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DIRECTION OF ELECTORAL VOLATILITY AND EUROPEAN PARLIAMENT ELECTIONS

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ABSTRACT. One of the features of electoral systems is electoral volatility, reflecting changes in voter preferences between two successive elections.

The well-known Pedersen volatility index takes into account a relative change of votes between two consecutive elections. A similar approach can be applied to the distribution of seats in a parliament, capturing the relative change in seats (i.e., mandates) after votes are transformed into seats. The indicator of relative seat volatility is examined using empirical data from the 2024 and 2019 European Parliament elections in each EU member state.

The direction of electoral volatility can be observed by means of a partition of the set of parties into two subsets according to their political positions: a subset of ‘the right-wing parties’ and a subset of ‘the left-wing parties’. (Alternatively, one may consider a partition of the set of parties into three subsets: ‘right-wing parties’, ‘centre parties’ and ‘left-wing parties’.) We propose the indicator of relative vote volatility direction, and the indicator of relative seat volatility direction. These indicators reveal whether the direction of relative vote or seat volatility between two consecutive elections is to ‘the right-wing parties’ or to ‘the left-wing parties’. The indicator of relative seat volatility direction is observed in the cases of elections in 2024 versus 2019 for European Parliament in each EU member state.

1. INTRODUCTION

There are various characteristics of electoral systems: for instance, electoral participation, number of parties (see [8], [13]), electoral volatility (see

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[11], [20]), proportionality (see [6], [8], [14]), party power and coalitions (see [15]) and government stability (see [12]) can be observed.

One of the key features of electoral systems is the change in electoral choices over time. Electoral volatility refers to the differences in outcomes between successive elections at time t and $t - 1$. It indicates whether there has been a substantial shift in party support between two consecutive elections and can reveal either a lack of party-system stabilization or relative stability in party positions (see [5], [7]). Low electoral volatility reflects stable interparty competition, whereas high electoral volatility suggests a greater likelihood that voters will alter their established electoral preferences (see [1]).

The question of the electoral volatility in an electoral system can be considered by means of vote fluctuation - change of votes in two consecutive elections at time t and $t - 1$ (see [8]). In addition, the electoral volatility can be viewed at the phase of posttransformation of votes into seats by means of seat fluctuation - change of seats (i.e., mandates) in a parliament in two consecutive elections (i.e., two consecutive inaugural sessions) at time t and $t - 1$.

Electoral volatility can depend on various criteria, such as electoral system, turnout levels, the number of parties competing in an election, and others (see [1], [2], [11], [19]). Electoral volatility can also be examined at the supranational level (for example, in European Parliament elections) or at the subnational level (see [7], [10]).

With respect to the electoral volatility in two consecutive elections at time t and $t - 1$, the votes each party obtains are considered. They refer to the pre-transformation phase of the electoral process, i.e., before the transformation of votes into seats. The following notation is used:

- $n \geq 2$ - the number of parties (or lists) in the elections at time t and time $t - 1$
- $v_i \geq 0$ - the number of votes that party (or list) i has got in the election at time t , $i \in I = \{1, \dots, n\}$, where I is the set of parties (or lists)
- $\omega_i = \frac{v_i}{\sum_{j=1}^n v_j}$ - the vote share of the party (or the list) i in the election at time t
- $v'_i \geq 0$ - the number of votes of the party (or the list) i in the election at time $t - 1$
- $\omega'_i = \frac{v'_i}{\sum_{j=1}^n v'_j}$ - vote share of the party (or the list) i in the election at time $t - 1$
- the condition: $v_i + v'_i > 0$, $i \in I = \{1, \dots, n\}$.

Some known indicators of electoral volatility are:

1) The indicator of electoral vote fluctuation (EF):

$$(1.1) \quad EF = \frac{1}{2} \sum_{i=1}^n |v_i - v'_i|.$$

The indicator EF is a simple measure for electoral volatility and represents an average change of votes between two consecutive elections (see [8]).

2) The indicator of relative vote volatility (irv):

$$(1.2) \quad irv = \frac{1}{2} \sum_{i=1}^n \left| \frac{v_i}{\sum_{j=1}^n v_j} - \frac{v'_i}{\sum_{j=1}^n v'_j} \right| = \frac{1}{2} \sum_{i=1}^n |\omega_i - \omega'_i|.$$

This indicator can be expressed by percentages (%) and it is called *the Pedersen volatility index* (see [17]). It is regarded as the most important measure of electoral volatility.

REMARK 1.1. One of the questions raised by the Pedersen volatility index concerns how to measure electoral systems in which parties split, merge, disappear, or are newly formed between two elections. Therefore, in many practical situations it is a challenging task to calculate Pedersen volatility index exactly. Although volatility formula (1.2) is not mathematically complex, an approximation problem often arises when volatility is computed from electoral databases (see [16]). So, various modifications of Pedersen volatility index are adopted, for example not to take into account the so-called tiny parties (or lists), and other modifications (see [18], [20]).

