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An embedding of the Cantor fan into the Lelek fan

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AN EMBEDDING OF THE CANTOR FAN INTO THE LELEK FAN

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Abstract. The Lelek fan L is usually constructed as a subcontinuum of the Cantor fan in such a way that the set of the end-points of L is dense in L. It easily follows that the Lelek fan is embeddable into the Cantor fan. It is also a well-known fact that the Cantor fan is embeddable into the Lelek fan, but this is less obvious. When proving this, one usually uses the well-known result by Dijkstra and van Mill that the Cantor set is embeddable into the complete Erdös space, and the well-known fact by Kawamura, Oversteegen, and Tymchatyn that the set of end-points of the Lelek fan is homeomorphic to the complete Erdös space. Then, the subcontinuum of the Lelek fan that is induced by the embedded Cantor set into the set of end-points of the Lelek fan, is a Cantor fan.

In our paper, we give an alternative straightforward embedding of a Cantor fan into the Lelek fan. We do not use the fact that the Cantor set is embeddable into the complete Erdös space and that it is homeomorphic to the set of end-points of the Lelek fan. Instead, we use our recent techniques of Mahavier products of closed relations to produce an embedding of the Cantor fan into the Lelek fan. Since the Cantor fan is universal for the family of all smooth fans, it follows that also the Lelek fan is universal for smooth fans.

1. Introduction

A continuum is a non-empty compact connected metric space. *A subcontinuum* is a subspace of a continuum, which is itself a continuum. Let X be a continuum. We say that X is *a Cantor fan*, if X is homeomorphic to the continuum $\bigcup_{c\in C} A_c$, where $C \subseteq [0,1]$ is a Cantor set and for each $c \in C$, A_c is the convex segment in the plane from $(0,0)$ to $(c,-1)$; see Figure 1. Let

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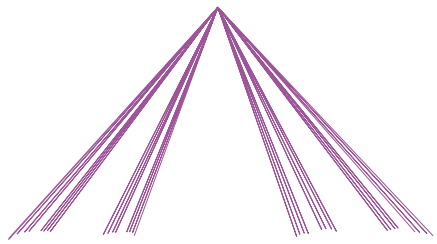

FIGURE 1. The Cantor fan

X be a Cantor fan and let Y be a subcontinuum of X. A point $x \in Y$ is called an *end-point of the continuum* Y, if for every arc A in Y that contains x, x is an end-point of A. The set of all end-points of Y will be denoted by $E(Y)$. We say that the subcontinuum Y of the Cantor fan X is *a Lelek fan*, if $\mathrm{Cl}(E(Y)) = Y$. The first example of a Lelek fan was constructed by

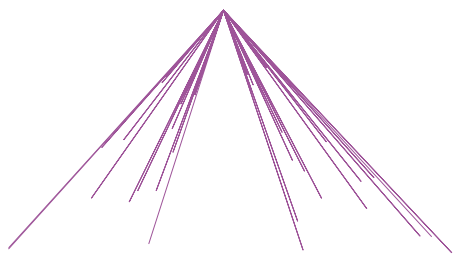

FIGURE 2. The Lelek fan

A. Lelek in [13]. He proved that the set of end-points of the Lelek fan is a one-dimensional set in the Lelek fan. The Lelek fan is also unique: any two non-degenerate subcontinua of the Cantor fan with a dense set of end-points are homeomorphic. This was proved independently by W. D. Bula and L. Oversteegen in [8] and by W. Charatonik in [10]. After the Lelek construction, there were many other constructions of the Lelek fan. For example, in 2013, D. Bartosova and A. Kwiatkowska constructed in [5] the Lelek fan as a quotient space of the projective Fraisse limit of a family that consists of finite rooted trees. In [2], the Lelek fan is constructed by I. Banič, G. Erceg and J. Kennedy as the inverse limit of inverse sequence of closed unit intervals with a single set-valued bonding function whose graph is an arc, and in [1], the Lelek fan is constructed by I. Banič, G. Erceg, J. Kennedy, C. Mouron and V. Nall as the inverse limit of an inverse sequence of Cantor fans and a single transitive continuous bonding function.

It easily follows from Lelek's construction that the Lelek fan is embeddable into the Cantor fan. However, it is not that obvious that the Cantor fan is embeddable into the Lelek fan. One can easily construct an embedding of the Cantor fan into the Lelek fan by using

1. the well-known result from [7] by J. J. Dijkstra and J. Mill that a space is almost zero-dimensional (a space is called almost zero-dimensional if every point of the space has a neighbourhood basis consisting of C-sets of the space, where a subset A of a space X is called a C-set in X if A can be written as an intersection of clopen subsets of X; see [7] for more details) if and only if it is embeddable into the complete Erdös space, and
2. the well-known result from [11] by K. Kawamura, L. G. Oversteegen, and E. D. Tymchatyn that the set of end-points of the Lelek fan is homeomorphic to the complete Erdös space.

