On K-types of irreducible representations of $SU(2,2)$

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$G = SU(2,2)$

$$G = SU(2,2) = \{g \in SL(4,\mathbb{C}) \mid g^* I_{2,2} g = I_{2,2}\}$$

$(\cdot)^*$ is conjugate transpose

$$I_{2,2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\mathfrak{g}_0 = \mathfrak{s}u(2,2) = \{X \in \mathfrak{s}l(4,\mathbb{C}) \mid X^* I_{2,2} + I_{2,2} X = 0\}$$

$$\mathfrak{g}_0 = \begin{bmatrix} Y & Z \\ Z^* & T \end{bmatrix}, \; Y, T \in \mathfrak{u}(2), \operatorname{tr} Y + \operatorname{tr} T = 0, \; Z \in \mathfrak{g}l(2,\mathbb{C})$$

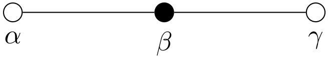
$$\mathfrak{g} = (\mathfrak{g}_0)^{\mathbb{C}} = \mathfrak{s}l(4,\mathbb{C})$$

$$\mathfrak{k}_0 = \mathfrak{s}u(2) \oplus \mathfrak{s}u(2) \oplus \mathbb{R}$$

$$\mathfrak{k} = (\mathfrak{k}_0)^{\mathbb{C}} = \mathfrak{s}l(2,C) \oplus \mathfrak{s}l(2,C) \oplus \mathbb{C}$$

Roots

Simple roots: α, β, γ, $\alpha + \beta$, $\beta + \gamma$, $\alpha + \beta + \gamma$



Compact roots: α, γ

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The unitary dual of $SU(2,2)$ is already found – [KnappSpeh1982].
We want to find the unitary dual of $SU(2,2)$ using $(\mathfrak{g}, K)$-modules.
We hope that it will lead us to unitary dual of some other groups.

We analyze finitedimensional modules.

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$V_{n,k,m}$, the highest weight is $v_{n,k,m}$,

$$H_\alpha.v_{n,k,m} = nv_{n,k,m},\ H_\beta.v_{n,k,m} = kv_{n,k.m},\ H_\gamma.v_{n,k,m} = mv_{n,k,m},\ \lambda = (n,k,m)$$

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$$H_\alpha . v_{n,k,m} = n v_{n,k,m}, \; H_\beta . v_{n,k,m} = k v_{n,k.m}, \; H_\gamma . v_{n,k,m} = m v_{n,k,m}, \; \lambda = (n, k, m)$$

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By their highest weight vectors.

$v_{p,r,q}$.

We need modified operators.

$$A_{\alpha+\beta+\gamma} = X_{\alpha+\beta+\gamma}$$
$$B_\beta = Y_\beta$$
$$A_{\alpha+\beta} = X_{\alpha+\beta}(H_\gamma + 1) + Y_\gamma X_{\alpha+\beta+\gamma}$$
$$A_{\beta+\gamma} = X_{\beta+\gamma}(H_\alpha + 1) - Y_\gamma X_{\alpha+\beta+\gamma}$$
$$A_\beta = X_\beta(H_\alpha + 1)(H_\gamma + 1) - Y_\alpha X_{\alpha+\beta}(H_\gamma + 1) + Y_\gamma X_{\beta+\gamma}(H_\alpha + 1) - Y_\alpha Y_\gamma X_{\alpha+\beta+\gamma}$$
$$B_{\alpha+\beta} = Y_{\alpha+\beta}(H_\alpha + 1) + Y_\alpha Y_\beta$$
$$B_{\beta+\gamma} = Y_{\beta+\gamma}(H_\gamma + 1) - Y_\gamma Y_\beta$$
$$B_{\alpha+\beta+\gamma} = Y_{\alpha+\beta+\gamma}(H_\alpha + 1)(H_\gamma + 1) + Y_\alpha Y_{\beta+\gamma}(H_\gamma + 1) - Y_\gamma Y_{\alpha+\beta}(H_\alpha + 1) - Y_\alpha Y_\gamma Y_\beta$$

