Interuniversity Centre, Dubrovnik, June 19 – 26, 2011

REPRESENTATION THEORY XII

ABSTRACTS

On the structure of Zhu's algebras for certain $\mathcal{W}$–algebras

Dražen Adamović, University of Zagreb, Croatia

In this talk we shall study the Zhu's algebras for a family of $\mathcal{W}$–algebras which appear in logarithmic conformal field theory. We shall completely describe the structure of Zhu's algebra for triplet vertex algebra $\mathcal{W}(p)$, the $N = 1$ triplet vertex superalgebra $\mathcal{SW}(m)$ and the $c = 0$ triplet vertex algebra $\mathcal{W}_{2,3}$. The existence of logarithmic representations is obtained from the structure of Zhu's algebra. We will also study the associated C_2–algebras. This talk is based on a joint work with Antun Milas.

Highest weight modules over pre-exp-polynomial Lie algebras

Punita Batra, Harish Chandra Research Institute, Allahabad, India

I will define pre-exp-polynomial Lie algebras and show that the non-graded and graded irreducible highest weight modules with the same highest weight simultaneously have all finite dimensional weight spaces if and only if the highest weight is an exp-polynomial highest weight.

Geometric structure in the representation theory of reductive p-adic groups

Paul Baum, Penn State University, USA

Let G be a reductive p-adic group. Examples are GL(n, F) SL(n, F) where n can be any positive integer and F can be any finite extension of the field Q_p of p-adic numbers. The smooth (or admissible) dual of G is the set of equivalence classes of smooth irreducible representations of G. The representations are on vector spaces over the complex numbers. The smooth dual has one point for each distinct smooth irreducible representation of G. Within the smooth dual there are subsets known as the Bernstein components, and the smooth dual is the disjoint union of the Bernstein components. This talk will explain a conjecture due to Aubert-Baum-Plymen (ABP) which says that each Bernstein component is a complex affine variety. These affine varieties are explicitly identified as certain extended quotients. The infinitesimal character of Bernstein and the L-packets which appear in the local Langlands conjecture are then described from this point of view. Recent results by a number of mathematicians (e.g. V. Heiermann, M. Solleveld) provide positive evidence for ABP.

Morita equivalence revisited

Paul Baum, Penn State University, USA

Let X be a complex affine variety and let k be its coordinate algebra. Equivalently, k is a unital algebra over the complex numbers which is commutative, finitely generated, and nilpotent-free. A k-algebra is an algebra A over the complex numbers which is a k-module (with an evident compatibility between the algebra structure of A and the k-module structure of A). A is of finite type if as a k-module it is finitely generated. This talk will review Morita equivalence for k-algebras and will then introduce a weakening of Morita equivalence called geometric equivalence. This new kind of equvalence preserves the primitive ideal space and the periodic cyclic homology. However, the new equivalence permits a tearing apart of strata in the primitive ideal space which is not allowed by Morita equivalence. The ABP conjecture asserts that the finite type algebra which Bernstein constructs for any given Bernstein component of a reductive p-adic group is geometrically equivalent to the coordinate algebra of the associated extended quotient — and that the geometric equivalence can be chosen such that the resulting bijection between the Bernstein component and and the extended quotient has properties as in the statement of ABP.

Frobenius Lie subalgebras of simple Lie algebras

Michel Duflo, Universit Paris 7, France

A Frobenius Lie algebra is a Lie algebra for which the coadjoint action has an open orbit. I present results on Frobenius Lie subalgebras of a simple complex Lie algebra which contain a Cartan subalgebra, with a special emphasis on the "Ooms spectrum" (the Ooms spectrum is an equivalent for Frobenius Lie algebras of the set of exponents of a simple Lie algebra).

On representations of affine Kac-Moody algebras at the critical level

Peter Fiebig, Erlangen University, Germany

I want to report on a joint research project with Tomoyuki Arakawa (RIMS, Kyoto) on the category $\mathcal{O}$ associated to an affine Kac-Moody algebra at the critical level. In his 1990 ICM address Lusztig conjectured a link between critical level representations and the representation theory of modular Lie algebras and quantum groups. Compatible with this is the Feigin-Frenkel conjecture on the characters of simple critical highest weight representations. Both conjectures motivated our approach, whose main goal is to establish a functorial connection between intersection cohomology sheaves on affine Grassmannians and the critical level category $\mathcal{O}$. Our approach should be thought of as being Koszul-dual to the results of Frenkel and Gaitsgory.