First, embed the Cantor set into the set of end-points of the Lelek fan and then, the subcontinuum of the Lelek fan that is induced by the embedded Cantor set, is a Cantor fan (among other things, this was already noted by G. Basso and R. Camerlo in [4], where another similar result is obtained).

In this paper, we give an alternative straightforward construction of a Cantor fan into the Lelek fan. In our approach, we do not use the well-known results from [7] or [11]. Instead, we use our recently developed techniques of Mahavier products of closed relations from [1], [2], and [3]. We proceed as follows. In Section 2, the basic definitions and results that are needed later in the paper are presented. In Section 3, our main result is proved.

2. Definitions and Notation

The following definitions, notation and well-known results will be needed in the paper.

Definition 2.1. *Let X be a non-empty compact metric space and let $F \subseteq X \times X$ be a relation on X. If F is closed in $X \times X$, then we say that F is* a closed relation on X.

Definition 2.2. *Let X be a non-empty compact metric space and let F be a closed relation on X. Then we call*

$$X_F^+ = \left\{(x_0, x_1, x_2, \ldots) \in \prod_{k=0}^{\infty} X \mid \text{ for each non-negative integer } k, (x_k, x_{k+1}) \in F\right\}$$

the Mahavier product of F.

Definition 2.3. *For each $(r, \rho) \in (0, \infty) \times (0, \infty)$, we define the sets L_r, L_ρ and $L_{r,\rho}$ as follows: $L_r = \{(x, y) \in [0, 1] \times [0, 1] \mid y = rx\}$, $L_\rho = \{(x, y) \in$*

$[0,1] \times [0,1] \mid y = \rho x\}$, *and* $L_{r,\rho} = L_r \cup L_\rho$. *We also define the set* $M_{r,\rho}$ *as follows:*

$$M_{r,\rho} = [0,1]^+_{L_{r,\rho}}.$$

Definition 2.4. *Let* $(r,\rho) \in (0,\infty) \times (0,\infty)$. *We say that* r and ρ never connect *or* $(r,\rho) \in \mathcal{NC}$, *if*

1. $r < 1$, $\rho > 1$ *and*
2. *for all integers* k *and* ℓ,

$$r^k = \rho^\ell \Longleftrightarrow k = \ell = 0.$$

In [2] , the following theorem is the main result; see [2, Theorem 14, page 21].

Theorem 2.5. *Let* $(r,\rho) \in \mathcal{NC}$. *Then* $M_{r,\rho}$ *is a Lelek fan with top* $(0,0,0,\ldots)$.

In Theorem 2.7, a characterization of end-points of $M_{r,\rho}$ is established; see [3, Theorem 3.5, page 8].

Definition 2.6. *For each non-negative integer* k, *we use* $\pi_k : \prod_{i=0}^{\infty}[0,1] \to [0,1]$ *to denote the* k*-th standard projection from* $\prod_{i=0}^{\infty}[0,1]$ *to* $[0,1]$. *For any non-negative integer* k *and for any* $\mathbf{x} \in \prod_{i=0}^{\infty}[0,1]$, *we also use* $\mathbf{x}(k)$ *to denote* $\pi_k(\mathbf{x})$.

Theorem 2.7. *Let* $(r,\rho) \in \mathcal{NC}$ *and let* $\mathbf{x} \in M_{r,\rho}$. *Then* $\mathbf{x} \in E(M_{r,\rho})$ *if and only if* $\sup\{\pi_n(\mathbf{x}) \mid n \text{ is a non-negative integer}\} = 1$.

The following theorem is also proved in [2, Theorem 9, page 18].

Theorem 2.8. *Let* $(r,\rho) \in \mathcal{NC}$. *Then for each* $x \in (0,1)$, *there is a sequence* $a \in \{r,\rho\}^{\mathbb{N}}$ *such that for each positive integer* n,

$$(a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n) \cdot x \in [0,1]$$

and

$$\sup\{(a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n) \cdot x \mid n \text{ is a positive integer}\} = 1.$$

3. An embedding of the Cantor fan into the Lelek fan

We show, using our recent techniques from [2] and [3], that the Cantor fan can be embedded into the Lelek fan.

