Regularity of Diophantine quadruples in $\mathbb{Q}(i)[X]$

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Introduction

- The concept of Diophantine m-tuples goes back to ancient Greek mathematics.
- Diophantus of Alexandria[1] investigated sets of rational numbers that fulfil a certain property, thereby laying the foundation for this area of number theory.
- The problem of identifying Diophantine m-tuples has occupied mathematicians for centuries and has led to numerous results and considerable progress in number theory in various of its classical and modern subfields.

[1] A mathematician active in the 3rd century.

Introduction

Definition 1 (Diophantine m-tuple with property $D(n)$)

Let $n \in \mathbb{Z}\backslash\{0\}$. A set of m positive integers $\{a_1, a_2, \ldots, a_m\}$ is called a Diophantine m-tuple with property $D(n)$[2] if

$$a_i a_j + n$$

is a perfect square for all $1 \leq i < j \leq m$.

Diophantine m-tuple with the property $D(1)$ is simply called Diophantine m-tuple.

[2] or a $D(n)$-m-tuple

Introduction

Diophantus found a set of four positive rational numbers with the property that the product of any two of them, increased by one, is a perfect square:

$$\left\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\right\}.$$

The set of integers

$$\{1, 3, 8, 120\},$$

known as *Fermat's set*, was discovered by Fermat.

Diophantine m-tuples

The most studied case is for $n = 1$.

- Baker & Davenport (1969) proved that Fermat's set cannot be extended by a positive integer to a Diophantine quintuple [1].
- Dujella (2004) proved that there is no Diophantine sextuple and that there are only finitely many Diophantine quintuples over the integers [6].
- He, Togbé & Ziegler have proved (2019) that there is no Diophantine quintuple solving a long-standing open problem [19].

Generalizations of the problem: the set of polynomials

Another very interesting generalization of the concept presented is when the domain we are dealing with is the set of polynomials.

Definition 2 (Diophantine m-tuple with the property $D(n)$ in R)

Let $m \geq 2$ and let R be a commutative ring with unity. Let $n \in R$ be a non-zero element and let $\{a_1, a_2, \ldots, a_m\}$ be a set of m distinct non-zero elements in R such that

$$a_i a_j + n$$

is a perfect square of an element in R for $1 \leq i < j \leq m$. The set $\{a_1, a_2, \ldots, a_m\}$ is called a Diophantine m-tuple with the property $D(n)$[3] in R.

[3] Or a $D(n)$-m-tuple in R

Generalizations of the problem: set of polynomials

Regularity of Diohantine quadruples was investigated.

- Dujella & Fuchs (2004) – $\mathbb{Z}[X]$,
- Filipin & Jurasić (2019) – $\mathbb{R}[X]$ and $\mathbb{Z}[i][X]$,
- S.B.B. and Jurasić – $\mathbb{Q}(i)[X]$,
- Dujella & Jurasić (2010) – $\mathbb{C}[X]$ (irregular quadruples exist).

By observing the problem of Diophantus in $\mathbb{Q}(i)[X]$, we will complete the observing of this problem in polynomial rings R.

Motivation

- In our previous work, Bliznac Trebješanin and S.B.B. have proved that every $D(4)$-quadruple in $\mathbb{Z}[i][X]$ is regular [4].
- Therefore, our current work is motivated both by our earlier research on $D(1)$-quadruples and on $D(4)$-quadruples over various polynomial rings.
- The results obtained for $D(1)$-quadruples in $\mathbb{Q}(i)[X]$ can also be applied to a $D(4)$-quadruples in $\mathbb{Q}(i)[X]$, i.e. we can transfer the proof of assertions, in the context of $n = 4$ by using proven results from the case $n = 1$.

Notation and ordering

Let $\{a, b, c\}$ be a Diophantine triple in $\mathbb{Q}(i)[X]$ such that

$$ab + 1 = r^2, \ ac + 1 = s^2, \ bc + 1 = t^2, \tag{1}$$

where $r, s, t \in \mathbb{Q}(i)[X]$.

In cases where R is a polynomial ring, in our case $R = \mathbb{Q}(i)[X]$, it is assumed that not all the polynomials a, b, c are constant polynomials.

Some differences in the approaches

- Since there is no explicit relation " $<$ " between the elements in $\mathbb{Z}[i][X], \mathbb{Q}[i][X]$ or $\mathbb{C}[X]$, we introduce the relation " $\leq$ " between the degrees of the polynomials.
- For example, within $\mathbb{R}[X]$, the set $\mathbb{R}^+[X]$ consists of all polynomials with a positive leading coefficient. An ordering in $\mathbb{R}[X]$ is then introduced by defining that $a < b$ if $b - a \in \mathbb{R}^+[X]$, for $a, b \in \mathbb{R}[X]$.

