REPRESENTATION THEORY XIII

IUC, Dubrovnik, Croatia, June 21–27, 2013

ABSTRACTS OF TALKS

On orbifold subalgebras of triplet vertex algebras

Dražen Adamović, University of Zagreb, Croatia

In this talk we shall present our results (and conjectures) on representation theory of certain vertex algebras realized as orbifold subalgebras of triplet vertex algebras important for logarithmic CFT. Particular emphasis will be put on type D–subalgebras, which can be treated as logarithmic generalizations of the vertex algebra V_L^+. This is joint work with Xianzu Lin and Antun Milas.

Degeneration of dynamical difference equations

Martina Balagović, University of York, UK

I will describe how, under Drinfelds degeneration of quantum loop algebras to Yangians, the trigonometric dynamical difference equations for the quantum affine algebra degenerate to the trigonometric Casimir differential equations for Yangians. I will explain all necessary prerequisites and results needed to state this claim precisely, and sketch the idea of the proof.

Permutation twisted modules for tensor product vertex operator superalgebras

Katrina Barron, University of Notre Dame, USA

We construct permutation twisted modules for tensor product vertex operator superalgebras for permutations in the alternating group, extending previous work by the author along with Dong and Mason. We discuss the obstructions to obtaining twisted modules in other cases such as those for an odd permutation which have important applications to mirror twisted modules for N=2 supersymmetric vertex operator superalgebras. We discuss further implications of these constructions.

An efficient algorithm for the energy function in type B

Carly Briggs, University at Albany, USA

Crystals are colored directed graphs encoding information about Lie algebra representations. Certain crystals for affine Lie algebras, called Kirillov-Reshetikhin (KR) crystals, are graded by the energy function, which is defined recursively. It is desirable to calculate the energy of a vertex using only data associated with that vertex. In recent work, C. Lenart, S. Naito, D. Sagaki, A. Schilling, and M. Shimozono gave such a formula, which is uniform across Lie types, based on the so-called quantum alcove model. In types A and C, this formula translates into a more efficient one, in terms of so-called Kashiwara-Nakashima columns. In this talk I will discuss partial results concerning the ex- tension of this approach to type B.

Quasi-particle bases of principal subspaces for affine Lie algebra of type $B_2^{(1)}$

Marijana Butorac, University of Rijeka, Croatia

We consider principal subspaces $W_{L(k\Lambda_0)}$ and $W_{N(k\Lambda_0)}$ of standard module $L(k\Lambda_0)$ and generalized Verma module $N(k\Lambda_0)$ at level $k \geq 1$ for affine Lie algebra of type $B_2^{(1)}$. By using the theory of vertex operator algebras, we find combinatorial bases of principal subspaces in terms of quasi-particles. From quasi-particle bases, we obtain character formulas for $W_{L(k\Lambda_0)}$ and $W_{N(k\Lambda_0)}$.

Vertex-algebraic structure of principal subspaces of standard $A_2^{(2)}$–modules

Corina Calinescu, City Tech, CUNY, USA

The principal subspaces of standard modules for untwisted affine Lie algebras, introduced by Feigin and Stoyanovsky, have been further developed by several authors from different standpoints. Extending our work on principal subspaces of level one standard modules for the untwisted affine Lie algebras of types A, D and E, we introduce and discuss the principal subspaces of certain standard modules for the twisted affine Lie algebra $A_2^{(2)}$. Our approach is based on vertex operator algebra theory.

This is joint work with J. Lepowsky and A. Milas.

The Dirac operator and discrete series for affine Hecke algebras

Dan Ciubotaru, University of Utah, Salt Lake City, U.S.A.

I will present the definition and main properties of a Dirac operator for the graded affine Hecke algebras which arise in the representation theory of reductive p-adic groups. As an application, I will give an L^2 construction of the discrete series of such Hecke algebras.

The talk is based on joint work with D. Barbasch, E. Opdam, and P. Trapa.

On quasi-reductive Lie algebras

Michel Duflo, University of Paris VII, France

By definition, a finite dimensional Lie algebra L is quasi-reductive if the generic stabilizer (in the adjoint Lie algebra) of the coadjoint representation is a diagonalizable subalgebra of End(L). I will present recent results on some classes of quasi-reductive Lie subalbegras of simple Lie algebra.

