1}{s+1}\int_\Omega \|x(\omega)\|\,d\mathbf{P}(\omega) &\leq N\left(\frac{t+1}{s+1}\right)^{-v}\ln\frac{t+1}{s+1}\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega)\\
&= \frac{N}{v}\frac{\ln\left(\frac{t+1}{s+1}\right)^v}{\left(\frac{t+1}{s+1}\right)^v}\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega)\\
&\leq \frac{N}{ve}\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega)\\
&\leq \left(1+\frac{N}{ve}\right)\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega),
\end{aligned}$$

where we use the fact that

$$\frac{\ln\xi}{\xi} \leq \frac{1}{e},\ \forall \xi \geq 1.$$

By (3.7) we can choose $L = 1 + \frac{N}{ve}$, and thus relation (3.6) holds for all $(t,s,x) \in Z$.

Sufficiency. Let c is an arbitrary constant belongs to $(1,\infty)$ and $r = e^{cL} - 1$, which implies $r > e - 1$. From (3.6) we get that

$$\begin{aligned}
\int_\Omega \|\Phi(rs+r+s,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega) &\geq L^{-1}\ln\frac{rs+r+s+1}{s+1}\int_\Omega \|x(\omega)\|\,d\mathbf{P}(\omega)\\
&= \frac{\ln(r+1)}{L}\int_\Omega \|x(\omega)\|\,d\mathbf{P}(\omega)\\
&= c\int_\Omega \|x(\omega)\|\,d\mathbf{P}(\omega)
\end{aligned}$$

for all $(s,x) \in Y$. Now applying Theorem 3.2, we obtain that (φ, Φ) is u.p.us.m. ◻

REMARK 3.5. Theorem 3.4 can be considered as a variant for uniform polynomial instability in mean of a result due to Boruga [6, Theorem 3.1] for polynomial stability in average of cocycles. We notice that the proof idea of Theorem 3.4 is different from Boruga's proof.

THEOREM 3.6. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform h-decay in mean. Then it is uniformly h-unstable in mean if and only if there exists a constant $L > 1$ such that*

$$(3.8)\quad L\int_\Omega \|\Phi(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega) \geq \ln\frac{h(t)}{h(s)}\int_\Omega \|x(\omega)\|\,d\mathbf{P}(\omega),\ \forall (t,s,x) \in Z.$$

Proof. Necessity. If (φ, Φ) is u.h.us.m., then from Proposition 2.17 we have that (ψ_h, Ψ_h) is u.p.us.m. Based on Theorem 3.4, it follows that there exists a constant $L > 1$ such that

$$L \int_\Omega \left\|\Phi(h^{-1}(u+1), h^{-1}(v+1), \omega)x(\omega)\right\| d\mathbf{P}(\omega) = L \int_\Omega \|\Psi_h(u, v, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq \ln\frac{u+1}{v+1} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)$$

for all $(u, v, x) \in Z$.

Let $(t, s) \in \Delta$. Then for $u = h(t) - 1$ and $v = h(s) - 1$ we have that

$$L \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) = L \int_\Omega \left\|\Phi(h^{-1}(u+1), h^{-1}(v+1), \omega)x(\omega)\right\| d\mathbf{P}(\omega) \geq \ln\frac{h(t)}{h(s)} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)$$

for all $(t, s, x) \in Z$.

Sufficiency. Let c is an arbitrary constant belongs to $(1, \infty)$ and $r = e^{cL}$, which implies $r > e$. From (3.8) it follows that

$$\begin{aligned}\int_\Omega \left\|\Phi(h^{-1}(re^s), h^{-1}(e^s), \omega)x(\omega)\right\| d\mathbf{P}(\omega) &\geq L^{-1}\ln\frac{h(h^{-1}(re^s))}{h(h^{-1}(e^s))} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\ &= \frac{\ln r}{L} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\ &= c \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)\end{aligned}$$

for all $(s, x) \in Y$. By Theorem 3.3, we conclude that (φ, Φ) is u.h.us.m. □

Corollary 3.7. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform exponential decay in mean. Then it is uniformly exponentially unstable in mean if and only if there exists a constant $L > 1$ such that*

$$L \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq (t - s) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t, s, x) \in Z. \tag{3.9}$$

Proof. It follows immediately from Theorem 3.6 for $h(t) = e^t$. □

3.3. *Majorization criteria.* In this subsection, we obtain some majorization criteria that characterize the uniform exponential instability in mean, the uniform polynomial instability in mean and the uniform h-instability in mean for stochastic skew-evolution semiflows.

