Interuniversity Centre, Dubrovnik, June 14 − 21, 2009

REPRESENTATION THEORY XI

ABSTRACTS

On the representation theory of affine Lie algebra $A_1^{(1)}$ at the critical level

Dražen Adamović, University of Zagreb, Croatia

In this talk we will investigate the representation theory of the affine Lie algebra $A_1^{(1)}$ at the critical level by using an infinite−dimensional Lie superalgebra $\mathcal{A}$. This algebra contains large center and can be considered as a certain limit of $N = 2$ superconformal algebras. We construct a functor from certain category of $\mathcal{A}$−modules to the category of $A_1^{(1)}$−modules at the critical level. Using this approach, we prove the irreducibility of a large family of $A_1^{(1)}$−modules at the critical level.

Vector−valued modular forms and generalized Moonshine

Peter Bantay, Eötvös Loránd University, Budapest, Hungary

We present an overview of the approach to the theory of vector−valued modular forms developed in joint work with T. Gannon. Besides the basic notions and results, we also discuss applications of the theory, in particular to generalized Moonshine.

Stable combinations of characters of unipotent representations

Dan Barbash, Cornell University, Ithaca, New York, USA

In his work, Arthur gives a conjectural description of the residual spectrum of automorphic forms. This motivated introducing the notion of special unipotent representations for real reductive groups; representations with maximal annihilator in the universal enveloping algebra, and particular infinitesimal character obtained from a nilpotent adjoint orbit in the dual Lie algebra. Forming stable combinations of such representations is important for applications of the Arthur Selberg trace formula. In the book "Langlands classification and irreducible characters for real reductive groups", we use geometric methods to construct stable combinations of characters. In this talk I will generalize these methods so as to give a basis of the space of stable combinations of unipotent representations. This is based on joint work with Peter Trapa.

The moduli space of $N = 2$ superconformal worldsheets arising in $N = 2$ superconformal field theory

Katrina Barron, University of Notre Dame, Indiana, USA

Motivated by the desire for an axiomatic definition of the algebra of correlations functions governed by the worldsheets swept out by propagating superstrings in $N = 2$ superconformal field theory, we discuss the moduli space of $N = 2$ superconformal super−Riemann surfaces with incoming and outgoing tubes. In particular, we show that there is a countably infinite family of genus−zero $N = 2$ superconformal super−Riemann surfaces up to $N = 2$ superconformal equivalence, classified by holomorphic line bundles over the Riemann sphere up to conformal equivalence. We give the group of $N = 2$ superconformal automorphisms for each equivalence class of $N = 2$ super−Riemann sphere and briefly discuss the implications for the algebraic structure imposed on the space of particle states governed by worldsheets arising in N=2 superconformal field theory.

The algebra of correlation functions arising from genus−zero, two−dimensional, holomorphic $N = 2$ superconformal field theory

Katrina Barron, University of Notre Dame, Indiana, USA

We show that the algebra of correlation functions arising from genus-zero, two−dimensional, holomorphic $N = 2$ superconformal field theory has a much richer structure than that of an $N = 2$ Neveu−Schwarz vertex operator superalgebra, V. In particular, we will discuss the V−modules that arise from the higher degree components of the moduli space of genus−zero $N = 2$ superconformal worldsheets.

Categorical Hecke algebra actions and applications

Roman Bezrukavnikov, MIT, Cambridge, USA

I will recall Soergel's approach to category $\mathcal{O}$ based on (what I call) a categorical Hecke algebra action and categorical Whittaker functional. I will discuss possible geometric interpretations of these structures and (time permitting) generalizations to real groups (joint with Kari Vilonen).

Categorical affine Hecke algebra actions and applications

Roman Bezrukavnikov, MIT, Cambridge, USA

Generalizing the categorical Hecke algebra action on representations of complex semi−simple Lie algebras, one gets a categorical affine Hecke algebra action on representations of similar Lie algebras in positive characteristic (studied in a joint project with Ivan Mirković). This is closely related to geometry of coherent sheaves on the Steinberg variety of triples. Time permitting, I will tell about a different realization of the categorical affine Hecke algebra and an equivalence between the two realizations inspired by (local) geometric Langlands duality.

Unitary functorial correspondences for quasisplit p−adic groups

Dan Ciubotaru, University of Utah, Salt Lake City, USA

I will present a generalization of the machinery of Barbasch and Moy for detecting the unitarity in Bernstein components of the category of smooth admissible representations of a p−adic group. In the case of quasisplit (nonsplit) groups and Iwahori−spherical representations, one consequence is a functorial correspondence of unitary representations with certain (endoscopic type) split p−adic groups. The talk is based on joint work with Dan Barbasch.