The passage of time is a variable that should be incorporated into analyses of electoral volatility. One can therefore examine several elections over a long period and calculate the corresponding average value of the Pedersen volatility index, or apply any other measure of volatility (see [7]). For example, in [2] patterns of electoral persistence and change are analysed in Western Europe between 1885 and 1985.

To measure the degree of vote transformation while taking into account the direction of electoral volatility, the following indicator of vote change is proposed in [8]:

$$(1.3) \quad VF = \frac{1}{d} \sum_{i=1}^n \sum_{j>i}^n |v_{ij} - v_{ji}| \cdot d_{ij},$$

where:

v_{ij} represents the number of voters who switched their vote from the former party i at time $t-1$ to the latter party j at time t ;
 $d_{ij} = |i - j|$ represents the ‘distance’ between the pair of parties (i, j) , $(i, j \in \{1, 2, \dots, n-1, n\})$, provided that the parties are ordered on ‘political axes’ from left to right according to their political positions: $1 - 2 - \dots - i - \dots - j - \dots - n$;

$\bar{d} = \max_i \{\sum_{j>i} d_{ij}\}$ is the maximum of the corresponding sum;
 $\frac{d_{ij}}{\bar{d}}$ is the weight, i.e., the ‘relative distance’ between the parties i and j .

It should be noted that in real electoral settings it is very difficult to determine the exact values of the quantities v_{ij} in expression (1.3).

With respect to the post-election phase when seats in a parliament are assigned, we consider parliamentary volatility between two successive elections based on the indicator of relative seat volatility *irsv*. This indicator *irsv* (see formula (2.5)) is analogous to the formula of Pedersen volatility index (1.2), whereby the values of seats are used in *irsv* instead of votes in (1.2).

The direction of electoral volatility can be examined by partitioning the set of parties (or lists) into two subsets according to their political positions: a subset of right-wing parties and a subset of left-wing parties. The division of parties into ‘left-wing’ and ‘right-wing’ is based on their ideological, economic and social views (see [3], [4]). We adopt the indicator of relative vote volatility direction and the indicator of relative seat volatility direction. These indicators show whether the direction of relative vote (or seat) volatility between two consecutive elections shifts toward ‘the right-wing’ or ‘the left-wing’ parties.

Section 2 adopts the indicator of relative seat volatility and examines it in the cases of the 2024 versus 2019 European Parliament (EP) elections in each member state of the European Union (EU). With respect to the direction of electoral volatility, Section 3 proposes the indicators of vote volatility direction and of seat volatility direction, and the latter is applied to the same set of the EP elections in 2024 and 2019. The final section provides concluding remarks.

2. THE INDICATOR OF RELATIVE SEAT VOLATILITY

With respect to electoral volatility in two consecutive elections at time t and $t - 1$, one can examine the distribution of seats obtained by each party (or list) in a parliament. This refers to the post-transformation phase of the electoral process, that is, the stage after votes have been converted into seats. The following notation is used:

- $n \geq 2$ - the number of parties (or lists) that have won seats in elections at time t or time $t - 1$
- $s_i \geq 0$ - the number of seats obtained by the party (or list) i in the parliament in the election at time t ($i = 1, \dots, n$)
- $\sigma_i = \frac{s_i}{\sum_{j=1}^n s_j}$ - the seat share of the party (or list) i in the parliament in the election at time t
- $s'_i \geq 0$ - the number of seats of the party (or list) i in the parliament in the election at time $t - 1$ ($i = 1, \dots, n$)

- $\sigma'_i = \frac{s'_i}{\sum_{j=1}^n s'_j}$ - the seat share of the party (or list) i in the parliament in the election at time $t - 1$
- the condition: $s_i + s'_i > 0$, $i = 1, \dots, n$.

Analogously to the definition of the indicator of relative vote volatility (1.2), it can be defined the corresponding set of indicators that measure electoral volatility in terms of parliamentary seats:

- 1) The indicator of electoral seat fluctuation (*EFs*):

$$(2.4) \quad EFs = \frac{1}{2} \sum_{i=1}^n |s_i - s'_i|$$

(the sum of the number of seats that are changed for each party in a parliament by the observed two consecutive elections)

- 2) The indicator of relative seat volatility (*irsv*):

$$(2.5) \quad irsv = \frac{1}{2} \sum_{i=1}^n \left| \frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j} \right| = \frac{1}{2} \sum_{i=1}^n |\sigma_i - \sigma'_i|.$$

This indicator can also be expressed by percentages (%) and takes values in the interval [0 %, 100 %].