Theorem 3.8. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform exponential decay in mean. Then it is uniformly exponentially unstable in mean if and only if there are a constant $L > 1$ and a nondecreasing* *function* $\rho : \mathbb{R}_+ \to \mathbb{R}_+$ *with* $\lim\limits_{t\to\infty} \rho(t) = \infty$ *such that*

$$L \int_\Omega \|\Phi(t,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq \rho(t-s) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t,s,x) \in Z. \tag{3.10}$$

Proof. Necessity follows easily by Corollary 3.7, if we consider $\rho(t) = t$.

Sufficiency. Since (φ, Φ) is u.e.d.m., it follows that there are $K > 1$ and $\alpha > 0$ such that relation (3.2) holds. From $\lim\limits_{t\to\infty} \rho(t) = \infty$ it results that there exists a constant $\delta > 0$ such that $\frac{\rho(\delta)}{L} > 1$. Moreover, for all $(t,s) \in \Delta$, there are $n \in \mathbb{N}$ and $l \in [0,\delta)$ such that $t - s = n\delta + l$. Using (3.2) and (3.10) we have that

$$\begin{aligned}
&\int_\Omega \|\Phi(t,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \int_\Omega \|\Phi(s+n\delta+l, s+n\delta, \varphi(s+n\delta,s,\omega))\Phi(s+n\delta,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{e^{-\alpha l}}{K} \int_\Omega \|\Phi(s+n\delta,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{e^{-\alpha \delta}}{K} \int_\Omega \|\Phi(s+n\delta,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{e^{-\alpha \delta}}{K} \int_\Omega \|\Phi(s+n\delta, s+(n-1)\delta, \varphi(s+(n-1)\delta,s,\omega))\Phi(s+(n-1)\delta,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{e^{-\alpha \delta}}{K} \frac{\rho(\delta)}{L} \int_\Omega \|\Phi(s+(n-1)\delta,s,\omega)x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \cdots \geq \frac{e^{-\alpha \delta}}{K} \left(\frac{\rho(\delta)}{L}\right)^n \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{e^{-\alpha \delta}}{K} e^{n \ln \frac{\rho(\delta)}{L}} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{e^{-\alpha \delta}}{K} e^{\frac{t-s-l}{\delta} \ln \frac{\rho(\delta)}{L}} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{e^{-\alpha \delta}}{K} e^{\frac{t-s}{\delta} \ln \frac{\rho(\delta)}{L}} e^{-\frac{\delta}{\delta} \ln \frac{\rho(\delta)}{L}} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&= \frac{Le^{-\alpha \delta}}{K\rho(\delta)} e^{\frac{t-s}{\delta} \ln \frac{\rho(\delta)}{L}} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\
&\geq \frac{1}{N} e^{v(t-s)} \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega)
\end{aligned}$$

for all $(t,s,x) \in Z$, where $N = \frac{K\rho(\delta)e^{\alpha\delta}}{L} + 1$ and $v = \frac{1}{\delta} \ln \frac{\rho(\delta)}{L}$. Hence, (φ, Φ) is u.e.us.m. $\square$

REMARK 3.9. Theorem 3.8 is a version of a result due to Stoica and Megan [27, Proposition 1] for uniform exponential instability in mean of stochastic skew-evolution semiflows.

THEOREM 3.10. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform polynomial decay in mean. Then it is uniformly polynomially unstable in mean if and only if there are a constant $L > 1$ and a nondecreasing function $\eta : [1, \infty) \to \mathbb{R}_+$ with $\lim_{t\to\infty} \eta(t) = \infty$ such that*

$$L \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq \eta\left(\frac{t+1}{s+1}\right) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t, s, x) \in Z. \tag{3.11}$$

PROOF. Necessity. It follows immediately from Theorem 3.4 for $\eta(t) = \ln t$.