Explicit formulas for singular vectors in Verma modules

Thomas Edlund, Lund University, Sweden

A way to interpret monomials with complex exponents in the universal enveloping algebra $\mathcal{U}(\mathfrak{g})$ of a Lie algebra $\mathfrak{g}$, has been introduced in an article by F. G. Malikov, B. L. Feigin and D. B. Fuchs. In the context of symmetrizable Kac–Moody algebras, they show that if the exponents are suitably chosen, the resulting expressions give rise to singular vectors in Verma modules. In this talk I will present a rigorous setting, in which these ideas are realized in a different manner. The construction includes Ore localization in $\mathcal{U}(\mathfrak{g})$ and the introduction of certain conjugation automorphisms.

Applications of vertex operator constructions and character theory to branching problems for level one representations of affine Kac-Moody algebras

Alex J Feingold, Binghamton University, USA

In 1991 Feingold, Frenkel and Ries gave a spinor construction of the vertex operator para-algebra $V = V^0 \oplus V^1 \oplus V^2 \oplus V^3$, whose summands are 4 level-1 irreducible representations (irreps) of the affine Kac−Moody (KM) algebra $D_4^{(1)}$. The triality group $S_3 = \langle \sigma, \tau \mid \sigma^3 = 1 = \tau^2, \tau\sigma\tau = \sigma^{-1} \rangle$ in $Aut(V)$ was constructed, preserving V^0 and permuting V^i, $i = 1, 2, 3$. V is $\frac{1}{2}\mathbb{Z}$−graded and V_n^i denotes the n-graded subspace of V^i. Vertex operators $Y(v, z)$ for $v \in V_1^0$ represent $D_4^{(1)}$ on V, while those for which $\sigma(v) = v$ represent $G_2^{(1)}$. V decomposes into the direct sum of $G_2^{(1)}$ irreps by a two-step process, first decomposing with respect to the intermediate algebra $B_3^{(1)}$ represented by $Y(v, z)$ for $\tau(v) = v$. There are three vertex operators, $Y(\omega_{D_4}, z)$, $Y(\omega_{B_3}, z)$, $Y(\omega_{G_2}, z)$, each representing the Virasoro algebra given by the Sugawara constructions from the three algebras. These give two coset Virasoro constructions, $Y(\omega_{D_4} - \omega_{B_3}, z)$ and $Y(\omega_{B_3} - \omega_{G_2}, z)$, with central charges 1/2 and 7/10, respectively, the first commuting with $B_3^{(1)}$, the second commuting with $G_2^{(1)}$, and each commuting with the other. This gives the space of highest weight vectors for $G_2^{(1)}$ in V as sums of tensor products of irreducible Virasoro modules $L(1/2, h_1) \otimes L(7/10, h_2)$.

The dissertation research of my student, Quincy Loney, explicitly constructs these coset Virasoro operators, and uses them to study the decomposition of V with respect to $G_2^{(1)}$. The character theory of affine KM algebras and of the Virasoro algebra, is used to prove that the summands explicitly found fill V, providing the complete decomposition. This requires using one of the forty identities among the Rogers−Ramanujan (RR) series found by Ramanujan in his famous notebook.

The dissertation research of another student, Christopher Mauriello, carries out a similar project for the decomposition of the three level-1 irreps of $E_6^{(1)}$ with respect to its subalgebra $F_4^{(1)}$ of fixed points under the order 2 Dynkin diagram automorphism. That investigation uses the bosonic lattice construction, and gives the highest weight vectors for $F_4^{(1)}$ as sums of Virasoro modules $L(4/5, h)$ which form a W−algebra and modules for it. A different one of the forty identities for the RR series is involved in showing that the summands found by explicit vertex operator computations provide the complete decomposition.

This is joint work with Quincy Loney and Christopher Mauriello.

Dirac cohomology and generalization of classical branching rules

Jing-Song Huang, The Hong Kong University of Science and Technology, China

We will discuss the branching rules that generalize the Littlewood restriction formulae. Our formulae are inspired by the work of Enright and Willenbring and in terms of Dirac cohomology.

This is joint work with Pavle Pandžić and Fuhai Zhu.