REMARK 2.1. Analogously to Pedersen volatility index and Remark 1.1, one of the questions raised by the indicator of relative seat volatility is how to measure parliament volatility in which parties split, merge, disappear, or are newly formed between two successive elections (i.e., between two successive inaugural sessions). Therefore, in many practical situations it is a demanding task to calculate the indicator of relative seat volatility exactly, although the parliamentary volatility formula (2.5) is not mathematically complex.

To illustrate the indicator of relative seat volatility, we provide the following example.

EXAMPLE 2.2. We calculate the values of the indicator of the relative seat volatility (2.5) for all the European Union (EU) member states by comparing the results of the two consecutive elections for the European Parliament (EP) in 2024 and 2019 (see [21], [22]). (The United Kingdom was excluded from the analysis due to its withdrawal from the EU in 2020.) The corresponding values of this indicator *irsv* per EU member states are given in Table 1. For the purpose of comparison, the values of the indicator of relative vote volatility *irv* (i.e., Pedersen volatility index) are also given in Table 1. Measuring seat volatility in a parliament in which parties (or lists) split, merge, disappear, or are newly formed new between two successive elections (i.e., two successive

inaugural sessions) presents a practical problem. Therefore, in accordance with Remarks 1.1 and 2.1, the values of the indicators irv and $irsv$ calculated in this example should be taken as approximate rather than exact.

TABLE 1. Values of the indicator of relative seat volatility $irsv$ per member states in two successive elections in 2024 and 2019 for the EU Parliament

member state	Pedersen volatility index (irv , %)	indicator of relative seat volatility ($irsv$, %)
Austria	14.74 %	18.95 %
Belgium	21.82 %	22.08 %
Bulgaria	41.91 %	35.29 %
Croatia	43.67 %	33.33 %
Cyprus	26.96 %	16.67 %
Czechia	27.70 %	19.05 %
Denmark	26.48 %	25.71 %
Estonia	23.32 %	28.57 %
Finland	16.47 %	18.10 %
France	29.50 %	26.07 %
Germany	17.64 %	14.58 %
Greece	32.93 %	23.81 %
Hungary	35.67 %	38.10 %
Ireland	21.45 %	40.66 %
Italy	37.98 %	35.53 %
Latvia	34.24 %	33.33 %
Lithuania	28.32 %	27.27 %
Luxembourg	16.01 %	16.67 %
Malta	11.80 %	16.67 %
Netherlands	29.00 %	34.82 %
Poland	23.17 %	25.91 %
Portugal	21.68 %	19.05 %
Romania	35.60 %	30.30 %
Slovakia	43.02 %	43.33 %
Slovenia	30.28 %	30.56 %
Spain	28.17 %	22.01 %
Sweden	10.04 %	4.76 %

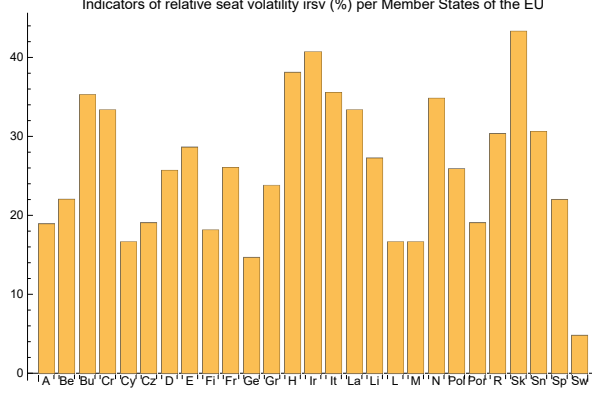
In Table 1 one can see that the values of indicators irv and $irsv$ for a particular member state are different, because various states use different proportional electoral methods, and for other reasons.

Extreme values (i.e., four minimal values and four maximal values) of the indicator of relative seat volatility from Table 1 are given in Table 2 .

The values of indicator of relative seat volatility for each EU member state from Table 1 are illustrated in Figure 1.

TABLE 2. Extreme values of the indicator of relative seat volatility from Table 1

min <i>irsv</i>	max <i>irsv</i>
4.76% Sweden	43.33% Slovakia
14.58% Germany	40.66% Ireland
16.67% Luxembourg	38.10% Hungary
16.67% Cyprus, Malta	35.53% Italy

FIGURE 1. Indicator of the relative seat volatility *irsv* for each EU member state

3. DIRECTION OF ELECTORAL VOLATILITY

Previously mentioned indicators of electoral volatility are not sensitive to the direction of electoral volatility.

To analyze the direction of electoral volatility (between two consecutive elections at time t and $t - 1$), let us assume that the set of parties (or lists) $I = \{1, 2, \dots, n\}$ is partitioned into two disjoint sets according to parties' political positions: the set of left-wing parties L and the set of right-wing parties R , where

$$(3.6) \quad I = L \cup R, \quad L \cap R = \emptyset.$$

There is, of course, the question of how 'left-wing parties' and 'right-wing parties' should be understood. The division of parties into 'left-wing' and 'right-wing' is based on their ideological, economic and social views. The sets L and R can be determined by political experts, although this is generally a challenging task (see [3], [4], [9]).