Proposition

Operators As and Bs send highest weight vectors to highest weight vectors

Proof

The vector v is the highest weight vector of some K-type if and only if it satisfies $X_\alpha.v=0$ and $X_\gamma.v=0$. Now, let v be the highest weight vector of some K-type. Then

$$X_\alpha.(A_{\alpha+\beta+\gamma}.v)=X_\alpha.(X_{\alpha+\beta+\gamma}.v)=X_{\alpha+\beta+\gamma}.(X_\alpha.v)+[X_\alpha,X_{\alpha+\beta+\gamma}].v=0$$

since $X_\alpha.v=0$ and $[X_\alpha,X_{\alpha+\beta+\gamma}]=0$. Similarly, $X_\gamma.(A_{\alpha+\beta+\gamma}.v)=0$.
Similarly, $B_\beta.v$ is the highest weight vector. Also,

$$X_\alpha.(A_{\alpha+\beta}.v)=X_\alpha.((X_{\alpha+\beta}(H_\gamma+1)+Y_\gamma X_{\alpha+\beta+\gamma}).v)=(X_{\alpha+\beta}(H_\gamma+1)+Y_\gamma X_{\alpha+\beta+\gamma}).(X_\alpha.v)=0.$$

Now,

$$X_\gamma.(A_{\alpha+\beta}.v)=X_\gamma.((X_{\alpha+\beta}(H_\gamma+1)+Y_\gamma X_{\alpha+\beta+\gamma}).v)=$$

$$=(X_{\alpha+\beta}X_\gamma(H_\gamma+1)-X_{\alpha+\beta+\gamma}(H_\gamma+1)+(H_\gamma+Y_\gamma X_\gamma)X_{\alpha+\beta+\gamma}).v=0$$

since $X_{\alpha+\beta}X_\gamma(H_\gamma+1).v=0$, $Y_\gamma X_\gamma X_{\alpha+\beta+\gamma}.v=0$ and

$$H_\gamma X_{\alpha+\beta+\gamma}=X_{\alpha+\beta+\gamma}(H_\gamma+1)\ \ ([H_\gamma,X_{\alpha+\beta+\gamma}]=X_{\alpha+\beta+\gamma}).$$

The proof is not complete, but, it is clear how to finish it.
For example,

$$X_\gamma.(B_{\alpha+\beta+\gamma}.v) =$$

$$= X_\gamma.((Y_{\alpha+\beta+\gamma}(H_\alpha+1)(H_\gamma+1) + Y_\alpha Y_{\beta+\gamma}(H_\gamma+1) - Y_\gamma Y_{\alpha+\beta}(H_\alpha+1) - Y_\alpha Y_\gamma Y_\beta).v) =$$

$$(Y_{\alpha+\beta}(H_\alpha+1)(H_\gamma+1) + Y_\alpha Y_\beta(H_\gamma+1) - H_\gamma Y_{\alpha+\beta}(H_\alpha+1) - Y_\alpha H_\gamma Y_\beta).v = 0$$

Again, we used that

$$H_\gamma X_{\alpha+\beta+\gamma} = X_{\alpha+\beta+\gamma}(H_\gamma+1).$$

What do we have?

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Operators $X_{\alpha+\beta}, \ldots$ send the highest weight vector v to the highest weight of another K-type and "something" from the third K-type, and we want to "clean" that "something".

$V_{3,2,1}$

Example

$V_{3,2,1}$ means that $v_{3,2,1}$ is the highest weight vector and

$$H_\alpha.v_{3,2,1} = 3v_{3,2,1},\ H_\beta.v_{3,2,1} = 3v_{3,2.1},\ H_\gamma.v_{3,2,1} = 3v_{3,2,1},\ \lambda = (3,2,1)$$

dim $V_{3,2,1} = 630$
The number of K-types is 57.

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Freudenthal's formula produces weights. Then computer calculates K-types.

$V_{3,2,1}$

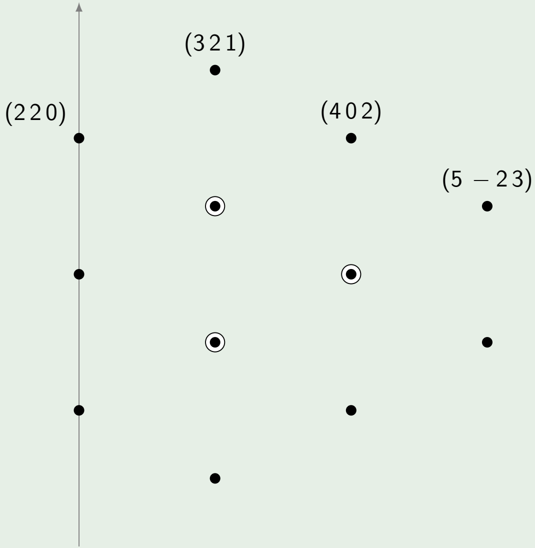
Example

We obtain

0	0	1	0	0	0	0
0	2	0	2	0	0	0
3	0	4	0	2	0	0
0	6	0	4	0	**2**	0
3	0	6	0	**4**	0	1
0	3	0	**6**	0	2	0
0	0	**3**	0	3	0	0

Numbers in table denote number of K-types.
Rows and columns go from 0.
Bolded 3 means 3 K-types with the highest weight $(2, 0)$.