The importance of mathematical theorems and conceptual understanding in the representation theory of vertex operator algebras

Yi-Zhi Huang, Rutgers University, USA

For the last twenty years, two very popular opinions have been widely accepted as the bases of the standard for evaluating mathematical research in the representation theory of vertex operator algebras: One states that the unpublished work of Beilinson-Feigin-Mazur had constructed the WZNW models and the minimal models (including, in particular, the modular tensor category structures) and that exactly the same method could be applied to all the other vertex operator algebras corresponding to rational conformal field theories. The other states that Zhu's theorem on the modular invariance of the q-traces of products of vertex operators for modules for a suitable vertex operator algebra had reduced the representation theory of vertex operator algebras satisfying suitable conditions, or equivalently, the study of rational conformal field theories, to the study of suitable finiteness conditions on the vertex operator algebra and complete reducibility conditions on its module category.

After around twenty years of research (some of it jointly with Lepowsky and some jointly with Kong), under strong objections from people with these opinions, I finally proved, a few years ago, the main fundamental conjectures made in the pioneering work of Moore and Seiberg, including, in particular, those on the operator product expansion and those on modular invariance. In this talk, I will discuss the importance of these mathematical theorems and the importance of the conceptual understanding that these results and the methods used have brought to us. In particular, I will use examples (including, in particular, the rigidity of the tensor categories associated to the WZNW models and the minimal models, and the modular invariance of the q-traces of the products of all the intertwining operators) to show that the opinions mentioned above are, unfortunately, misunderstandings of the important works of Beilinson-Feigin-Mazur and Zhu and are therefore not correct, as is now being acknowledged by a number of people, so far.

Wedge modules for two-parameter quantum algebras

Naihuan Jing, North Carolina State University, USA

I will discuss the natural construction of all fundamental modules for two parameter quantum enveloping algebras using the fusion procedure.

Localization of Generalized Harish-Chandra Modules

Sarah Kitchen, University of Freiburg, Germany

Localization in representation theory is well known to be a powerful tool which allows us to apply geometric methods to representation theoretic problems. In this talk, I will discuss the localization of generalized Harish-Chandra Modules and a geometric approach to a conjecture of Penkov and Zuckerman.

Real forms of dual pairs $\mathfrak{g}_2 \times \mathfrak{h}$ in $\mathfrak{g}$ of type E_6, E_7 and E_8

Domagoj Kovačević, University of Zagreb, Croatia

Let $\mathfrak{g}$ be a complex Lie algebra of type E_6, E_7 or E_8 and let $\mathfrak{g}_2 \times \mathfrak{h}$ be a dual pair in $\mathfrak{g}$. We look for possible real forms of $\mathfrak{g}_2 \times \mathfrak{h}$. It turns out that for each $n(=6,7,8)$ and for all real forms, say $\mathfrak{a}_0 \times \mathfrak{h}_0$ of $\mathfrak{g}_2 \times \mathfrak{h}$, there exists a real form $\mathfrak{g}_0$ of $\mathfrak{g}$ such that $\mathfrak{a}_0 \times \mathfrak{h}_0$ embedds into $\mathfrak{g}_0$. The full description is given in Theorem 3.1.

Principal subspaces for quantum affine algebra $U_q(A_n^{(1)})$

Slaven Kožić, University of Zagreb, Croatia

We consider principal subspace $W(\Lambda)$ of irreducible highest weight module $L(\Lambda)$ for quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_{n+1})$. We introduce quantum analogues of the quasi-particles associated with the principal subspaces for $\widehat{\mathfrak{sl}}_{n+1}$ and discover certain relations among them. By using theese relations we find, for certain highest weight Λ, combinatorial bases of principal subspace $W(\Lambda)$ in terms of monomials of quantum quasi-particles.

Kirillov−Reshetikhin crystals, Macdonald polynomials, affine Demazure characters, and combinatorial models

Cristian Lenart, University at Albany, USA

In recent work with S. Naito, D. Sagaki, A. Schilling, and M. Shimozono, we show that in all untwisted affine types the specialization of a Macdonald polynomial at $t = 0$ is the graded character of a tensor product of one-column Kirillov-Reshetikhin (KR) modules. We also obtain two uniform models for the corresponding KR crystals, namely a generalization of the Lakshmibai-Seshadri (canonical Littelmann) paths based on the so-called parabolic quantum Bruhat graph, and the quantum alcove model of myself and A. Lubovsky. I will also mention other closely related topics: affine Demazure crystals (extending the work of Ion and Fourier-Littelmann), expressing the energy function, and a uniform realization of the combinatorial R-matrix, which commutes factors in a tensor product of KR crystals (with A. Lubovsky).