Theorem 3.1. *The Cantor fan is embeddable into the Lelek fan.*

Proof. Let $X = [0,1]$, let $(r,\rho) \in \mathcal{NC}$ and let

$$F = L_{r,\rho} \cup \{(t,t) \mid t \in [0,1]\} \text{ and } G = L_r \cup \{(t,t) \mid t \in [0,1]\}.$$

It follows from [2, Example 1, page 7] that X_G^+ is a Cantor fan. Since $X_G^+ \subseteq X_F^+$, it suffices to see that X_F^+ is a Lelek fan. To do that, let

$$B_{\mathbf{a}} = \{(t, \mathbf{a}(1) \cdot t, \mathbf{a}(2)\mathbf{a}(1) \cdot t, \mathbf{a}(3)\mathbf{a}(2)\mathbf{a}(1) \cdot t, \ldots) \mid t \in [0,1]\}$$

and

$$A_{\mathbf{a}} = B_{\mathbf{a}} \cap X_F^+$$

for each $\mathbf{a} = (\mathbf{a}(1), \mathbf{a}(2), \mathbf{a}(3), \ldots)) \in \{1, r, \rho\}^{\mathbb{N}}$. Note that for each $\mathbf{a} \in \{1, r, \rho\}^{\mathbb{N}}$, $B_{\mathbf{a}}$ is a straight line segment in Hilbert cube $\prod_{k=1}^{\infty}[0, \rho^{k-1}]$ from $(0, 0, 0, \ldots)$ to $(1, \mathbf{a}(1) \cdot 1, \mathbf{a}(2)\mathbf{a}(1) \cdot 1, \mathbf{a}(3)\mathbf{a}(2)\mathbf{a}(1) \cdot 1, \ldots)$, and that for all $\mathbf{a}, \mathbf{b} \in \{1, r, \rho\}^{\mathbb{N}}$,

$$B_{\mathbf{a}} \cap B_{\mathbf{b}} = \{(0, 0, 0, \ldots)\} \iff \mathbf{a} \neq \mathbf{b}.$$

Since $\{(1, \mathbf{a}(1) \cdot 1, \mathbf{a}(2)\mathbf{a}(1) \cdot 1, \mathbf{a}(3)\mathbf{a}(2)\mathbf{a}(1) \cdot 1, \ldots) \mid \mathbf{a} \in \{1, r, \rho\}^{\mathbb{N}}\}$ is a Cantor set, it follows that $\bigcup_{\mathbf{a} \in \{1,r,\rho\}^{\mathbb{N}}} B_{\mathbf{a}}$ is a Cantor fan. Therefore, X_F^+ is a subcontinuum of the Cantor fan $\bigcup_{\mathbf{a} \in \{1,r,\rho\}^{\mathbb{N}}} B_{\mathbf{a}}$. Note that for each $\mathbf{a} \in \{1, r, \rho\}^{\mathbb{N}}$, $A_{\mathbf{a}}$ is either degenerate or it is an arc from $(0, 0, 0, \ldots)$ to some other point, denote it by $\mathbf{e}_{\mathbf{a}}$. Let

$$\mathcal{U} = \{\mathbf{a} \in \{1, r, \rho\}^{\mathbb{N}} \mid A_{\mathbf{a}} \text{ is an arc}\}.$$

Then

$$X_F^+ = \bigcup_{\mathbf{a} \in \mathcal{U}} A_{\mathbf{a}} \quad \text{and} \quad E(X_F^+) = \{\mathbf{e}_{\mathbf{a}} \mid \mathbf{a} \in \mathcal{U}\}.$$

Next, we show that for each $\mathbf{x} \in X_F^+$,

$$\mathbf{x} \in E(X_F^+) \iff \sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = 1.$$

Let $\mathbf{x} \in X_F^+$. We treat the following possible cases.

Case 1. For each non-negative integer k, there is a positive integer ℓ such that $\ell > k$ and $\mathbf{x}(k) \neq \mathbf{x}(\ell)$. Without loss of generality we may assume that $\mathbf{x} \in M_{r,\rho}$. First, suppose that $\mathbf{x} \in E(X_F^+)$. Then $\mathbf{x} \in E(M_{r,\rho})$ and by Theorem 2.7, $\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = 1$. Next, suppose that $\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = 1$. Since $\mathbf{x} \in M_{r,\rho}$, it follows from Theorem 2.7 that $\mathbf{x} \in E(M_{r,\rho})$. Since $E(M_{r,\rho}) \subseteq E(X_F^+)$, it follows that $\mathbf{x} \in E(X_F^+)$.