Example



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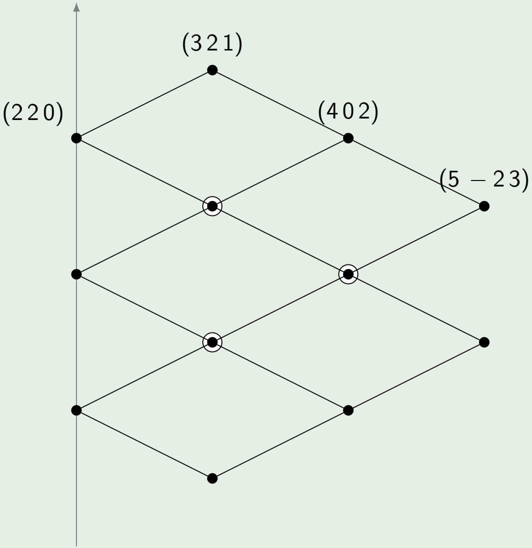

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Let us consider operators

$$Q = X_{\alpha+\beta} X_{\beta+\gamma} - X_{\beta} X_{\alpha+\beta+\gamma}, \quad R = Y_{\alpha+\beta} Y_{\beta+\gamma} - Y_{\beta} Y_{\alpha+\beta+\gamma}$$

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The vector v is the highest weight vector of some K-type if and only if it satisfies $X_\alpha.v = 0$ and $X_\gamma.v = 0$. Now, let v be the highest weight vector of some K-type. Then

$$X_\alpha.(Q.v) = X_\alpha.((X_{\alpha+\beta}X_{\beta+\gamma} - X_\beta X_{\alpha+\beta+\gamma}).v) = (X_{\alpha+\beta}X_{\alpha+\beta+\gamma} - X_{\alpha+\beta}X_{\alpha+\beta+\gamma}).v = 0$$

$$X_\gamma.(Q.v) = X_\gamma.((X_{\alpha+\beta}X_{\beta+\gamma} - X_\beta X_{\alpha+\beta+\gamma}).v) = (-X_{\alpha+\beta+\gamma}X_{\beta+\gamma} + X_{\beta+\gamma}X_{\alpha+\beta+\gamma}).v = 0$$

$$X_\alpha.(R.v) = X_\alpha.((Y_{\alpha+\beta}Y_{\beta+\gamma} - Y_\beta Y_{\alpha+\beta+\gamma}).v) = (-Y_\beta Y_{\beta+\gamma} + Y_\beta Y_{\beta+\gamma}).v = 0$$

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Remark

Actually, $[X_\alpha, Q] = [X_\gamma, Q] = [X_\alpha, R] = [X_\gamma, R] = 0$.

Commutation relations

They satisfy

$$[A_\beta, R] = B_{\alpha+\beta+\gamma}(1 - H_\beta), \;\; [A_{\alpha+\beta+\gamma}, R] = -B_\beta(H_\alpha + H_\beta + H_\gamma),$$

$$[B_\beta, Q] = A_{\alpha+\beta+\gamma}(H_\beta - 1), \;\; [B_{\alpha+\beta+\gamma}, Q] = A_\beta(H_\alpha + H_\beta + H_\gamma),$$

$$[A_{\alpha+\beta+\gamma}, A_\beta] = Q(H_\alpha + H_\gamma + 2), \;\; [B_\beta, B_{\alpha+\beta+\gamma}] = R(H_\alpha + H_\gamma + 2),$$

$$[A_\beta, Q] = [A_{\alpha+\beta+\gamma}, Q] = [B_\beta, R] = [B_{\alpha+\beta+\gamma}, R] = [A_\beta, B_{\alpha+\beta+\gamma}] = [B_\beta, A_{\alpha+\beta+\gamma}] = 0,$$

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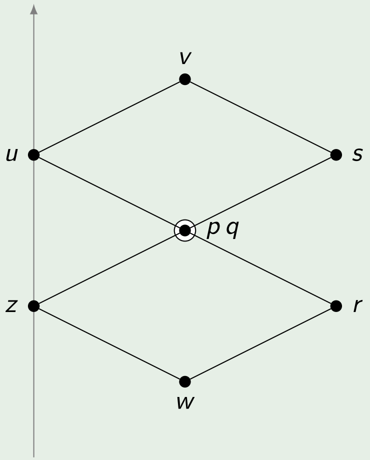
Let us consider $V_{1,1,1}$.