An introduction to logarithmic tensor category theory

James Lepowsky, Rutgers University, USA

In joint work with Yi-Zhi Huang and Lin Zhang, we have developed a "logarithmic tensor category theory" for suitable categories of generalized modules for a vertex operator algebra. I will motivate and sketch this theory in an introductory way. In the context of the analogy between Lie algebra theory and vertex operator algebra theory, logarithmic tensor category theory is the analogue of the representation theory of an arbitrary finite-dimensional Lie algebra in the sense that the earlier tensor category theory for suitable module categories for a vertex operator algebra developed with Huang was the analogue of the representation theory of a semisimple Lie algebra (for which all modules are completely reducible). Tensor category structure on the module category for a Lie algebra is of course trivial to construct, whether or not the Lie algebra is semisimple. In vertex operator algebra theory, logarithmic tensor category theory is much more elaborate than the earlier semisimple theory, but this is the natural generality for the theory. The constructed tensor categories are vertex tensor categories and in particular braided tensor categories.

Vertex algebraic structure in integral forms of standard affine Lie algebra modules

Robert McRae, Rutgers University, USA

Integral forms of the universal enveloping algebras of affine Lie algebras have been constructed by Garland and others, allowing the construction of integral forms in standard modules for affine Lie algebras. We show that these integral forms have the structure of vertex algebras over $\mathbb{Z}$, and we give sets of generators for this vertex algebraic structure.

Ramanujan-type formulas for $\frac{1}{\pi}$

Arne Meurman, Lund University, Sweden

In a paper 1914 S. Ramanujan stated 17 formulas for $\frac{1}{\pi}$, of which we give one example:

$$\sum_{n=0}^{\infty}(1+8n)\frac{(1/4)_n(1/2)_n(3/4)_n}{(n!)^3}\frac{1}{9^n}=\frac{2\sqrt{3}}{\pi}.$$

Here $(a)_n$ denotes the Pochhammer symbol

$$(a)_n = a(a+1)\cdots(a+n-1).$$

We present recent results of Z.-W. Sun, and G.Almkvist, A.Aycock concerning generalizations of Ramanujan's formulas.

Beyond C_2-cofinite vertex algebras

Antun Milas, University at Albany, USA

C_2-cofiniteness is an important internal property of vertex algebras which, among other things, guarantees modular invariance type property of generalized characters of modules. If we relax the C_2–cofiniteness condition not much can be said, unless our vertex algebra admits extra symmetries (eg. an affine Lie algebra action). In this talk I'll discuss examples of vertex algebras that are not C_2–cofinite and discuss their properties of characters. It is partially based on an ongoing joint project with T. Creutzig.

Kazhdan-Lusztig conjectures for Whittaker modules

Dragan Miličić, University of Utah, Salt Lake City, U.S.A.

We are going to discuss joint work with Wolfgang Soergel on geometrization of categories of Whittaker modules. Using a recent result of Mochizuki on decomposition theorem for direct images of irreducible holonomic modules under projective morphisms, we prove the analogue of Kazhdan-Lusztig conjectures in this situation.

Dirac cohomology for modules without infinitesimal character

Pavle Pandžić, University of Zagreb, Croatia

The usual definition of Dirac cohomology does not have good properties if the module in question does not have infinitesimal character but only generalized infinitesimal character. I will discuss alternative definitions which have better behavior in this case, and give the same notion for modules with infinitesimal character.

This is joint work with Petr Somberg.

Generalized Harish−Chandra modules

Ivan Penkov, Jacobs University Bremen, Germany

Generalized Harish−Chandra modules are $(\mathfrak{g},\mathfrak{k})-$modules of finite type for a not necessarily symmetric subalgebra $\mathfrak{k}\subset\mathfrak{g}$. In this talk I will state a classification result for generic simple generalized Harish-Chandra modules. I will illustrate the result in the case of $\mathfrak{sl}(2)-$subalgebras corresponding to a long root.

This is joint work with Gregg Zuckerman.

Some coset vertex algebras with central charge one

Ozren Perše, University of Zagreb, Croatia

In this talk, we study certain coset vertex algebras associated to affine vertex algebras of orthogonal and symplectic type. We show that these cosets are isomorphic to $\mathbb{Z}_2-$orbifold vertex algebras of lattice and Heisenberg vertex algebras. The construction is based on fermionic and bosonic realization of modules for associated affine Lie algebras, and certain conformal embeddings.