REMARK 3.1. This paper considers a restriction to mainly two blocks, 'left' and 'right', i.e., a simple left-right dichotomy. However, many party

systems contain parties, lists, or groups that do not naturally fit into a simple left-right dichotomy, as they combine ideas from the ‘left’ and ‘right’. Therefore, in such cases, one can consider a partition of the set of parties (or lists) $I = \{1, 2, \dots, n\}$, where a third subset C is added which consists of parties (or lists) with a centrist political position: $I = L \cup C \cup R$ (see Subsection 3.3).

3.1. Direction of relative vote volatility.

In addition, the following terms are defined.

DEFINITION 3.2. *Party i is said to be ‘relative vote increasing’ between two successive elections, if it holds:*

$$(3.7) \quad \frac{v_i}{\sum_{j=1}^n v_j} - \frac{v'_i}{\sum_{j=1}^n v'_j} > 0.$$

DEFINITION 3.3. *The sum R_{rv} of differences $\frac{v_i}{\sum_{j=1}^n v_j} - \frac{v'_i}{\sum_{j=1}^n v'_j}$ for the right-wing parties is defined as follows:*

$$(3.8) \quad R_{rv} = \sum_{i \in R} \left(\frac{v_i}{\sum_{j=1}^n v_j} - \frac{v'_i}{\sum_{j=1}^n v'_j} \right),$$

and it represents the sum of the relative vote volatility of right-wing parties between two successive elections. Analogously, the sum L_{rv} of the corresponding differences for the left-wing parties is defined as follows:

$$(3.9) \quad L_{rv} = \sum_{i \in L} \left(\frac{v_i}{\sum_{j=1}^n v_j} - \frac{v'_i}{\sum_{j=1}^n v'_j} \right),$$

and it represents the sum of the relative vote volatility of left-wing parties between two successive elections.

DEFINITION 3.4. *The quantity*

$$(3.10) \quad DRV = \frac{1}{2}(R_{rv} - L_{rv})$$

represents the difference between the relative vote volatility of right-wing parties and the relative vote volatility of left-wing parties. It can be called ‘the indicator of relative vote volatility direction’ between two successive elections.

PROPOSITION 3.5. *Under the above assumptions, the following relations hold: $L_{rv} + R_{rv} = 0$, and $DRV = R_{rv} = -L_{rv}$.*

PROOF. The first relation follows from (3.6), (3.8) and (3.9), and hence the second from (3.10). $\square$

Therefore, one can conclude the following.

If $DRV > 0$, then the relative vote volatility between two successive elections is increased to the right-wing parties.

If $DRV < 0$, then the relative vote volatility between two successive elections is increased to the left-wing parties.

3.2. Direction of relative seat volatility.

Analogously to the definition of the indicator of relative vote volatility direction (3.10), the indicator of relative seat volatility direction can be defined with respect to seats of parties (or lists) in the parliament between two successive elections (i.e., two successive inaugural sessions).

Let $n \geq 2$ be the number of parties (or lists) that have won seats in elections at time t or time $t - 1$. Let a partition of the set of parties (or lists) $I = \{1, 2, \dots, n\}$ in a parliament into two disjoint sets defined according to parties' political positions be given (provided that $s_i > 0$ or $s'_i > 0$ for all $i \in I$):

$$(3.11) \quad I = L \cup R, \quad L \cap R = \emptyset,$$

where L is the set of left-wing parties and R is the set of right-wing parties. On this basis, the following terms can be defined:

DEFINITION 3.6. *Party i is said to be 'relative seat increasing' between two successive elections (i.e., two successive inaugural sessions), if it holds:*

$$(3.12) \quad \frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j} > 0.$$

DEFINITION 3.7. *The sum R_{rs} of differences $\frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j}$ for the right-wing parties is defined as follows:*

$$(3.13) \quad R_{rs} = \sum_{i \in R} \left(\frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j} \right),$$

and it represents the sum of the relative seat volatility of right-wing parties between two successive elections. Analogously, the sum L_{rs} of the corresponding differences for the left-wing parties is defined as follows:

$$(3.14) \quad L_{rs} = \sum_{i \in L} \left(\frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j} \right),$$

and it represents the sum of the relative seat volatility of left-wing parties between two successive elections.

DEFINITION 3.8. *The quantity*

$$(3.15) \quad DRSV = \frac{1}{2}(R_{rs} - L_{rs})$$

represents the difference between the relative seat volatility of right-wing parties and the relative seat volatility of left-wing parties. It can be called 'the indicator of relative seat volatility direction' between two successive elections.