Duality between representations of classical real groups and affine graded Hecke algebras

Dan Ciubotaru, University of Utah, Salt Lake City, USA

We define exact functors from certain categories of Harish−Chandra modules for classical real groups $(GL(n, \mathbb{R}), U(p, q), Sp(2n, \mathbb{R}), O(p, q))$ to the category of finite dimensional modules for affine graded Hecke algebras naturally attached to the systems of restricted roots for these groups, and study some of their basic properties, in particular relative to the preservation of unitarity. The talk is based on joint work with Peter Trapa.

On Poincare−Birkhoff−Witt theorem

Michel Duflo, Université Paris 7, France

Let L be a Lie algebra (or a Lie super−algebra) over a commutative ring R. The PBW theorems provide, under suitable hypotheses, isomorphisms of coalgebras between the symmetric algebra and the enveloping algebra of L. In this lecture, I consider some functorial aspects, and some explicit formulas, for these isomorphisms.

Vector−valued modular forms

Terry Gannon, University of Alberta, Canada

This talk is a sequel to that of Peter Bantay. In it I describe how to work out highly nontrivial examples, and I describe applications to e.g. Generalized Moonshine, vertex operator algebras (existence of exotic VOAs), and conformal field theory (e.g. 4−point conformal blocks on a sphere and 1− and 2−pt blocks on a torus).

Zhu's algebra, the C_2 algebra, and twisted modules

Terry Gannon, University of Alberta, Canada

In his landmark paper, Zhu associated two associative algebras to a vertex operator algebra: Zhu's algebra and the C_2 algebra. The former has a nice interpretation in terms of representation theory of the VOA, while the latter only serves as a finiteness condition. In this talk I review these two algebras, and I try to explain the algebraic significance of the C_2 space, namely that it sees and controls the "twisted" representations. This sheds considerable light on an old problem: understanding when C_2 and Zhu are the same size. This is joint work with Matthias Gaberdiel.

Gelfand pairs and symmetric pairs

Dmitry Gourevitch, Weizmann Institute, Rehovot, Israel

First we will introduce the notion of Gelfand pair and its connection to invariant distributions. Then we will discuss some tools to work with invariant distributions. At the end we will discuss the question when a symmetric pair is a Gelfand pair.

Multiplicity one Theorems – a uniform proof

Dmitry Gourevitch, Weizmann Institute, Rehovot, Israel

Let F be a local field of characteristic 0. We consider distributions on $GL(n+1,F)$ which are invariant under the adjoint action of $GL(n,F)$. We prove that such distributions are invariant under transposition. This implies that an irreducible representation of $GL(n+1,F)$, when restricted to $GL(n,F)$ "decomposes" with multiplicity one. For non-Archimedean F this theorem was proven in [AGRS]. For Archimedean F this theorem was proven in [AG] and independently in [SZ]. In the lecture we will present a new proof which is uniform for both cases. This proof is based on the above papers and an additional new tool due to Aizenbud. If time permits we will discuss similar theorems that hold for orthogonal and unitary groups.

[AG] A. Aizenbud, D. Gourevitch: "Multiplicity one theorem for $GL(n+1,\mathbb{R}), GL(n,\mathbb{R})$", to appear in Selecta Mathematica, arXiv:0808.2729v1 [math.RT]

[AGRS] A. Aizenbud, D. Gourevitch, S. Rallis, G. Schiffmann: "Multiplicity One Theorems", arXiv:0709.4215v1 [math.RT], to appear in the Annals of Mathematics.

[SZ] B. Sun and C.-B. Zhu" "Multiplicity one theorems: the archimedean case", arXiv:0903.1413

Conformal Geometry and Schrodinger Model of Minimal Representations

Toshiyuki Kobayashi, University of Tokyo, Japan

Minimal representations are the ”smallest” infinite dimensional unitary representations. The Weil representation, which plays a prominent role in number theory, is a classic example. Most of these are isolated among the set of all unitary representations, and cannot be built up by induction. Highlighting indefinite orthogonal groups, I plan to discuss two models of minimal representations, namely, the one is the conformal geometric construction by using the Yamabe operator, and the other is an analogue of the Schrodinger model. The latter model leads us to a natural generalization of the ”Fourier transform” on the isotropic cone.