This is joint work with Dražen Adamović.

The $(\mathfrak{sp}(4),\mathfrak{sl}(2))-$modules of finite type

Alexey Petukhov, Moscow, Russia

A $(\mathfrak{g},\mathfrak{k})-$module of a generic pair of Lie algebras is a straightforward generalization of a $(\mathfrak{g},\mathfrak{k})-$module for a symmetric pair of Lie algebras, i.e. of Harish−Chandra modules. It turns out that to achieve a reasonable theory of $(\mathfrak{g},\mathfrak{k})-$modules one has to introduce additional condition(s) on them. One such a condition, being of finite type, was introduced by I. Penkov and G. Zuckerman and is a straightforward generalization of a well-known admissibility condition for Harish−Chandra modules.

If $\mathfrak{k}$ is a symmetric subalgebra of $\mathfrak{g}$, $\mathfrak{k}$ has finitely many orbits on the full flag variety of $\mathfrak{g}$ and this is one of the key points in the description of $(\mathfrak{g},\mathfrak{k})-$modules for symmetric pairs $(\mathfrak{g},\mathfrak{k})$, i.e. of Harish−Chandra modules. The smallest pair of semisimple Lie algebras for which this finitness condition fails (say, by dimension reasons) is $\mathfrak{k}=\mathfrak{sl}(2)$ in $\mathfrak{g}=\mathfrak{sp}(4)$.

In my talk I will provide some general facts about $(\mathfrak{g},\mathfrak{k})-$modules of finite type and show how using these facts one can achieve a description of $(\mathfrak{sp}(4),\mathfrak{sl}(2))-$modules (of finite type) via the SL(2)−geometry of the full flag variety of $\mathfrak{sp}(4)$.

Difference conditions in combinatorial bases of representations of affine Lie algebras

Mirko Primc, University of Zagreb, Croatia

In this talk I'll review some known constructions of combinatorial bases of standard level k modules of affine Lie algebras which are based on a defining relation

$$x_\theta(z)^{k+1}=0.$$

I'll also comment on a type of combinatorial difference conditions one should expect in general for higher rank algebras.

Dirac induction for the large discrete series of $\mathrm{SU}(2,1)$

Ana Prlić, University of Zagreb, Croatia

In a joint work, P. Pandžić and D. Renard proved that holomorphic and anti-holomorphic discrete series can be obtained using Dirac induction. The group $SU(2,1)$ except those two discrete series has also a third type, neither holomorphic nor anti-holomorphic, the so-called large discrete series. In this talk, we are going to show that large discrete series of the group $SU(2,1)$ can also be constructed via Dirac induction.

A geometric translation principle for Dirac operators

Nicolas Prudhon, Université Lorraine − Metz, France

We study the smooth kernel of the cubic Dirac operator on a homogeneous space G/H. First, I will show that tensoring by a finite dimensional representation of G sends „the unitary part of the kernel" to the kernel of the corresponding Dirac operator. Second, I will consider the quotient of the kernel by those elements that also are in its range. I will prove that this quotient has finitely many primary components under the action of the center of the universal Lie algebra of G.

Subsingular vectors in Verma modules, and tensor product of weight modules over the $W(2,2)$ algebra

Gordan Radobolja, University of Split

Lie algebra $W(2,2)$ was first introduced by W. Zhang and C. Dong in 2009. as a part of classification of simple vertex operator algebras generated by two weight two vectors. It is an extension of a well known Virasoro algebra Vir, and its representation theory is somewhat similar to that of Vir. Criterion for irreducibility of Verma modules over $W(2,2)$ was given by Zhang and Dong. In this talk we will show that subsingular vectors may exist in Verma modules over the $W(2,2)$ and will present a subquotient structure of these modules.
Furthermore, we will prove conditions for irreducibility of a tensor product of intermediate series module with a highest weight module. Relations to intertwining operators over the vertex operator algebra associated to $W(2,2)$ will be discussed.

Presentations of principal subspaces for the affine Lie algebra $A_2^{(1)}$

Christopher Sadowski, Rutgers University, USA

In this talk, we review some known results about principal subspaces of standard modules associated to affine Lie algebras. We consider principal subspaces associated to $A_2^{(1)}$ standard modules, and give a new set of presentations for these principal subspaces in terms of left ideals.