Case 2. There is a non-negative integer k such that for each non-negative integer $\ell \geq k$, $\mathbf{x}(\ell) = \mathbf{x}(k)$. In this case,

$$\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = \max\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\}.$$

Let $\mathbf{x} \in E(X_F^+)$ and suppose that $\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = m < 1$. Also, let k_0 be a non-negative integer such that $\mathbf{x}(k_0) = m$ and let $\mathbf{a} \in \{1, r, \rho\}^{\mathbb{N}}$ be such that

$$\mathbf{x} = (\mathbf{x}(0), \mathbf{a}(1) \cdot \mathbf{x}(0), \mathbf{a}(2)\mathbf{a}(1) \cdot \mathbf{x}(0), \mathbf{a}(3)\mathbf{a}(2)\mathbf{a}(1) \cdot \mathbf{x}(0), \ldots).$$

Then

$$\mathbf{x} \in \Big\{\Big(\frac{1}{\mathbf{a}(1)\cdot\mathbf{a}(2)\cdot\mathbf{a}(3)\cdot\ldots\cdot\mathbf{a}(k_0-1)}\cdot t,\ldots,\frac{1}{\mathbf{a}(k_0-2)\cdot\mathbf{a}(k_0-1)}\cdot t,\frac{1}{\mathbf{a}(k_0-1)}\cdot t,$$
$$t,\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+2)\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\ldots\Big) \mid t\in[0,m]\Big\},$$

which is a proper subset of the arc

$$\Big\{\Big(\frac{1}{\mathbf{a}(1)\cdot\mathbf{a}(2)\cdot\mathbf{a}(3)\cdot\ldots\cdot\mathbf{a}(k_0-1)}\cdot t,\ldots,\frac{1}{\mathbf{a}(k_0-2)\cdot\mathbf{a}(k_0-1)}\cdot t,\frac{1}{\mathbf{a}(k_0-1)}\cdot t,$$
$$t,\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+2)\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\ldots\Big) \mid t\in[0,1]\Big\}$$

in X_F^+ and is, therefore, not an endpoint of X_F^+. It follows that the supremum $\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\}$ equals 1. To prove the other implication, suppose that $\sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = 1$. Then $\mathbf{x}$ is the end-point of the arc

$$\Big\{\Big(\frac{1}{\mathbf{a}(1)\cdot\mathbf{a}(2)\cdot\mathbf{a}(3)\cdot\ldots\cdot\mathbf{a}(k_0-1)}\cdot t,\ldots,\frac{1}{\mathbf{a}(k_0-2)\cdot\mathbf{a}(k_0-1)}\cdot t,\frac{1}{\mathbf{a}(k_0-1)}\cdot t,$$
$$t,\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\mathbf{a}(k_0+2)\mathbf{a}(k_0+1)\mathbf{a}(k_0)\cdot t,\ldots\Big) \mid t\in[0,1]\Big\}$$

in X_F^+, which is not equal to $(0,0,0,\ldots)$. Therefore, it is an end-point of X_F^+.

Therefore, $\mathbf{x} \in E(X_F^+) \iff \sup\{\mathbf{x}(k) \mid k \text{ is a non-negative integer}\} = 1$ follows.

To see that X_F^+ is a Lelek fan, let $\mathbf{x} \in X_F^+$ be any point and let $\varepsilon > 0$. We prove that there is a point $\mathbf{e} \in E(X_F^+)$ such that $\mathbf{e} \in B(\mathbf{x},\varepsilon)$ by considering the following possible cases.

Case 1. For each non-negative integer k, there is a positive integer ℓ such that $\ell > k$ and $\mathbf{x}(k) \neq \mathbf{x}(\ell)$. Again, without loss of generality we assume that $\mathbf{x} \in M_{r,\rho} \setminus \{(0,0,0,\ldots)\}$. Then $\mathbf{x}(n) \neq 0$ for each positive integer n. For each positive integer n, by Theorem 2.8, there is a sequence $\mathbf{a}^n = (a_1^n, a_2^n, a_3^n, \ldots) \in \{r,\rho\}^{\mathbb{N}}$ such that for each positive integer k,

$$a_1^n \cdot a_2^n \cdot a_3^n \cdot \ldots \cdot a_k^n \cdot \mathbf{x}(n) \in [0,1]$$

and

$$\sup\{a_1^n \cdot a_2^n \cdot a_3^n \cdot \ldots \cdot a_k^n \cdot \mathbf{x}(n) \mid k \text{ is a positive integer}\} = 1.$$