α, β, γ denote the degrees of the polynomials a, b, c, respectively.

From a Diophantine pair to triple

- The problem of extending a Diophantine pair $\{a, b\}$ to a Diophantine triple $\{a, b, c\}$ is closely related to the solution of a certain Pellian equation.
- If the Diophantine pair $\{a, b\}$ is extended to the Diophantine triple $\{a, b, c\}$, then the Pellian equation

$$bs^2 - at^2 = b - a$$

  is satisfied.
- There is always a possibility where $c = a + b + 2r$ and $r = \sqrt{ab + 1}$, which leads us to a so-called **regular triple**.

Every Diophantine pair can be extended to a Diophantine triple.

Regularity of a Diophantine triple

Definition 3 (Regularity of a Diophantine triple $\{a, b, c\}$)

A Diophantine triple $\{a, b, c\}$ in $\mathbb{Q}(i)[X]$ is called regular if

$$(c - b - a)^2 = 4(ab + 1). \tag{2}$$

Or, more explicitly, if

$$c = c_{\pm} = a + b \pm 2r, \tag{3}$$

$$ac_{\pm} + 1 = (a \pm r)^2, \ bc_{\pm} + 1 = (b \pm r)^2. \tag{4}$$

A simple example of a Diophantine triple in $\mathbb{Q}(i)[X]$

For the triple

$$\{a, b, c\} = \left\{1, \quad X^2 + (1+i)X + \frac{i}{2} - 1, \quad X^2 + (3+i)X + \frac{3}{2}i + 1\right\}$$

it is easy to verify

$$ab + 1 = \left(X + \frac{1+i}{2}\right)^2, \quad ac + 1 = \left(X + \frac{3+i}{2}\right)^2,$$

$$bc + 1 = \left(X^2 + (2+i)X + i - \frac{1}{2}\right)^2.$$

Regularity of a Diophantine quadruple in $\mathbb{Q}(i)[X]$

Definition 4 (Regularity of a Diophantine quadruple $\{a, b, c, d\}$)

A Diophantine quadruple $\{a, b, c, d\}$ in $\mathbb{Q}(i)[X]$ is called regular if

$$(a + b - c - d)^2 = 4(ab + 1)(cd + 1), \tag{5}$$

or equivalently if

$$d = d_{\pm} = a + b + c + 2(abc \pm rst). \tag{6}$$

In this case, we have

$$ad_{\pm} + 1 = u_{\pm}^2, \quad bd_{\pm} + 1 = v_{\pm}^2, \quad cd_{\pm} + 1 = w_{\pm}^2,$$

where $u_{\pm}, v_{\pm}, w_{\pm} \in \mathbb{Q}(i)[X]$.

Regularity of a Diophantine quadruple

Remark 1

There always exist regular Diophantine quadruples $\{a, b, c, d_{\pm}\}$[4], *where* $d_{\pm}$ *is defined by (6). Specifically, any Diophantine pair* $\{a, b\}$ *in* $\mathbb{Q}(i)[X]$ *can be extended to regular Diophantine quadruples of the form*

$$\{a, b, a + b \pm 2r, r(r \pm a)(b \pm r)\}. \tag{7}$$

[4] d_+ is the polynomial with the larger degree among two polynomials $d_{\pm}$.

An **irregular** Diophantine tuple is one that is not regular.

Assume now that $\{a, b, c, d'\}$ with $\deg(d') = \delta$ and $\gamma \leq \delta$, is an irregular Diophantine quadruple with minimal δ among all irregular Diophantine quadruples in $\mathbb{Q}(i)[X]$. According to Lemma from [12], we have

$$\delta \geq \frac{3\beta + 5\gamma}{2}. \tag{8}$$

Our goal is to investigate whether such quadruple exists in $\mathbb{Q}(i)[X]$.

The Common Strategy

Common strategy is to

- find the system of simultaneous Pellian equations,
- find the intersection of the binary recurrent sequences,
- apply congruence relations and the gap principle.

1. We aim to answer the question whether every Diophantine quadruple in $\mathbb{Q}(i)[X]$ is regular.
2. Consequently, we will answer the question whether every $D(4)$ quadruple in $\mathbb{Q}(i)[X]$ is regular.