A rigidity property of the automorphic representations of adele groups

Hadi Salmasian, University of Ottawa, Canada

Let G be a classical $Q-$isotropic algebraic group and $G(A)$ be the group of adele$-$points of G. In the 1980's Roger Howe defined a notion of rank for irreducible unitary representations of $G(A)$ and its local components $G(R)$ and $G(Q_p)$. Among many other results, he proved that for an automorphic representation of $G(A)$ all of these ranks are equal. The latter technique has found a number of applications, namely in the study of multiplicities of automorphic forms, Howe duality, etc. In this talk we extend the above rigidity lemma of Howe in a uniform and conceptual way to include exceptional groups. Our approach is based on the orbit method for nilpotent (real and $p-$adic) Lie groups. In the real case, one needs functional calculus on Lie groups, and in the $p-$adic case one needs to analyze representations of certain algebras of bi-invariant functions.

Nichols algebras as logarithmic CFTs

Alexei Semikhatov, Lebedev Physics Institute, Moscow, Russia

Finite-dimensional braided Hopf algebras of a certain class — so-called Nichols algebras — may be parameterizing the set of "good" logarithmic models of two-dimensional conformal field theory.

Construction of combinatorial bases for basic modules of affine Lie algebras of type $C_n^{(1)}$

Tomislav Šikić, University of Zagreb, Croatia

In a joint work, A. Meurman and M. Primc constructed a combinatorial bases of integrable highest weight modules for affine Lie algebra $A_1^{(1)}$. In this construction they used vertex operator algebra theory and combinatorial arguments. A "representation theory part" of that construction has been extended to all affine Lie algebras, whereas the "combinatorial part" remained to be an open problem for general affine Lie algebras. In this talk I'll describe the method to construct combinatorial bases for basic modules of affine Lie algebras of type $C_n^{(1)}$ obtained in a joint work with Mirko Primc. Special accent of this talk will be devoted to the combinatorial parametrization of leading terms of defining relations for all standard modules for affine Lie algebra of type $C_n^{(1)}$.

This is joint work with M. Primc.

TBA

David Vogan, Massachussetts Institute of Technology, Cambridge, USA

(2A,3A)-generated subalgebras and 3-transposition subgroups on a vertex operator algebra

Hiroshi Yamauchi, Tokyo Woman's Christian University, Japan

In this talk I will consider a series of subalgebras of a vertex operator algebra generated by simple $c = 1/2$ Virasoro vectors (which we will call Ising vectors) satisfying the $(2A, 3A)$-generated configuration. Then I will show that the corresponding commutant subalgebras afford actions of 3-transposition subgroups where transposition automorphisms are realized by several Virasoro vectors of unitary series. As an application, I will explain how the second largest Fischer 3-transposition group arises as an automorphism group of a vertex operator algebra.

Zhu's algebra and a new characterization of modules for a vertex operator algebra.

Jinwei Yang, Rutgers University, USA

It is known that associativity for vertex operators gives a formula which expresses products of components of vertex operators as linear combinations of iterates of components of vertex operators. On the other hand, the top level of a module gives a module for Zhu's algebra. In particular, the actions of the generators of the two-sided ideal defining Zhu's algebra on the top level are zero, giving us another formula. However, any one of these formulas cannot be used to replace the associativity as the main axiom for modules.

In this talk, I will discuss a recent result showing that these two formulas together are equivalent to associativity. Using these two formulas, we obtain a new construction of modules for a vertex operator algebra from modules for the corresponding Zhu's algebra. The advantage of this new construction is that it does not use the commutator formula and thus can be generalized to much more difficult situations in which the commutator formula does not hold anymore. For example, in a forthcoming paper jointly with Prof. Huang, these results will be generalized to the case of generalized twisted modules associated to general automorphisms in the sense of Huang.

Identities for modular forms generated by vertex operator superalgebras

Alexander Zuevsky, Academy of Science, Prague, Czech Republic

Recent results in construction of identities for modular forms in the vertex algebra approach are reviewed. We show how to introduce, calculate explicitly, and establish modular properties of the (twisted) partition and $n-$point correlation functions on sewn and self-sewn Riemann surfaces of genus $g \geq 1$ for vertex operator superalgebras (via generalized vertex algebra of intertwiners). Twisted elliptic functions, Szegö kernels, and genus two Jacobi product formulas naturally appear as applications.