Example

- $u \to B_{\alpha+\beta+\gamma}.v$
- $p \to B_{\beta}.u$
- $z \to A_{\beta}.w$
- $q \to A_{\alpha+\beta+\gamma}.z$
- $R.v \to \frac{2}{5}p - \frac{3}{5}q$
- $Q.w \to -\frac{3}{5}p + \frac{2}{3}q$



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Multiplicities of all K-types are 1 ($m = 0$).

We do not want to construct a basis. We calculate expressions

$$A_\beta B_\beta, A_{\alpha+\beta} B_{\alpha+\beta}, A_{\beta+\gamma} B_{\beta+\gamma}, A_{\alpha+\beta+\gamma} B_{\alpha+\beta+\gamma}$$

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If the multiplicity is 1, these expressions are numbers.

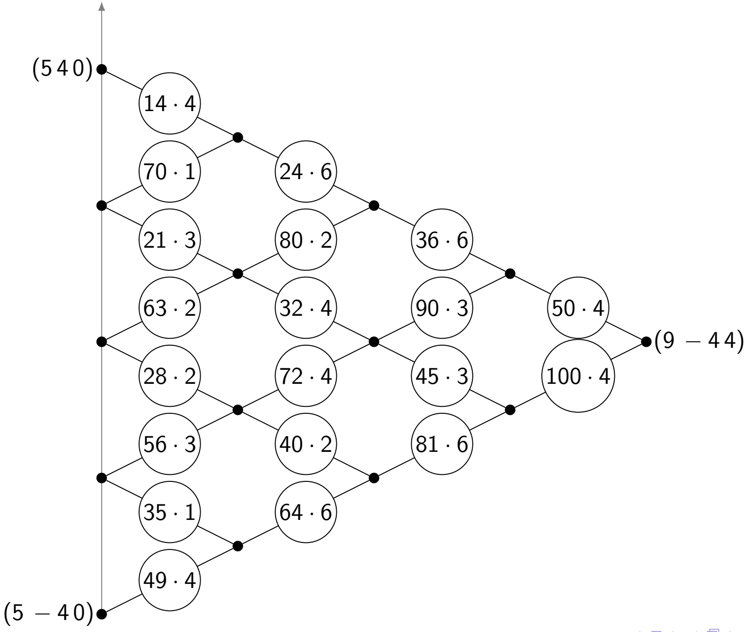
They are given in the following example.

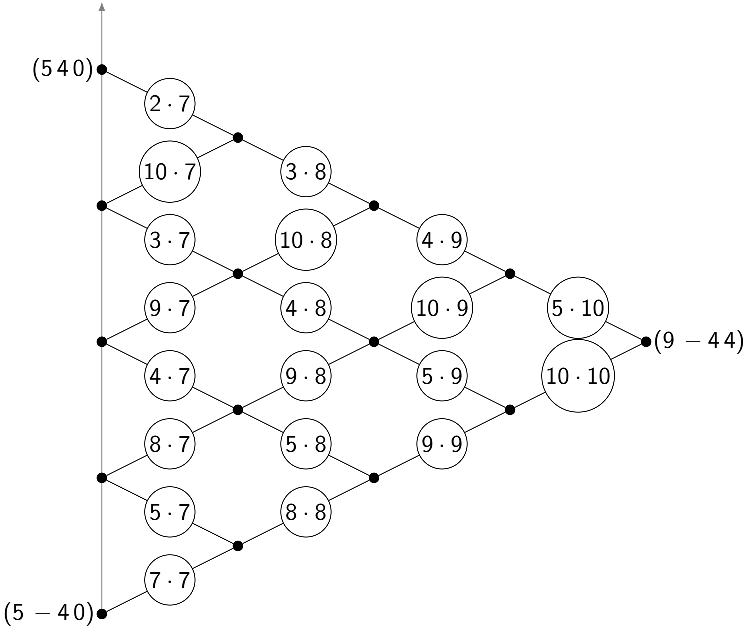
$V_{5,4,0}$

Example

0	0	0	0	1	0	0	0	0	0
0	0	0	2	0	1	0	0	0	0
0	0	3	0	2	0	1	0	0	0
0	4	0	3	0	2	0	1	0	0
5	0	4	0	3	0	2	0	1	0
0	5	0	4	0	3	0	2	0	**1**
0	0	5	0	4	0	3	0	**2**	0
0	0	0	5	0	4	0	**3**	0	0
0	0	0	0	5	0	**4**	0	0	0
0	0	0	0	0	**5**	0	0	0	0

$m = 0$ ensures that all multiplicities are equal to 1.