On B-orbit decompositions of orbital varieties

Lucas Fresse, Hebrew University, Israel

Let G be a reductive algebraic group and B a Borel subgroup. We consider the intersection of a nilpotent (adjoint) orbit in $Lie(G)$ with the subalgebra $Lie(B)$. The irreducible components of this intersection are called orbital varieties and they arise in some problems in representation theory. In this talk, we study the (adjoint) action of B on this intersection. The main result presented is a necessary and sufficient condition for this action to have finitely many orbits, in the case of $G = GL(n)$.

Lie Group Representations and the Baum-Connes Conjecture

Nigel Higson, Penn State University, USA

I shall give an introduction to the Baum-Connes conjecture that I hope will be congenial to representation theorists. The conjecture has to do with group C*-algebras and topological K-theory, and because of that it might appear to be a bit remote from representation theory as representation theorists know it. So I'll start with compact Lie groups, where the connection between K-theory and representations was made clear by Raoul Bott a long time ago. I'll move from there to real reductive groups, and I'll try to argue that Baum-Connes carries Bott's K-theoretic view of the Weyl character formula (or, if you like, his representation-theoretic view of index theory on the flag variety) into the realm of noncompact groups. If there's time I'll conclude by describing a phenomenon first noticed by Mackey that is strongly suggested by, although not implied by, the Baum-Connes conjecture. Neither K-theory nor C*-algebras is involved here at all.

Dirac cohomology of highest weight modules

Jing-Song Huang, HKUST, Hong Kong, China

We show that Dirac cohomology coincides with nilpotent Lie algebra cohomology for simple highest weight modules up to a twist of one-dimensional representation. As a consequence we determine the Dirac cohomology of simple highest weight modules. This is a joint work with Wei Xiao.

The q-characters and blocks at roots of unity

Dijana Jakelić, University of North Carolina Wilmington, USA

The notion of q-characters, introduced by E. Frenkel and N. Reshetikhin in the context of finite-dimensional representations of quantum affine algebras, is a generalization of the notion of characters of finite-dimensional representations of simple Lie algebras. The q-characters have been studied extensively in the last 10 years using geometric, combinatorial, and representation-theoretic methods.

In this talk, we will focus on a joint work with A. Moura where we consider the q-characters in the roots of unity setting. We will review the basics on finite-dimensional representations of quantum affine algebras at roots of unity including the classification of the irreducible modules in terms of dominant ℓ-weights, the existence of the Weyl modules, and specialization of modules. We will then proceed to present results concerning the q-characters and the block decomposition of the underlying abelian category. It is known that the q-characters are not invariant under the action of the braid group in general. However, we will present a result saying that, if the underlying Lie algebra is of classical type, the q-characters of fundamental representations satisfy a certain invariance property with respect to the braid group action. Finally, we will give an example in type D_n describing how this result and the theory of specialization of modules can be used to obtain explicit formulae for the q-characters of fundamental representations in the root of unity setting.

Quasi-particle fermionic formulas for $(k, 2)-$admissible configurations

Miroslav Jerković, University of Zagreb, Croatia

In joint work with Mirko Primc, we establish the quasi-particle bases for Feigin-Stoyanovsky's type subspaces of standard $\tilde{\mathfrak{sl}(3, \mathbb{C})}-$modules, together with corresponding combinatorially constructed fermionic characters. It turns out that these formulas equal previously known formulas obtained by solving systems of recurrence relations for characters. In the end, we look at some new developments considering a different kind of quasi-particles and propose new fermionic-type character formulas.

Restrictions of Verma modules to symmetric pairs and some applications to differential geometry

Toshiyuki Kobayashi, University of Tokyo, Japan

I will discuss a "framework" of branching problems for generalized Verma modules with respect to reductive symmetric pairs from the viewpoint of "discrete decomposability", and explain some basic results on the size of irreducible summands and multiplicities. As an application, I plan to explain a new and simple method to obtain Cohen-Rankin operators for holomorphic automorphic forms and Juhl's conformally equivariant differential operators together with their generalizations.

Harmonic spinors, Dirac cohomology and representation theory

Salah Mehdi, University of Metz, France

We will discuss several aspects of representation theory of semisimple Lie groups involving harmonic spinors. In particular we will describe recent results with Pavle Pandžić on the behavior of Dirac cohomology under translation functors.