Sufficiency. Let (φ_1, Φ_1) be defined as in Corollary 2.14. Since (φ, Φ) is u.p.d.m., from Corollary 2.16 we have that (φ_1, Φ_1) is u.e.d.m. Let $(t, s, x) \in Z$. Using (3.11) we deduce that

$$\begin{aligned} L \int_\Omega \|\Phi_1(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) &= L \int_\Omega \left\|\Phi(e^t - 1, e^s - 1, \omega)x(\omega)\right\| d\mathbf{P}(\omega) \\ &\geq \eta\left(\frac{e^t}{e^s}\right) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega) \\ &= \rho(t - s) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \end{aligned}$$

where $\rho(t) = \eta(e^t)$.

From Theorem 3.8 we have that (φ_1, Φ_1) is u.e.us.m. which implies from the Corollary 2.14 that (φ, Φ) is u.p.us.m., the proof completes. ∎

REMARK 3.11. Theorem 3.10 is a generalization to the case of uniform polynomial instability in mean of a result proved by Boruga in [7, Theorem 3.6] for the deterministic case of uniform polynomial instability of evolution operators.

THEOREM 3.12. *Let (φ, Φ) be a stochastic skew-evolution semiflow with uniform h-decay in mean. Then it is uniformly h-unstable in mean if and only if there are a constant $L > 1$ and a nondecreasing function $\eta : [1, \infty) \to \mathbb{R}_+$ with $\lim_{t\to\infty} \eta(t) = \infty$ such that*

$$L \int_\Omega \|\Phi(t, s, \omega)x(\omega)\| \, d\mathbf{P}(\omega) \geq \eta\left(\frac{h(t)}{h(s)}\right) \int_\Omega \|x(\omega)\| \, d\mathbf{P}(\omega), \ \forall (t, s, x) \in Z. \tag{3.12}$$

PROOF. Necessity. It follows immediately from Theorem 3.6 for $\eta(t) = \ln t$.

Sufficiency. Let $(t, s, x) \in Z$ and (ψ_h, Ψ_h) be defined as in Proposition 2.17. Since (φ, Φ) is u.h.d.m., from Proposition 2.18 we have that (ψ_h, Ψ_h) is u.p.d.m. By (3.12), we have

$$\begin{aligned}
L\int_{\Omega}\|\Psi_h(t,s,\omega)x(\omega)\|\,d\mathbf{P}(\omega) &= L\int_{\Omega}\left\|\Phi(h^{-1}(t+1),h^{-1}(s+1),\omega)x(\omega)\right\|\,d\mathbf{P}(\omega)\\
&\geq \eta\left(\frac{h(h^{-1}(t+1))}{h(h^{-1}(s+1))}\right)\int_{\Omega}\|x(\omega)\|\,d\mathbf{P}(\omega)\\
&= \eta\left(\frac{t+1}{s+1}\right)\int_{\Omega}\|x(\omega)\|\,d\mathbf{P}(\omega).
\end{aligned}$$

From Theorem 3.10 we have that (ψ_h,Ψ_h) is u.p.us.m. which implies from the Proposition 2.17 that (φ,Φ) is u.h.us.m. This completes the proof. □

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References

[1] L. Arnold, Random Dynamical Systems. Springer-Verlag, Berlin, Germany, 1998.

[2] L. Barreira, D. Dragičević and C. Valls, *Admissibility for exponential dichotomies in average,* Stoch. Dyn. **15** (2015), 1550014, 16pp.

[3] L. Barreira, D. Dragičević and C. Valls, *Exponential dichotomies in average for flows and admissibility,* Publ. Math. Debrecen **89** (2016), 415–439.

[4] L. Barreira and C. Valls, *Polynomial growth rates,* Nonlinear Anal. **71(11)** (2009), 5208–5219.

[5] A. J. G. Bento and C. Silva, *Stable manifolds for nonuniform polynomial dichotomies,* J. Funct. Anal. **257(1)** (2009), 122–148.

[6] R. Boruga, *Polynomial stability in average for cocycles of linear operators,* Theory Appl. Math. Comput. Sci. **9** (2019), 8–13.

[7] R. Boruga, *Majorization criteria for polynomial stability and instability of evolution operators,* Sci. Bull. Politeh. Univ. Timiş. **64** (2019), 55–62.