For each positive integer n, choose such a sequence $\mathbf{a}^n$ and let

$$\mathbf{x}_n = (\mathbf{x}(1),\mathbf{x}(2),\mathbf{x}(3),\ldots,\mathbf{x}(n),a_1^n\cdot\mathbf{x}(n),a_1^n\cdot a_2^n\cdot\mathbf{x}(n),a_1^n\cdot a_2^n\cdot a_3^n\cdot\mathbf{x}(n),\ldots).$$

By Theorem 2.7, $\mathbf{x}_n \in E(M_{r,\rho})$ for each positive integer n. It follows from $E(M_{r,\rho}) \subseteq E(X_F^+)$ that for each positive integer n, $\mathbf{x}_n \in E(X_F^+)$. Since $\lim\limits_{n\to\infty} \mathbf{x}_n = \mathbf{x}$, it follows that there is a point $\mathbf{e} \in E(X_F^+)$ such that $\mathbf{e} \in B(\mathbf{x},\varepsilon)$.

Case 2. There is a non-negative integer k such that for each non-negative integer $\ell \geq k$, $\mathbf{x}(\ell) = \mathbf{x}(k)$. Without loss of generality, we assume that $\mathbf{x} \neq (0, 0, 0, \ldots)$. Let k_0 be a positive integer such that $\sum_{k=k_0}^{\infty} \frac{1}{2^k} < \varepsilon$ and such that for each positive integer $k \geq k_0$, $\mathbf{x}(k) = \mathbf{x}(k_0)$. It follows from Theorem 2.8 that there is a sequence $(a_1, a_2, a_3, \ldots) \in \{r, \rho\}^{\mathbf{N}}$ such that

$$\sup\{(a_1 \cdot a_2 \cdot a_3 \cdot \ldots \cdot a_n) \cdot \mathbf{x}(k_0) \mid n \text{ is a positive integer}\} = 1.$$

Choose and fix such a sequence $(a_1, a_2, a_3, \ldots)$. Let

$$\mathbf{e} = (\mathbf{x}(0), \mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(k_0), a_1 \cdot \mathbf{x}(k_0), a_2 a_1 \cdot \mathbf{x}(k_0), a_3 a_2 a_1 \cdot \mathbf{x}(k_0), \ldots).$$

Then $\mathbf{e} \in E(X_F^+)$ and

$$D(\mathbf{e}, \mathbf{x}) \leq \sum_{k=k_0}^{\infty} \frac{1}{2^k} < \varepsilon,$$

where D is the metric on X_F^+.

This proves that X_F^+ is a Lelek fan. □

Observation 3.2. *It is a well-known fact that the Cantor fan is universal for smooth fans, i.e., every smooth fan embeds into it (for details see* [9, Theorem 9, p. 27], [12, Corollary 4], *and* [6]*). Since the Lelek fan contains a Cantor fan, it follows also that the Lelek fan is a universal continuum for smooth fans.*

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Ulaganje Cantorove lepeze u Lelekovu lepezu

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Sažetak. Lelekova lepeza L obično se konstruira kao potkontinuum Cantorove lepeze na način da je skup krajnjih točaka od L gust u L. Lako slijedi da je Lelekova lepeza uloživa u Cantorovu lepezu. Takodjer je dobro poznata činjenica da se Cantorova lepeza može uložiti u Lelekovu lepezu, ali to je manje očito. U dokazu te tvrdnje, obično se koristi dobro poznati rezultat Dijkstre i van Milla da je Cantorov skup uloživ u potpuni Erdösev prostor, te dobro poznata činjenica Kawamure, Oversteegena i Tymchatyna da je skup krajnjih točaka Lelekove lepeze homeomorfan potpunom Erdösevom prostoru. Zatim, potkontinuum Lelekove lepeze koji je induciran uloženim Cantorovim skupom u skup krajnjih točaka Lelekove lepeze je Cantorova lepeza.

U našem radu dajemo alternativnu konstrukciju ulaganja Cantorove lepeze u Lelekovu lepezu. Ne koristimo se činjenicom da je Cantorov skup moguće uložiti u potpun Erdösev prostor i da je homeomorfan skupu krajnjih točaka Lelekove lepeze. Umjesto toga, koristimo naše nedavne tehnike Mahavierovih produkta zatvorenih relacija za ulaganje Cantorove lepeze u Lelekovu lepezu. Budući da je Cantorova lepeza univerzalna za klasu svih glatkih lepeza, slijedi da je i Lelekova lepeza univerzalna za glatke lepeze.

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