PROPOSITION 3.9. *Under the above assumptions, the following relations hold: $L_{rs} + R_{rs} = 0$, and $DRSV = R_{rs} = -L_{rs}$.*

PROOF. The first relation follows from (3.11), (3.13) and (3.14), and hence the second from (3.15). $\square$

Thus, the following conclusion applies:

If $DRSV > 0$, then relative seat volatility is increased to the right-wing parties.

If $DRSV < 0$, then relative seat volatility is increased to the left-wing parties.

If $DRSV = 0$, then there are no change of the direction of relative seat volatility to the ‘left’ or to the ‘right’.

REMARK 3.10. Analogously to the indicator of relative seat volatility (2.5) and Remark 2.1, there arises the question of how to measure the indicator of relative seat volatility direction (3.15) in a parliament in which parties split, merge, disappear, or are newly formed new between two successive elections. Therefore, in many practical situations, calculating the indicator of relative seat volatility direction ($DRSV$) exactly is a demanding task.

On the other hand, an additional practical problem in calculating the $DRSV$ indicator is how to classify the parties (or lists) into left-wing and right-wing parties. As a result, such constraints complicate the practical calculation of the $DRSV$ indicator.

To illustrate the indicator of relative seat volatility direction, we present the following examples.

EXAMPLE 3.11. We consider the elections in Croatia for the European Parliament (EP) in 2024 versus 2019.

In the 2024 election in Croatia for the EP, the left-wing parties obtained the following number of seats in the EP: $s_L = (SDP, M, IDS, NIL) = (4, 1, 0, 0)$, and the right-wing parties obtained the following number of seats: $s_R = (HDZ, DP, HS, NIr) = (6, 1, 0, 0)$.

In the 2019 election in Croatia for the EP, the left-wing parties obtained the following number of seats in the EP:

$s'_L = (SDP, M, IDS, NIL) = (4, 0, 1, 1)$, and the right-wing parties obtained the following number of seats: $s'_R = \{HDZ, DP, HS, NIr\} = \{4, 0, 1, 1\}$. Two seats were won by the party ‘PiP’, but they were the non-attached members of the group NI in the EP; for simplicity, it is assumed here that half of the non-attached members belong to the left group ‘NIL’ (i.e., 1 member), and half belong to the right group ‘NIr’ (i.e., 1 member).

Then one obtains that $R_{rs} = 0.0833$, $L_{rs} = -0.0833$, and from (3.15) it follows $DRSV = 8.33\% > 0$.

Therefore we can conclude that the direction of relative seat volatility shifts towards the right-wing parties. It can be interpreted that the right-wing parties R gained 8.33% more seats (i.e., $+1/12$) and the left parties L lost 8.33% seats (i.e., $-1/12$), in the election 2024 with respect to the election 2019 in Croatia for the EU Parliament. In addition, this relative seat fluctuation to the right-wing parties is significant, because the right-wing parties won the majority of seats in the election 2024 (i.e., $\sum_{i \in R} s_i = 7 > 5 = \sum_{i \in L} s_i$) in comparison with the election 2019 for the EP when the right-wing parties and the left-wing parties obtained equal number of seats ($\sum_{i \in R} s'_i = \sum_{i \in L} s'_i = 6$).

EXAMPLE 3.12. We consider two consecutive elections for the European Parliament in 2024 and 2019. The values of the indicator of the direction of relative seat volatility (3.15) are calculated in these two consecutive elections with respect to party groups in the EP. In 2019 the total number of members of the EP was 751, and since 2024 it has been 720 (the United Kingdom exited the EU in 2020).

In 2024, numbers of seats for the left-wing groups in European Parliament were

$s_L = (\text{Renew}, \text{Greens}/\text{EFA}, \text{S\&D}, \text{TheLeft}, \text{NI}) = (77, 53, 136, 46, 16)$, and numbers of seats for the right-wing groups in European Parliament were

$s_R = (\text{EPP}, \text{ECR}, \text{Pfe}, \text{ESN}, \text{NIr}) = (188, 78, 84, 25, 17)$.

In 2019, numbers of seats for the left-wing groups in European Parliament were

$s'_L = (\text{Renew}, \text{Greens}/\text{EFA}, \text{S\&D}, \text{TheLeft}, \text{NI}) = (108, 74, 154, 41, 28)$, and numbers of seats for the right-wing groups in European Parliament were

$s'_R = (\text{EPP}, \text{ECR}, \text{Pfe}, \text{ESN}, \text{NIr}) = (182, 62, 0, 73, 29)$.

For simplicity, it is assumed here that approximately half of the non-attached members belong to the left-wing group 'NI' (16 and 28 members, respectively), and half belong to the right-wing group 'NIr' (17 and 29 members, respectively). In accordance with Remark 3.10, the values of the $DRSV$ indicator calculated in this example should also be taken as approximate rather than exact.

