Hecke symmetries and quantum vertex algebras

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action of $S_2 = \{ I, P\}$ on $\mathbb{C}^N \otimes \mathbb{C}^N$, $N \geq 2$:

$$I = \sum_{i,j} e_{ii} \otimes e_{jj}, \quad P = \sum_{i,j} e_{ij} \otimes e_{ji}$$

Yang R-matrix $R(u) = I - \frac{\hbar}{u} P$

YBE: $R_{12}(u) R_{13}(u+v) R_{23}(v) = \longleftrightarrow$

2000. Etingof, Kazhdan $\rightsquigarrow$ qVAs

$$\begin{array}{ccccccc} P & \rightsquigarrow & R(u) & \rightsquigarrow & DY(\mathfrak{gl}_N) & \rightsquigarrow & \mathcal{V}_c(\mathfrak{gl}_N) \text{ qVA} \\ & & & & \downarrow & \underset{\hbar \to 0}{\text{cl lim}} & \downarrow \\ & & & & U(\mathfrak{gl}_N[t]) & & V_c(\mathfrak{gl}_N) \text{ affine VA} \end{array}$$

Q: $\text{?} \sim \ldots \text{—\textquotedbl—} \ldots \rightsquigarrow \text{?}$

R .. a skew inv Hecke symmetry :

- br rel.: $R_{12} R_{23} R_{12} = R_{23} R_{12} R_{23}$ ✓
- $(R - e^{\hbar} I)(R + e^{-\hbar} I) = 0$ ✓
- $\exists \Psi$ s.t. $\operatorname{tr}_2 R_{12} \Psi_{23} = P$ ✓

S-locality:

$\forall n$ $\forall a,b \in V$ $\exists r \geq 0$ s.t

$$(z_1 - z_2)^r Y(z_1)(1 \otimes Y(z_2)) S(z_1 - z_2) a \otimes b$$
$$= (z_1 - z_2)^r Y(b, z_2) Y(a, z_1) \mod \hbar^n$$

for $S(z) : V \otimes V \to V \otimes V [[z^{\pm 1}]]$ ... YBE

Gurevich, Saponov, 2019:

	Yangians of RTT-type	Braided Yangians
$\bar{R}(X) = PR + \frac{e^{\hbar}-e^{-\hbar}}{1-X}\cdot X \cdot P$ ... mult.-YBE	$Y_{RTT}(\bar{R})$	$Y(\bar{R})$
$R(u) = \left(\bar{R}(x)\right)\Big\vert_{x=e^{-2u/a}}$ .. (addit.)-YBE	$Y_{RTT}(R)$	$Y(R)$

Algebra $Y^{+}_{RTT}(R)$:

<u>gen.</u> $t_{ij}^{(-r)}$, $i,j = 1,\dots,N$, $r \geq 1$

$\rightsquigarrow t^{+}_{ij}(u) = \sum_{r\geq 1} t_{ij}^{(-r)} u^{r-1}$

$\rightsquigarrow T^{+}(u) = \sum e_{ij} \otimes t^{+}_{ij}(u)$

<u>rel</u> $R_{12}(u-v)\, T^{+}_{13}(u)\, T^{+}_{23}(v) = \longleftrightarrow$

Thm. (SK. 2022)

(a) $\exists\, Y_{RTT}(R)$-action of lvl. $c \in \mathbb{C}$ on

$$\mathcal{V}_c(R) := Y^{+}_{RTT}(R)_{\hbar}.$$

(b) $\mathcal{V}_c(R)$ is $\hbar$-adic qVA st

$$Y(T^{+}(u), z) = T^{+}(z+u)\, T(z+u+\hbar c/2)^{-1}.$$

V .. VA / $\hbar$-adic qVA

W ... V-module / V-module (Li, '08) / ϕ-coord V-module (Li '11)

w. assoc. (Li, Tan, Wang, '09)

$\forall n \geq 0 \quad \forall u, v \in V \quad \exists r \geq 0$ s.t.