Screening operators and vertex algebras

Antun Milas, University at Albany (SUNY), USA

I will first define a family of vertex algebras coming from multiples of root lattices. Various facets of their representation theory will be discussed including explicit construction of (logarithmic) modules, classification of irreps and related combinatorial identities, computation of q-dimensions and modular invariance, etc. This talk is an overview of a joint project with D. Adamović.

Geometry and unitarity

Dragan Miličič, University of Utah, USA

$\mathcal{D}-$module theory allows us to approach a number of problems in representation theory of real reductive groups using methods of algebraic geometry. For a long time, all attempts to study the classification of unitary representations in this framework failed. Recently, some new ideas due to Vogan and his collaborators, and Schmid and Vilonen, enabled that this problem is put in that framework.

Characters and extensions of finite-dimensional representations of classical and quantum affine algebras

Adriano Moura, University of Campinas, Brazil

There are a few notions of characters which can be associated to a finite-dimensional representation of an affine Kac-Moody algebra or of its quantum group. The finer one, among the character theories we will be considering in this talk, is that of q-characters introduced by E. Frenkel and N. Reshetikhin. The quest for obtaining general formulas for the q-characters of the simple modules is one of the main topics being investigated in the area. Also, one can consider the branching problem associated to the inclusion of the underlying finite-type algebra which is equivalent to the study of the usual characters of these modules.

The $q \to 1$ limits of the simple modules are typically reducible and always indecomposable giving rise to examples of nontrivial extensions in the classical setting. The block decomposition of an abelian category is a first approximation to the study of extensions. The blocks of the categories we are considering are parameterized by a third type of characters. We may also consider certain graded characters attached to these modules.

In the talk we will survey several results related to the aforementioned character theories, some of which were obtained in joint works with V. Chari and D. Jakelić.

Finiteness of orbits on double flag varieties for symmetric pairs

Kyo Nishiyama, Aoyama Gakuin University, Japan

Let G be a reductive algebraic group over C, and B a Borel subgroup. It is well known that a symmetric subgroup K fixed by an involution has finitely many orbits on the full flag variety G/B. This fact is an important tool of the theory of Harish-Chandra modules of the symmetric pair (G, K) through the K-equivariant D-modules.

We consider a generalization to a symmetric pair. Namely, we consider partial flag varieties G/P of G and K/Q of K, and study K orbits on the product $G/P \times K/Q$. The number of orbits is infinite in general, but in many good cases, there are finitely many orbits. We give some sufficient conditions for finiteness of orbits, and discuss examples.

This talk is based on a joint work with Hiroyuki Ochiai (Kyushu Univeriesty) and Xuhua He (Hong Kong University of Science and Technology).

Dirac cohomology of some unipotent representations

Pavle Pandžić, University of Zagreb, Croatia

We will show how to calculate Dirac cohomology for a class of unipotent representations of real symplectic groups. A special feature of these examples is the high multiplicity of K-types in the Dirac cohomology. This is joint work with Dan Barbasch.

Conformal embeddings of affine vertex operator algebras

Ozren Perše, University of Zagreb, Croatia

Conformal embeddings have been studied in conformal field theory, the representation theory of affine Kac-Moody Lie algebras and the theory of affine vertex operator algebras. The construction and classification of conformal embeddings have mostly been studied in the case of positive integer levels. In this talk, we present a general criterion for conformal embeddings at arbitrary levels, within the framework of vertex operator algebra theory. Using that criterion, we construct new conformal embeddings at admissible rational and negative integer levels. In particular, we construct all remaining conformal embeddings associated to automorphisms of Dynkin diagrams of simple Lie algebras. The talk is based on joint work with Dražen Adamović.

Some representations over Virasoro algebra with infinite-dimensional weight spaces

Gordan Radobolja, University of Split, Croatia

In this talk we shall present certain new results and constructions for representations of the Virasoro algebra with infinite-dimensional weight spaces. Motivated by the representation theory of Virasoro vertex algebras, we consider tensor products of irreducible modules from intermediate series with irreducible highest weight modules for Virasoro algebra. We present a new irreducibility criterion, and give several examples. We shall also study generalizations of this construction for some other infinite-dimensional Lie algebras.

On Omega regular unitary representations

Susana Salamanca–Riba, New Mexico State University, USA
Joint with Alessandra Pantano and Annegret Paul

In this lecture we will present a construction of unitary representations of $Mp(2n)$, show that they parameterize a subset of the unitary dual with some regularity condition on the infinitesimal character and discuss similar constructions for other split groups.