Then one obtains that $R_{rs} = 0.0837$, $L_{rs} = -0.0837$, and from (3.15) it follows $DRSV = 0.0837 = 8.37\% > 0$.

Therefore we can conclude that the direction of relative seat volatility is increased to the right-wing party groups. It can be interpreted that the right-wing party groups R have won 8.37% more seats and the left-wing party groups L have lost 8.37% seats, in the election 2024 with respect to the election 2019 in the EU Parliament.

In Example 3.13 the values of the indicator of relative seat volatility direction are calculated in the elections 2024 versus 2019 for the EU Parliament in the EU member states.

EXAMPLE 3.13. We compare the 2024 and 2019 European Parliament elections and calculate the corresponding values for the indicator of relative seat volatility direction (3.15) in each member state. All member states of the EU used the proportional electoral system in these elections (see [21], [22]).

Let us also note that there are several cases where parties split, merge, or are newly formed between two elections (the United Kingdom withdrew from the EU in 2020). Therefore, in accordance with Remark 3.10, the values of the $DRSV$ indicator and the results calculated in this example should also be considered approximate rather than exact.

The corresponding values in the cases where $DRSV < 0$ (i.e., for those member states that have the relative change in the number of seats to the left) are given in Table 3. For instance, Sweden has got the direction of relative seat volatility to the left, but it is not significant since the right-wing parties have got majority of seats. Additionally, Slovakia exhibited a leftward direction of relative seat volatility, and this shift is significant because left-wing parties obtained a majority of seats in 2024, whereas they lacked a majority in 2019.

In addition, member states that had no relative seat fluctuation between the ‘left’ and the ‘right’ in two successive elections 2024 vs. 2019 for the EU Parliament (i.e., $DRSV = 0$) are given in Table 4. For instance, Greece has a value of $DRSV = 0$, with right-wing parties holding a majority of seats.

The corresponding values in cases where $DRSV > 0$ (i.e., for those countries that have the relative change in the number of seats to the right) are given in Table 5. For instance, Hungary showed the direction of relative seat volatility to the right, but it is not significant since the right-wing parties had majority of seats in 2019, too. Portugal also showed the direction of relative seat volatility to the right and this seats fluctuation is significant since the right-wing parties obtained majority seats in 2024 but the right-wing parties had not got majority of seats in 2019. In Example 3.11 it is shown that Croatia also exhibited a relative seat volatility toward the right, and this shift is significant.

TABLE 3. Member states that have the relative change in the number of seats to the left ($DRSV < 0$) in two successive elections 2024 vs. 2019 for the EU Parliament

Relative seat fluctuation in 2024 vs. 2019 to the left ($DRSV < 0$)	
not significant:	significant, the left majority:
Sweden ($DRSV = -4.76\%$, right maj.)	Slovakia ($DRSV = -30.00\%$)
Poland ($DRSV = -2.98\%$, right maj.)	Finland ($DRSV = -3.33\%$)
Belgium ($DRSV = -2.92\%$, right maj.)	
Italy ($DRSV = -2.63\%$, right maj.)	
Austria ($DRSV = -0.56\%$, right maj.)	

TABLE 4. Member states that had no relative seat fluctuation between the left and the right ($DRSV = 0$) in two successive elections 2024 vs. 2019 for the EU Parliament

$DRSV = 0$ for 2024 vs. 2019
Greece (right maj.)
Lithuania (left maj.)
Cyprus (equal)
Luxembourg (right maj.)

TABLE 5. Member states that have the relative change in the number of seats to the right ($DRSV > 0$) in two successive elections 2024 vs. 2019 for the EU Parliament

Relative seats fluctuation in 2024 to the right ($DRSV > 0$)	
not significant:	significant, the right majority:
Hungary ($DRSV = 23.81\%$, right maj.)	Portugal ($DRSV = 19.05\%$)
Bulgaria ($DRSV = 17.65\%$, right maj.)	Latvia ($DRSV = 16.67\%$)
Malta ($DRSV = 16.67\%$, equal)	France ($DRSV = 11.38\%$)
Romania ($DRSV = 15.15\%$, right maj.)	Croatia ($DRSV = 8.33\%$)
Estonia ($DRSV = 14.29\%$, left maj.)	Netherlands ($DRSV = 8.06\%$)
Ireland ($DRSV = 10.44\%$, right maj.)	Slovenia ($DRSV = 5.56\%$)
Germany ($DRSV = 5.21\%$, equal)	Spain ($DRSV = 5.00\%$)
Czech Republic ($DRSV = 4.76\%$, right maj.)	
Denmark ($DRSV = 3.81\%$, left maj.)	

The values of indicator of relative seat volatility direction for each member state of the EU from Table 3, Table 4 and Table 5 are illustrated in Figure 2.