We partially follow strategies used by

- Dujella and Fuchs [2004] for $\mathbb{Z}[X]$,
- Filipin and Jurasić [2019] for $\mathbb{Z}[i][X]$ and $\mathbb{R}[X]$,
- Dujella and Jurasić [2010] for $\mathbb{C}[X]$.

The System of Pellian equations

Extending a Diophantine triple $\{a, b, c\}$ to a Diophantine quadruple $\{a, b, c, d\}$ in $\mathbb{Q}(i)[X]$, where d is a non-zero polynomial such that $d \neq a, b, c$, means there exist $x, y, z \in \mathbb{Q}(i)[X]$ satisfying

$$ad + 1 = x^2, \ bd + 1 = y^2, \ cd + 1 = z^2. \tag{9}$$

Eliminating d from (9), we obtain the system of simultaneous Pellian equations

$$az^2 - cx^2 = a - c, \tag{10}$$

$$bz^2 - cy^2 = b - c. \tag{11}$$

Some basic properties

Lemma 5 (Dujella, Luca)

Let $\{a_1, a_2, \ldots, a_m\}$ be a Diophantine m-tuple in $\mathbb{C}[X]$, such that not all a_i's are constant polynomials. Then $a_i \neq a_j$ for $i \neq j$ and at most one of the polynomials a_i, $i = 1, \ldots, m$, is constant.

So, in our case within $\mathbb{Q}(i)[X]$ we also conclude

$$0 \leq \alpha \leq \beta \leq \gamma, \quad \beta, \gamma > 0.$$

Some basic properties

Lemma 6

Let (z, x) and (z, y) be the solutions of (10) and (11), respectively, with $x, y, z \in \mathbb{Q}(i)[X]$. Then:

1. *We have $z \neq 0$ and if z is a constant polynomial, then $z = x = \pm 1$.*
2. *If z is not a constant polynomial, there holds:*
   a) *if x is a constant polynomial, then $\deg z = \frac{\gamma - \alpha}{2}$ and $a \mid c$ and*
   b) *if y is a constant polynomial, then $\deg z = \frac{\gamma - \beta}{2}$ and $b \mid c$.*

Relations on degrees of the polynomials $a, b, c \in \mathbb{R}[X]$

Lemma 7 (adapted result of Jurasić & Filipin [17])

Let (z, x) and (z, y) be the solutions of (10) and (11), respectively, with $x, y, z \in \mathbb{Q}(i)[X]$.
*Then, there **exist** solutions (z_0, x_0) and (z_1, y_1) of (10) and (11), respectively, such that*

$$\deg(z_0) \leq \frac{3\gamma - \alpha}{4}, \quad \deg(x_0) \leq \frac{\alpha + \gamma}{4}, \tag{12}$$

$$\deg(z_1) \leq \frac{3\gamma - \beta}{4}, \quad \deg(y_1) \leq \frac{\beta + \gamma}{4}, \tag{13}$$

$z_0, z_1, x_0, y_1 \in \mathbb{Q}(i)[X]$.

Lemma 8 (adapted result of Jurasić & Filipin [17])

Let (z, x) and (z, y) be the solutions of (10) and (11), respectively, with $x, y, z \in \mathbb{Q}(i)[X]$. Then, there exist non-negative integers m and n such that

$$z\sqrt{a} + x\sqrt{c} = (z_0\sqrt{a} + x_0\sqrt{c})\left(s + \sqrt{ac}\right)^m, \tag{14}$$

$$z\sqrt{b} + y\sqrt{c} = (z_1\sqrt{b} + y_1\sqrt{c})\left(t + \sqrt{bc}\right)^n. \tag{15}$$

Binary recurrence sequences $(v_m)_{m\geq 0}$ and $(w_n)_{n\geq 0}$ are given by

$$v_0 = z_0, \quad v_1 = sz_0 + cx_0, \quad v_{m+2} = 2sv_{m+1} - v_m, \tag{16}$$

$$w_0 = z_1, \quad w_1 = tz_1 + cy_1, \quad w_{n+2} = 2tw_{n+1} - w_n. \tag{17}$$

Binary recurrence sequences

We can conclude that there exist $d_0, d_1 \in \mathbb{Q}(i)[X]$ such that

$$ad_0 + 1 = x_0^2 \quad \text{and} \quad cd_0 + 1 = z_0^2, \tag{18}$$

and

$$bd_1 + 1 = y_1^2 \quad \text{and} \quad cd_1 + 1 = z_1^2. \tag{19}$$

We reduce the problem of finding the extensions of Diophantine triple to solving the equation

$$v_m = w_n$$

for some $m, n \geq 0$.