I $\quad \underbrace{(z_1 - z_2)^r Y_W(u, z_1) Y_W(v, z_2)} \in \mathrm{Hom}(W, W((z_1, z_2)))$ mod $\hbar^n$

II $\quad \big(\underbrace{\qquad}_{\text{mod } \hbar^n}\big)\Big|_{\underbrace{z_1 = z_2 + z_0}} = \underbrace{z_0^r} Y_W(Y(u, z_0)v, z_2)$ mod $\hbar^n$

$\rightsquigarrow z_1 = \phi(z_2, z_0) \qquad \rightsquigarrow (\phi(z_2, z_0) - z_2)^r$

$\phi(z_2, z_0) \in \mathbb{C}((z_2))[[z_0]]$ s.t.

- $\phi(z_2, 0) = z_2$
- $\phi(\phi(z_2, z_0), z) = \phi(z_2, z + z_0)$

VAs .. $\dfrac{1}{(z_1 - z_2)^r}$ ✓

$\hbar$-qVAs .. $\dfrac{1}{(z_1 - z_2 + b\hbar)^r} = \sum_{\ell \geq 0} \binom{-r}{\ell} (z_1 - z_2)^{-r-\ell} (b\hbar)^\ell$ ✓

... $\dfrac{1}{\left(1 - e^{b\hbar} \frac{x_1}{x_2}\right)^r}$?! $\xrightarrow[x_1 = e^{z_1}]{\text{subst}} \dfrac{1}{(z_1 - z_2 + b\hbar)^r}$ · T-series ✓

Examples

- $\phi(z_2, z_0) = z_2 + z_0 \rightsquigarrow V$-module
- $\phi(z_2, z_0) = z_2 e^{-2z_0/a}$ ✓

Thm. (S.K, 2022)

(a) $\mathcal{M}^c(R) = Y^+(R)_\hbar$ ... $\mathcal{V}_c(R)$-module

(b) $\mathcal{M}^c_{RTT}(\bar{R}) = Y^+_{RTT}(\bar{R})_\hbar$ ... ϕ-coord. $\mathcal{V}_c(R)$-module
for $\phi(z_2, z_0) = z_2 = e^{-2z_0/a}$

(c) $\mathcal{M}^c_{RTT}(R) = Y^+_{RTT}(R)_\hbar$ ... — // —

- R of brank $(M|0)$
- $\overset{R}{\mathrm{tr}}\, A = \mathrm{tr}\,(A \cdot \mathrm{tr}_2 \Psi)$
- $P^{(k+1)}_{1\cdots k+1} \sim \hat{R}_{12}(e^{2\hbar})\hat{R}_{23}(e^{4\hbar})\cdots\hat{R}_{k\,k+1}(e^{2k\hbar})P^{(k)}_{1\cdots k}$
  & $P^{(1)} = I$ (skew-symmetrizer)
- $\hat{R}(x) = P\bar{R}(x)$

notation

(d) $\mathrm{qdet}\, T^+(u) := \overset{R}{\mathrm{tr}}_{1,\dots,M}\, P^{(M)} \overset{+}{T}_{1\,M+1}(u)\, \overset{+}{T}_{2\,M+1}(u+a\hbar) \cdots \overset{+}{T}_{M\,M+1}(u+a(M-1)\hbar)\,\mathbb{1} \in \mathfrak{Z}(\mathcal{V}_c(R))[[u]]$

only for certain Hecke symmetries R

(e) For $M = \mathcal{M}^c(R), \mathcal{M}^c_{RTT}(\bar{R}), \mathcal{M}^c_{RTT}(R)$

$$\mathrm{qdet}_M := Y_M(\mathrm{qdet}\, T^+(0), u)\mathbb{1}_M \in \mathfrak{Z}(M)[[u]].$$

— Thank you —