From Table 5 and Figure 2, it can be seen that 16 member states have values of $DRSV > 0$, meaning their relative change of seats is to the ‘right’. Table 3 shows that Slovakia is an outlier with a value of $DRSV = -30\%$; however, some parties in Slovakia are here classified as ‘left-wing parties’ according to ‘standard’ criteria, even though they combine certain ideas from the ‘right’.

3.3. An alternative variant of relative seat (or vote) volatility direction.

With regard to Remark 3.1, one can consider a partition of the set of parties (or lists) $I = \{1, 2, \dots, n\}$, where a third subset C is added which consists of parties (or lists) with a centrist political position:

$$(3.16) \quad I = L \cup C \cup R, \text{ where } L, C, R \text{ are mutually disjoint subsets of } I.$$

One can then analogously extend expressions (3.13) and (3.14), and use the following definition.

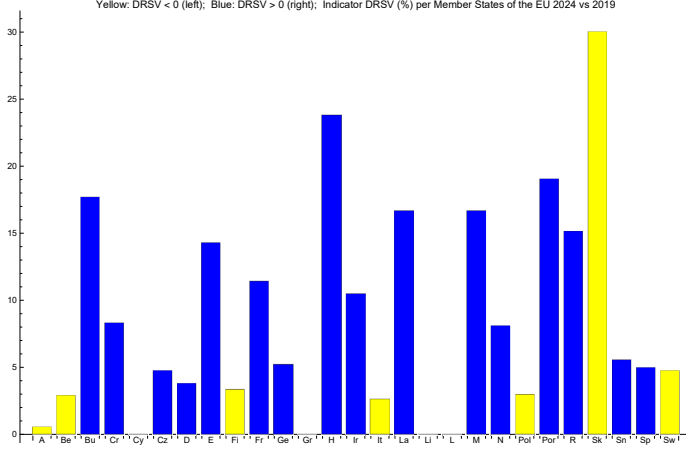


FIGURE 2. The direction of relative seat volatility for each member state of the EU: yellow: $DRSV < 0$ (to ‘left’); ‘no change’: $DRSV = 0$; blue: $DRSV > 0$ (to ‘right’)

DEFINITION 3.14. The sum C_{rs} of the differences $\frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j}$ for the centrist parties is defined as follows:

$$(3.17) \quad C_{rs} = \sum_{i \in C} \left(\frac{s_i}{\sum_{j=1}^n s_j} - \frac{s'_i}{\sum_{j=1}^n s'_j} \right),$$

and it represents the sum of the relative seat volatility for centrist parties between two successive elections (i.e., two successive inaugural sessions).

In this case, the corresponding indicator of relative seat volatility direction is analogously defined as follows.

DEFINITION 3.15. The quantity

$$(3.18) \quad DRSV_3 = \max\{L_{rs}, C_{rs}, R_{rs}\}$$

represents the maximum of the relative seat volatility for the left-wing, centrist, and right-wing parties. It can be called ‘the indicator of relative triple-seat volatility direction’ (towards the right, the centre, or the left) between two successive elections (i.e., two successive inaugural sessions).

PROPOSITION 3.16. Under the above assumptions, we have $L_{rs} + C_{rs} + R_{rs} = 0$.

PROOF. The relation follows from (3.16), (3.13), (3.14), and (3.17). $\square$

Thus, in the case of the partition (3.16), the following conclusion applies.

If $DRSV_3 = L_{rs} > \max\{C_{rs}, R_{rs}\}$, then relative seat fluctuation is increased to the left-wing parties.

If $DRSV_3 = C_{rs} > \max\{L_{rs}, R_{rs}\}$, then relative seat fluctuation is increased to the centrist parties.

If $DRSV_3 = R_{rs} > \max\{C_{rs}, L_{rs}\}$, then relative seat fluctuation is increased to the right-wing parties.

If $DRSV_3 = 0$, then there are no change of relative seat fluctuation to the left, to the centrist, or to the right.

There are also other possibilities for values of ‘the indicator of relative triple-seat volatility direction’ (3.18), for example: $DRSV_3 = R_{rs} = L_{rs} > 0 > C_{rs}$, or similarly. In such cases, appropriate conclusions can be drawn.

To illustrate the $DRSV_3$ indicator (3.18), we consider the following example.

EXAMPLE 3.17. As in Example 3.11, we consider the European Parliament (EP) elections in Croatia in 2024 versus 2019. However, we assume here that the partition of the set of parties is $I = L \cup C \cup R$, where $L = \{SDP, M, IDS\}$, $C = \{PiP\}$, and $R = \{HDZ, DP, HS\}$.

In the 2024 European Parliament elections in Croatia, the seat allocations for the left-wing, centrist, and right-wing parties were $s_L = (SDP, M, IDS) = (4, 1, 0)$, $s_C = (PiP) = (0)$, and $s_R = (HDZ, DP, HS) = (6, 1, 0)$, respectively.