(5 4 0)
14 · 4
70 · 1
24 · 6
21 · 3
80 · 2
36 · 6
63 · 2
32 · 4
90 · 3
50 · 4
(9 − 4 4)
28 · 2
72 · 4
45 · 3
100 · 4
56 · 3
40 · 2
81 · 6
35 · 1
64 · 6
49 · 4
(5 − 4 0)


(5 4 0)
2 · 7
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(9 − 4 4)
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7 · 7
(5 − 4 0)

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We have

$$[B_{\alpha+\beta+\gamma}, A_{\alpha+\beta+\gamma}] + [Q, R] =$$

$$\frac{1}{8}\left(c - 3(H_\alpha + H_\gamma + 2)^2 - 4(H_{\alpha+H_\beta})(H_\beta + H_\gamma)\right)(H_\alpha + H_\gamma + H_\gamma + 1)$$

and

$$[B_\beta, A_\beta] + [Q, R] =$$

$$\frac{1}{8}\left(c - 3(H_\alpha + H_\gamma + 2)^2 - 4(H_\alpha + H_\beta)(H_\beta + H_\gamma)\right)(H_\beta - 1)$$

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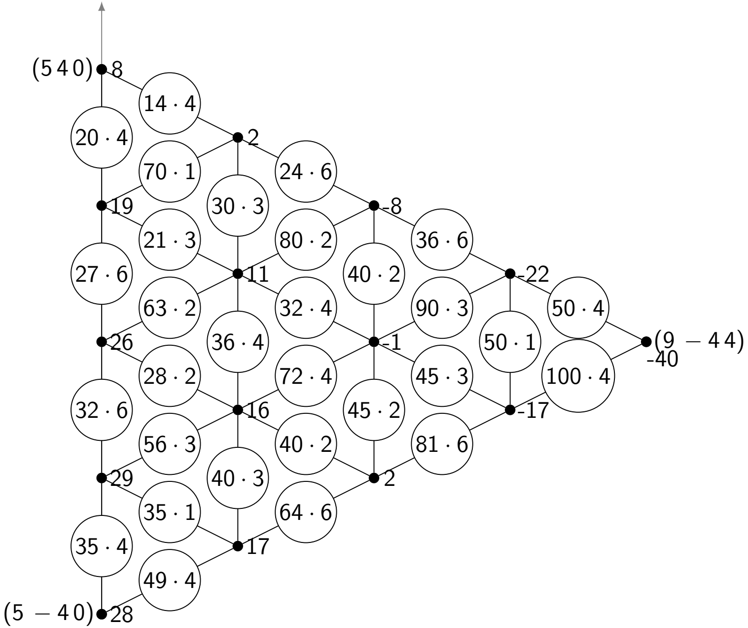
In our example, $c = 355$.


(5 4 0)
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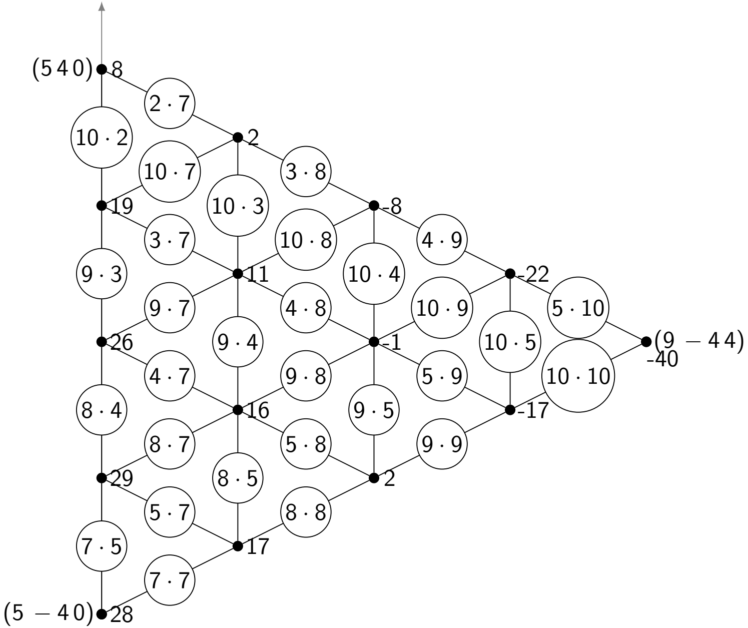

(5 4 0)
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-40
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7 · 7
(5 − 4 0)
28

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Irreducible Unitary Representations of $SU(2,2)$,
Journal of Functional Analysis **45**, 41-73 (1982)

Thank you!