Degrees of solutions of binary recurrence sequences

Lemma 9 (adapted version of Dujella & Luca, [11])

Let $\{a, b, c, d\}$ be a Diophantine quadruple in $\mathbb{Q}(i)[X]$ and (v_m) and (w_n) be sequences defined as in (16) and (17). Then for $m \geq 1$ and $n \geq 1$

$$\deg(v_m) = (m-1)\frac{\alpha+\gamma}{2} + \deg(v_1), \tag{20}$$

$$\deg(w_n) = (n-1)\frac{\beta+\gamma}{2} + \deg(w_1). \tag{21}$$

Also,

$$\frac{\gamma}{2} \leq \deg(v_1) \leq \frac{\alpha+5\gamma}{4}, \tag{22}$$

$$\frac{\gamma}{2} \leq \deg(w_1) \leq \frac{\beta+5\gamma}{4}. \tag{23}$$

Binary recurrence sequences

The next lemma [12] shows an unconditional relationship between m and n, when $v_m = w_n$.

Lemma 10

If $v_m = w_n$, then

$$n - 1 \leq m \leq 2n + 1.$$

About d_-

The polynomial d_- plays an important role in our analysis because by observing its properties we conclude even more about polynomials a, b and c and their possible degrees.

A similar gap principle is known in the classical and polynomial case ([11, 17]).

About d_-

Lemma 11

Let $\{a, b, c\}$ be a Diophantine triple in $\mathbb{Q}(i)[X]$ such that (1) holds. Then, for

$$d_- = a + b + c + 2(abc \pm rst), \tag{24}$$

we have

$$ad_- + 1 = u^2, \quad bd_- + 1 = v^2, \quad cd_- + 1 = w^2, \tag{25}$$

where

$$u = at \pm rs, \quad v = bs \pm rt, \quad w = cr \pm st. \tag{26}$$

About d_-

Lemma 12

Let $\{a, b, c\}$ be a Diophantine triple in $\mathbb{Q}(i)[X]$ and let d_- be defined by (6). Then

$$d_- = 0 \quad \text{or} \quad \deg(d_-) = \gamma - \alpha - \beta.$$

If $d_- \neq 0$, then:

- $\alpha = 0$, $\beta = \gamma$ *and* $d_- = a = \pm i$ *or*
- $\alpha = 0$, $\beta = \gamma$ *and* $d_- = a = \pm\frac{1}{2}\left(f - \frac{1}{f}\right)$, *for* $f \in \mathbb{Q}(i)\backslash\{0\}$ *or*
- $d_- = c + sv + tu$, *where* s, t *are given with (1) and* u, v *with (25), and*
  - $\alpha = 0$, $\beta < \gamma$ *and* $\deg(d_-) = \gamma - \beta > 0$ *or*
  - $\alpha > 0$, $\gamma = \alpha + \beta$ *and* $\deg(d_-) = 0$.

For $f \in \mathbb{Q}(i)\backslash\{0\}$ we get

$$\{a, b, c\} = \left\{\frac{1}{2}\left(f - \frac{1}{f}\right), \quad f(1-t), \quad -\frac{1}{f}(t+1)\right\},$$

For example, for $f = 2i$ we have

$$\{a, b, c\} = \left\{\frac{5i}{4}, \quad -\frac{4}{5}i(X^2-1), \quad \frac{1}{5}i(X^2+4)\right\}.$$

Lemma 13 (all the possible relations between degrees α, β, γ)

Let $\{a, b, c\}$ be a Diophantine triple in $\mathbb{Q}(i)[X]$. The following may occur:

1. *If $d_- = 0$, then $z_0 = z_1 = \pm 1$. In this case $c = a + b \pm 2r$ and $\beta = \gamma$. If $\alpha < \beta$, then $C = B$ and, if $\alpha = \beta$, then $C = A + B \pm 2\sqrt{AB}$.*
2. *If $\deg(d_-) = 0$, then:*
   1. *If $d_- = a$, then $(z_0, z_1) = (\pm s, \pm s)$, $\alpha = 0$, $\beta = \gamma$.*
   2. *If $d_- \in \mathbb{Q}(i)\backslash\{0, a\}$, then $z_0 = z_1 = \pm cr \pm st$, $\alpha > 0$ and $\gamma = \alpha + \beta$ and $C = ABD_-$.*
3. *If $\deg(d_-) > 0$, then $C = ABD_-$ and:*
   1. *$z_0 = z_1 = \pm cr \pm st$, with $\deg(d_-) \leq \alpha$, $\alpha > 0$ and $\alpha + \beta < \gamma \leq 2\alpha + \beta$,*
   2. *$(z_0, z_1) = (\pm cr \pm st, \pm s)$, where $\alpha \leq \deg(d_-) \leq \beta$, $\alpha \geq 0$ and $2\alpha + \beta \leq \gamma \leq \alpha + 2\beta$,*
   3. *$(z_0, z_1) = (\pm t, \pm cr \pm st)$, with $\deg(d_-) = \alpha$, $\alpha = \beta$ and $\gamma = 3\alpha$,*
   4. *$(z_0, z_1) = (\pm t, \pm s)$ where $\beta \leq \deg(d_-) < \gamma$, $\alpha \geq 0$ and $\gamma \geq \alpha + 2\beta$.*

Congruence relations

The next lemma will give us the relations between initial terms $z_0, z_1, x_0, y_1 \in \mathbb{Q}(i)[X]$ of the equations (10) and (11).

Lemma 14

Let the sequences (v_m) and (w_n) are given by (16) and (17), respectively. Then

$$v_{2m} \equiv z_0 \pmod{2c}, \quad v_{2m+1} \equiv sz_0 + cx_0 \pmod{2c},$$

$$w_{2n} \equiv z_1 \pmod{2c}, \quad w_{2n+1} \equiv tz_1 + cy_1 \pmod{2c}.$$

All possible initial terms

- Dujella and Jurasić [2010] described all possible relations between initial terms z_0 and z_1 of the reccuring sequences (v_m) and (w_n) in $\mathbb{C}[X]$.
- The main goal of this step is to determine all possible initial terms of the recurring sequences (v_m) and (w_n) for the extension of the Diophantine triple $\{a, b, c\}$ in $\mathbb{Q}(i)[X]$.

The possible cases are distinguished depending on the parity of the indices m, n.

Lemma 15

1. *If* $v_{2m} = w_{2n}$, *then either*
   1. $z_0 = z_1 = \pm 1$ *or*
   2. $z_0 = z_1 = \pm s$ *and* $\alpha = 0$,
   3. $z_0 = z_1 = \pm cr \pm st$ *and* $\alpha > 0,\ \ \alpha + \beta \leq \gamma \leq 2\alpha + \beta$,
2. *If* $v_{2m+1} = w_{2n}$, *then either*
   1. $(z_0, z_1) = \pm(1, s)$ *and* $\gamma \geq \alpha + 2\beta$ *or*
   2. $(z_0, z_1) = \pm(s, 1)$ *and* $\alpha = 0,\ \beta = \gamma$ *or*
   3. $(z_0, z_1) = (\pm t, \pm cr \pm st),\ \alpha = \beta,\ \gamma = 3\alpha$.
3. *If* $v_{2m} = w_{2n+1}$, *then*
   1. $(z_0, z_1) = \pm(t, 1)$ *and* $\gamma \geq \alpha + 2\beta$ *or*
   2. $(z_0, z_1) = (\pm s, \pm 1),\ \alpha = 0,\ \beta = \gamma$ *or*
   3. $(z_0, z_1) = \pm(1, 1),\ \alpha = 0,\ \beta = \gamma$ *or*
   4. $(z_0, z_1) = (\pm cr \pm st, \pm s)$ *and* $\alpha \geq 0,\ 2\alpha + \beta \leq \gamma \leq \alpha + 2\beta$

      (special case is $(z_0, z_1) = (\pm s, \pm s)$ *and* $\alpha = 0,\ \beta = \gamma$*).*
4. *If* $v_{2m+1} = w_{2n+1}$, *the following options can happen:*
   1. $(z_0, z_1) = \pm(1, cr + st)$ *and* $\gamma \leq 2\alpha + \beta$ *or*
   2. $(z_0, z_1) = \pm(cr + st, 1)$ *and* $\gamma \leq \alpha + 2\beta$ *or*
   3. $(z_0, z_1) = (\pm t, \pm s)$ *and* $\gamma \geq \alpha + 2\beta$.

Future Work

- The examination of all possibilities from the Lemma 15.
- We introduce some more very useful congruences that will help exclude some of the cases
- If we obtain that none of the possible initial terms introduced by Lemma 15 leads to an irregular Diophantine quadruple with $d = d'$, then we can conclude that every Diophantine quadruple in $\mathbb{Q}(i)[X]$ is regular.

Thank you for your attention!



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