The Generating Condition for the Extension of the Classical Gauss Series-product Identity

Tomislav Šikić, University of Zagreb, Croatia

In this talk will be presented the condition on parameters (n_1, n_2, Λ_k) which guarantees that equation for fundamental modules $L(\Lambda_k)$ of the affine Kac-Moody Lie algebra $\widehat{\mathfrak{sl}}_n$

$$\mathcal{F}_{\mathbf{s}}(ch_{L(\Lambda_k)}) = q^{const} \prod_{j\geq 1}(1-q^{jN}) \frac{\sum_{k_1+k_2=k} q^{\frac{N}{2}(\frac{k_1^2}{n_1}+\frac{k_2^2}{n_2})}}{\prod_{j\geq 1}(1-q^{\frac{jN}{n_1}})\prod_{j\geq 1}(1-q^{\frac{jN}{n_2}})}$$

generate new series-product identities based on the classical Gauss identity following the methodology as in the paper [1].

It is very important to accentuate that this generating condition discover infinitely many new examples of extended classical Gauss identities which are not presented in the mentioned paper.

[1] T. Šikić, *An extension of the classical Gauss series−product identity by boson−fermionic realization of the affine algebra* $\widehat{\mathfrak{gl}}_n$, J. Algebra Appl. No.1, Vol 9(2010), 123-133.

Pairs of Lie algebras: On self-normalizing subalgebras, Cartan subalgebras and compatible pairs of Borel subalgebras

Boris Širola, University of Zagreb, Croatia

Let $\mathfrak{g}$ be a semisimple (complex) Lie algebra, and $\mathfrak{g}_1$ a subalgebra reductive in $\mathfrak{g}$. First we will explain why it is useful to know the following: For which such pairs, $(\mathfrak{g}, \mathfrak{g}_1)$, $\mathfrak{g}_1$ is self-normalizing in $\mathfrak{g}$? In particular, this self-normalizing condition characterizes the distinguished nilpotent orbits. Next we will characterize those pairs $(\mathfrak{g}, \mathfrak{g}_1)$ satisfying the following condition: For any Cartan subalgebra $\mathfrak{h}_1$ of $\mathfrak{g}_1$ there is a unique Cartan subalgebra $\mathfrak{h}$ of $\mathfrak{g}$ such that $\mathfrak{h}_1 \subseteq \mathfrak{h}$. For any such pair and any Borel subalgebra $\mathfrak{b}_1$ of $\mathfrak{g}_1$ define $\mathcal{S}^{\mathfrak{g}}_{Bo}(\mathfrak{b}_1)$ as the set of all Borel subalgebras $\mathfrak{b}$ of $\mathfrak{g}$ containing $\mathfrak{b}_1$; the later set is finite. We will present some new basic results (and recover some old ones as well) concerning those pairs for which every $\mathcal{S}^{\mathfrak{g}}_{Bo}(\mathfrak{b}_1)$ is a singleton.

Combinatorial bases of principal subspaces for affine lie algebras

Goran Trupčević, University of Zagreb, Croatia

We consider a particular type of principal subspace $W(\Lambda)$ of a standard module $L(\Lambda)$ for an affine Lie algebra of the type $C_\ell^{(1)}$. We find a combinatorial basis of $W(\Lambda)$ given in terms of difference and initial conditions. Linear independence of the generating set is proved inductively by using coefficients of intertwining operators. (This talk is based on joint work with I. Baranović and M. Primc.)

TBA

Kari Vilonen, Northwestern University, Evanston IL, USA

Finite maximal tori

David Vogan, MIT, Cambridge, USA

A maximal torus in a compact Lie group G is a maximal connected abelian subgroup T. Attached to G and T there is a combinatorial structure called a root datum. Grothendieck's formulation of the Cartan-Killing theory of root systems says that G_0 is determined uniquely by the root datum, and that all root data appear in this way. Easy inspection of the root datum exhibits many interesting subgroups of G.

I will present joint work with Gang Han of Zhjiang University in which we start instead with a maximal finite abelian subgroup A of G, which we call a finite maximal torus. We attach to G and A a combinatorial structure that we call a finite root datum; again, inspection of the finite root datum exhibits interesting subgroups of G (which in many cases were not so obvious from the classical root system perspective). We do not know how to classify finite root data, or how to see which ones arise from compact groups; but the examples suggest that this is an interesting theory.