Generalized Bernstein−Reznikov integrals

Toshiyuki Kobayashi, University of Tokyo, Japan

I plan to explain an explicit formula for the integral of a triple product of powers of symplectic forms on the triple product of spheres in $\mathbb{R}^{6n}$.

Crystals and Macdonald polynomials via alcove walks

Cristian Lenart, University at Albany (SUNY), New York, USA

I will present some applications of a combinatorial model in the representation theory of semisimple Lie algebras (and, more generally, of symmetrizable Kac−Moody algebras), which is based on so−called alcove walks. The alcove model can be used to study the combinatorics of Kashiwara's crystal graphs; these encode the bases of representations of the quantized universal enveloping algebra $U_q(g)$ as q goes to zero. The alcove model was also used (by Schwer and Ram−Yip) to give combinatorial formulas for Macdonald spherical functions (or Hall−Littlewood polynomials) in the theory of p−adic groups, as well as for Macdonald polynomials; these are one− and two−parameter generalizations of irreducible characters. I will present a ”compression” procedure, which allows one to derive simpler formulas from the mentioned formulas of Schwer and Ram−Yip. These simpler formulas have important applications, particularly to Lusztig's t−analog of weight multiplicities and its two−parameter generalization in type A.

Representation theoretic harmonic spinors

Salah Mehdi, Université Paul Verlaine, Metz, France

We will describe various results on harmonic spinors for cubic Dirac operators on reductive homogeneous spaces.

Plancherel formula for differential forms on symmetric spaces and applications

Salah Mehdi, Université Paul Verlaine, Metz, France

We explain how one can use a Plancherel formula for differential forms on symmetric spaces to obtain fine estimates for the heat kernel and for the bottom of the spectrum of the form laplacian on locally symmetric spaces. One can also deduce some information on L^2−cohomology of locally symmetric spaces of infinite volume.

Logarithmic modules for vertex operator (super)algebras

Antun Milas, University at Albany (SUNY), New York, USA

This talk will be divided in three parts. In part one I'll describe a general framework for constructing logartihmic modules and related structures in vertex algebra theory, with an eye on irrational C_2-cofinite vertex algebras coming from lattice vertex (super)algebras. Our main examples are certain (projective) logarithmic modules for the triplet vertex algebra $W(p)$. In the second part I will explain how the notion of Huang's generalized twisted modules associated to automorphisms of infinite order fit into our framework. I will end by discussing more recent developments, such as the C_2-cofiniteness of certain $W-$algebras obtained as extensions of Virasoro minimal models. This is an ongoing project with Dražen Adamović.

Lusztig's conjecture on modular representations of semisimple Lie algebras

Ivan Mirković, University of Massachusetts, Amherst, USA

This is a joint work with Roman Bezrukavnikov. Lusztig's conjecture predicts in great detail the numerical structure of (stable) representation theory of semisimple Lie algebras in positive characteristic. The verification here is based on a − mysteriously close − relation with perverse sheaves (or modules for differential operators) on the affine flag variety of the Langlands dual group. One translates the problem from representations of a Lie algebra to $\mathcal{D}-$modules on its flag variety, then to coherent sheaves on the cotangent bundle to the flaf variety and finally to perverse sheaves on the affine flag variety of the dual group. In this last setting one can check the deepest part of Lusztig's conjectural characterization of irreducible representations by using Hodge theory. The steps in the translation process are equivalences of derived categories (categories of complexes over abelian categories). The deeper part here is to follow what happens with abelian categories themselves under the equivalences of categories of complexes. This is done in terms of actions of affine braid groups on derived categories.

Existence and Construction of Cusp Forms

Goran Muić, University of Zagreb, Croatia

Let G be a semisimple algebraic group over defined over rational integers $\mathbb{Z}$. It was an open and difficult question of existence of infinitely many cusp forms in $L^2(G(\mathbb{Z})\backslash G(\mathbb{R}))$. In their recent work Venkatesh and Lindenstrauss showed how to establish this fact via Weyl law. In this talk we discuss different approach which is based on a spectral decomposition of compactly supported Poincaré series. We will also present some further applications of our approach in obtaining some improvements in the compact case as well as discuss some applications to the questions of existence of cuspidal automorphic representations in adelic set−up.

Dirac cohomology of some unitary highest weight modules

Pavle Pandžić, University of Zagreb, Croatia

I will start by reviewing the definition, motivation and basic facts about Dirac cohomology of Harish−Chandra modules. Then I will move to describing how Dirac cohomology can be calculated in some concrete examples, like certain unitary highest weight modules. The results are joint work with J.−S. Huang and V. Protsak.