In the 2019 European Parliament elections in Croatia, the seat allocations for the left-wing, centrist, and right-wing parties were $s'_L = (SDP, M, IDS) = (4, 0, 1)$, $s'_C = (PiP) = (2)$, and $s'_R = (HDZ, DP, HS) = (4, 0, 1)$, respectively.

Then one obtains that $R_{rs} = 0.16667$, $C_{rs} = -0.16667$, $L_{rs} = 0$, and from (3.18) it follows $DRSV_3 = R_{rs} = 0.16667 = 16.667\% > 0$.

Therefore, we can conclude that the direction of relative triple-seat volatility is toward the right-wing parties. This can be interpreted as the right-wing parties R gaining 16.667% more seats (i.e., $+2/12$) and the centrist parties C losing 16.667% of their seats (i.e., $-2/12$) in the 2024 EP elections compared to the 2019 EP elections in Croatia. In addition, this relative seat volatility toward the right-wing parties is significant because the right-wing parties won the majority of seats in the 2024 EP elections (i.e., $\sum_{i \in R} s_i = 7 > 6$) compared to the 2019 EP elections, when the right-wing and left-wing parties obtained an equal number of seats (i.e., $\sum_{i \in R} s'_i = \sum_{i \in L} s'_i = 5$ seats, while the centrist parties obtained $\sum_{i \in C} s'_i = 2$ seats).

Let us note that, with regard to the direction of relative vote volatility and DRV indicator (3.10), we can analogously define ‘the indicator of relative triple-vote volatility direction’ (towards the left, the centre, or the right) between two successive elections, following Definition 3.15.

4. CONCLUDING REMARKS

Electoral volatility is a feature of electoral system that refers to a change of choice made by electoral body in time. It can indicate whether there is a significant change with respect to party positions in two consecutive elections and can show the lack of party system stabilization.

The Pedersen volatility index as the indicator of relative vote volatility is regarded as the most important measure of electoral volatility and it takes into account a change of votes by percentage between two successive elections. At the phase of electoral process after the transformation of votes into seats (i.e., mandates), a relative change of seats (i.e., mandates) in a parliament can be observed. Both the Pedersen volatility index and the indicator of relative seat volatility are considered in the cases of elections in 2024 versus 2019 for the European Parliament (EP) in each member state of the European Union (EU), provided that there exist certain constraints in calculation of exact values of these indicators.

With respect to the political positions of parties, the direction of electoral volatility is considered by means of a partition of the set of parties (or lists, or groups) into a subset of ‘left-wing parties’ and a subset of ‘right-wing parties’. Hence, the indicator of relative vote volatility direction and the indicator of relative seat volatility direction between two consecutive elections are proposed. It can show if the direction of relative seat volatility (or, respectively, the direction of relative vote volatility) is turned to the ‘right-wing parties’ or to the ‘left-wing parties’. The indicator of relative seat volatility direction is observed in the cases of elections in 2024 versus 2019 for the EP in each member state of the EU, provided that there also exist various constraints in calculation of exact values of this indicator. The indicator of relative seat volatility direction can be extended, provided that the partition of the set of parties consists of three subsets: the left-wing, centrist, and right-wing parties.

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Smjer izborne volatilnosti u izbornim sustavima

Tomislav Marošević, Josip Miletić, Marija Miloloža Pandur

SAŽETAK. Jedno od obilježja izbornih sustava je izborna volatilnost, koja odražava promjene u odabirima glasača između dvaju uzastopnih izbora.

Poznati Pedersen indeks volatilnosti uzima u obzir relativnu promjenu glasova između dvaju uzastopnih izbora. Sličan pristup može se primijeniti na raspodjelu zastupničkih mjesta (mandata) u parlamentu, uzimajući u obzir promjenu u zastupničkim mjestima nakon što su glasovi preinačeni u mjesta. Promatramo indikator relativne volatilnosti mjesta za svaku državu članicu Europske unije, u primjeru korištenjem empirijskih podataka iz uzastopnih izbora za Europski parlament godine 2024. i 2019.

Smjer izborne volatilnosti može se promatrati pomoću particije skupa stranaka na dva podskupa s obzirom na njihove političke pozicije: podskup 'desnih stranaka' i podskup 'lijevih stranaka'. (Alternativno, može se razmatrati particija skupa stranaka na tri podskupa: 'desnih stranaka', 'stranaka centra' i 'lijevih stranaka'.) Predlažemo indikator smjera relativne volatilnosti glasova, te indikator smjera relativne volatilnosti mjesta. Ovi indikatori pokazuju je li smjer relativne volatilnosti glasova, te je li smjer relativne volatilnosti mjesta, prema 'desnim strankama' ili prema 'lijevim strankama'. Indikator smjera relativne volatilnosti mjesta se razmatra u slučajevima uzastopnih izbora u 2024. prema 2019. godini za Europski parlament u svakoj državi članici Europske unije.

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