[8] H. Damak, *On uniform h-stability of non-autonomous evolution equations in Banach spaces,* Bull. Malays. Math. Sci. Soc. **44** (2021), 4367–4381.

[9] R. Datko, *Uniform asymptotic stability of evolutionary processes in a Banach space,* SIAM J. Math. Anal. **3(3)** (1972), 428–445.

[10] D. Dragičević, *A version of a theorem of R.Datko for stability in average,* Systems Control Lett. **96** (2016), 1–6.

[11] D. Dragičević, *Admissibility and polynomial dichotomies for evolution families,* Commun. Pure Appl. Anal. **19** (2020), 1321–1336.

[12] D. Dragičević and C. Preda, *Lyapunov type theorems for exponential stability of linear skew-product three-parameter semiflows with discrete time,* Axioms **9** (2020), 47, 12pp.

[13] T. M. S. Fülöp, M. Megan and D. I. Borlea, *On uniform stability with growth rates of stochastic skew-evolution semiflows in Banach spaces,* Axioms **10** (2021), 182, 11pp.

[14] P. V. Hai, *Continuous and discrete characterizations for the uniform exponential stability of linear skew-evolution semiflows,* Nonlinear. Anal. **72** (2010), 4390–4396.

[15] P. V. Hai, *Discrete and continuous versions of Barbashin-type theorem of linear skew-evolution semiflows,* Appl. Anal. **90** (2011), 1897–1907.

[16] P. V. Hai, *On two theorems regarding exponential stability,* Appl. Anal. Discrete Math. **5** (2011), 240–258.

[17] P. V. Hai, *On the polynomial stability of evolution families,* Appl. Anal. **95** (2016), 1239–1255.

[18] P. V. Hai, *Polynomial stability of evolution cocycles and Banach function spaces,* Bull. Belg. Math. Soc. Simon Stevin **26** (2019), 299–314.

[19] P. V. Hai, *Polynomial stability and polynomial instability for skew-evolution semiflows,* Results Math. **74** (2019), 175, 19pp.

[20] P. V. Hai, *Polynomial behavior in mean of stochastic skew-evolution semiflows,* arXiv:1902.04214 (2019).

[21] M. Megan and C. Stoica, *Discrete asymptotic behaviors for skew-evolution semiflows on Banach spaces,* Carpathian J. Math. **24** (2008), 348–355.

[22] M. Megan, C. Stoica and L. Buliga, *On asymptotic behaviors for linear skew-evolution semiflows in Banach spaces,* Carpathian J. Math. **23** (2007), 117–125.

[23] M. Pinto, *Perturbation of asymptotically stable differential systems,* Analysis **4** (1984), 161–175.

[24] C. Preda and A.-P. Popiţiu, *A discrete-time approach in the qualitative theory of skew-product three-parameter semiflows,* Bull. Belg. Math. Soc. Simon Stevin **24** (2017), 367–379.

[25] C. Preda and P. Preda, *Some results on the qualitative theory of semiflows,* Bull. Belg. Math. Soc. Simon Stevin **18** (2011), 173–186.

[26] C. Preda, P. Preda and A.-P. Petre, *On the uniform exponential stability of linear skew-product three-parameter semiflows,* Bull. Math. Soc. Sci. Math. Roumanie **54** (2011), 269–279.

[27] C. Stoica and M. Megan, *On uniform exponential stability for skew-evolution semiflows on Banach spaces,* Nonlinear Anal. **72** (2010), 1305–1313.

[28] D. Stoica, *Uniform exponential dichotomy of stochastic cocycles,* Stochastic Process. Appl. **120** (2010), 1920–1928.

[29] D. Stoica and M. Megan, *On nonuniform dichotomy for stochastic skew-evolution semiflows in Hilbert spaces,* Czechoslovak Math. J. **62** (2012), 879–887.

[30] T. Yue, *Some Datko and Barbashin type characterizations for the uniform h-instability of evolution families,* Glas. Mat. Ser. III **55(77)** (2022), 265–280.

[31] T. Yue, *Barbashin type characterizations for the uniform polynomial stability and instability of evolution families,* Georgian Math. J. **29** (2022), 953–966.

T. Yue
School of Mathematics, Physics and Optoelectronic Engineering
Hubei University of Automotive Technology
442002 Shiyan, China
E-mail: `yuet@huat.edu.cn`