Unitary Genuine Principal Series of the Metaplectic Group

Alessandra Pantano, University of California at Irvine, USA

The purpose of this talk is to present some recent progress on the classification of the unitary genuine irreducible representations of the metapectic group $Mp(2n)$ (the nontrivial double cover of the symplectic group $Sp(2n)$). A representation of $Mp(2n)$ is called genuine if it does not factor to a representation of $Sp(2n)$. Our focus will be on Langlands quotients of genuine minimal principal series of $Mp(2n)$. The main result is an embedding of the set of unitary parameters of such representations into the union of spherical unitary parameters for certain split orthogonal groups. The latter are known from work of Barbasch, hence we obtain the non−unitarity of a large (conjecturally complete) set of parameters for Langlands quotients of genuine principal series of $Mp(2n)$. This is joint work with Annegret Paul and Susana Salamanca Riba.

Conformal embeddings of vertex operator algebras associated to orthogonal affine Lie algebras

Ozren Perše, University of Zagreb, Croatia

Let $L_{D_\ell}(-\ell+\frac{3}{2},0)$ (resp. $L_{B_\ell}(-\ell+\frac{3}{2},0)$) be the simple vertex operator algebra associated to affine Lie algebra of type $D_\ell^{(1)}$ (resp. $B_\ell^{(1)}$) with the lowest admissible half−integer level $-\ell+\frac{3}{2}$. We show that $L_{D_\ell}(-\ell+\frac{3}{2},0)$ is a vertex subalgebra of $L_{B_\ell}(-\ell+\frac{3}{2},0)$ with the same conformal vector. For $\ell=4$, $L_{D_4}(-\frac{5}{2},0)$ is a vertex subalgebra of three copies of $L_{B_4}(-\frac{5}{2},0)$ contained in $L_{F_4}(-\frac{5}{2},0)$, and all five of these vertex operator algebras have the same conformal vector.

Combinatorial bases of representations of affine Lie algebras $A_1^{(1)}$ and $B_2^{(1)}$

Mirko Primc, University of Zagreb, Croatia

In this talk I'll discuss different constructions of combinatorial bases of representations for some low rank affine Lie algebras and different approaches to proving linear independence of these bases. A particular connection of constructions for $A_1^{(1)}$ and $B_2^{(1)}$ illustrates advantages and disadvantages of Groebner basis type method compared to Capparelli−Lepowsky−Milas approach by using simple currents and coefficients of intertwining operators.

On the omega regular unitary representations of the metaplectic group

Susana A. Salamanca−Riba, New Mexico State University, USA

This work is joint work with A. Pantano and A. Paul. We propose a program to classify these representations using a generalized definition of $A_q(\lambda)$ representations. The proofs use a combination of arguments originally developed by Barbasch and Vogan and detailed in other results by Pantano, Paul and Salamanca−Riba. We will focus on the genuine representations for this talk.

News from the logarithmic Kazhdan–Lusztig duality. What Hopf algebras know about LCFT

Alexei Semikhatov, Lebedev Physics Institute, Moscow, Russia

I will discuss an extended algebraic setting that parallels the structure of logarithmic conformal field theories.

Some distinguished Lie subalgebras and subgroups and the role of Kostant's TDS theory

Boris Širola, University of Zagreb, Croatia

Given a (semisimple) $\mathbb{K}$−Lie algebra $\mathfrak{g}$, we consider some (reductive) subalgebras $\mathfrak{g}_1$ which satisfy certain additional conditions; here $\mathbb{K}$ is a field of characteristic zero. The class of such obtained pairs $(\mathfrak{g}, \mathfrak{g}_1)$ contains a number of interesting (nonsymmetric) pairs. For these we can obtain various useful information (e.g., for the purposes of the geometry of (co)adjoint orbits, for better understanding of some group normalizers and self−normalizing subgroups, etc.) We will explain how some old results and ideas of Kostant enter into the picture. (Besides we will present some instructive basic examples that suggest certain interesting general results.)

TBA

Kari Vilonen, Northwestern University, Evanston IL, USA

On Lipsman's conjecture

Taro Yoshino, University of Tokyo, Japan

In the study of discontinuous groups, proper action in the sense of Palais is a nice class of actions. Lipsman conjectured that the properness of action can be checked "locally" if the action is unipotent. However, in this talk, we will see a counterexample to the conjecture.