# FUNCTIONAL ANALYSIS X

## REPRESENTATION THEORY

Dubrovnik, Croatia, June 29- July 6, 2008

# ABSTRACTS OF TALKS

## On the representation theory of a family of $N = 1$ vertex superalgebras

Dražen Adamović

In this talk we shall present the free-fields realization of two new families of super $W$-algebras: the singlet and the triplet $N = 1$ vertex superalgebras. These vertex superalgebras are defined by using screening operators and they are certain extensions of the Neveu-Schwarz vertex operator superalgebras. We shall discuss their representation theory. In particular, we shall demonstrate that the super-triplet $\mathcal{SW}(m)$ is $C_2$ cofinite and irrational vertex operator superalgebra. (This talk is based on joint work with A. Milas).

## K-types of small unitary representations and Dirac cohomology

Dan Barbasch

## Stable Nilpotent Orbital Integrals

Mladen Božičević

We shall discuss some applications of the theory of characteristic cycles of equivariant sheaves on the flag variety, developed by Rossmann, Schmid and Vilonen, to the problems of harmonic analysis on a real reductive Lie algebra. Our goal will be to understand the space of stably invariant distributions associated with complex nilpotent orbit. The dimension of this space was computed by Kottwitz and Assem conjectured how to obtain a natural basis. Assem's conjecture will be examined in a number of special cases.

## Unitarizable ramified principal series

Dan Ciubotaru

Let G denote the F-points of a connected adjoint semisimple algebraic group, where F is the real or p-adic field. Assume G is split, and the split torus is H. Let K be the maximal compact subgroup. We consider the ramified (nonspherical) principal series $X(\delta, \nu)$, where $\delta$ is a smooth character of $M = K \cap H$. I will explain how the unitarizability of the Langlands quotient of $X(\delta, \nu)$ is related to the spherical unitary dual of a certain endoscopic group $G(\delta)$. In the p-adic case, this relation is obtained by applying the techniques of Barbasch and Moy in the framework of the theory of types (in particular, in Roche's work on ramified principal series). In the real case, the relation is the work of Barbasch and Pantano on (nonspherical) petite K-types. If time permits, I will try to say what happens when one considers non-adjoint groups G. In that case, the correspondence is with the quasi-spherical unitary dual of an extension of $G(\delta)$ by an R-group.

## Weyl's functional calculus and equivariant cohomology.

Michel Duflo

Forty years ago, Edward Nelson wrote a beautiful formula which gives explicitly Weyl's distribution functional calculus $f \to f(A_1, \ldots, A_d)$, associated to $d$ Hermitian matrices of size $n$, as the derivative of a compactly supported probability measure on $\mathbb{R}^d$. We will explain how this formula fits in the setting of Hamiltonian geometry for compact Lie groups, and in particular, equivariant cohomology.

## Boundaries of $K$-types in discrete series

Miki Havlickova

A fundamental problem about irreducible representations of a reductive Lie group $G$ is understanding their restriction to a maximal compact subgroup $K$. In case of discrete series, the Blattner character formula gives the multiplicity of any given irreducible $K$-representation (or $K$-type) as an alternating sum. It is not immediately clear from this formula which $K$-types, indexed by their highest weights, have non-zero multiplicity. Evidence suggests that the collection is very close to a set of lattice points in a convex polyhedron. I shall describe a recursive algorithm for finding the boundary facets of this polyhedron.

## Dirac cohomology of $(g, K)$-modules

Jing-Song Huang

We will discuss joint work with Pavle Pandzic, David Renard and Yifang Kang on Dirac cohomology and its relation with Lie algebra cohomology. The aim of this talk is to show that Dirac cohomology can be employed as an unifying theme in solving many problems in representation theory

## Finite-dimensional representations of hyper loop algebras

Dijana Jakelić

Hyper loop algebras are certain Hopf algebras associated to affine Kac-Moody algebras. We will focus on finite-dimensional representations of hyper loop algebras over arbitrary fields. The main results concern the classification of the irreducible representations, their tensor products, the construction of the Weyl modules, and base change. We will also address multiplicity problems for the underlying tensor category. Time permitting, we may discuss a conjecture saying that the Weyl modules of a hyper loop algebra over an algebraically closed field of characteristic p are the reduction modulo p of suitable characteristic zero Weyl modules. This is a joint work with Adriano Moura.

## Recurrence relations for characters of affine Lie algebra $A_\ell^{(1)}$

Miroslav Jerković

By using the known description of combinatorial bases for Feigin-Stoyanovsky's type subspaces of standard modules for affine Lie algebra $\mathfrak{sl}(l+1,\mathbb{C})\tilde{}$, as well as certain intertwining operators between standard modules, we obtain exact sequences of Feigin-Stoyanovsky's type subspaces at fixed level $k$. This directly leads to systems of recurrence relations for formal characters of those subspaces. Particularly, by solving the above mentioned system for affine Lie algebra $\mathfrak{sl}(3,\mathbb{C})\tilde{}$ we obtain fermionic type character formulas for all Feigin-Stoyanovsky's type subspaces at general level.

## Visible actions and multiplicity-free representations

Toshiyuki Kobayashi

## The homogeneous realization of level one $A_2^{(1)}$-modules and rational solutions of Painlevé VI

Johan van de Leur

We show that the Riemann-Hilbert factorization of certain elements in the loop group of type $GL(3,\mathbb{C})$ is related to the Jimbo-Miwa-Okamoto sigma-form of the Painlevé VI equation. The action of these loop group elements on the highest weight vector of a level 1 affine $gl(3,\mathbb{C})$ module, in its homogeneous realization, gives a family of tau functions. Each such tau function provides a collection of rational solutions of the Painlevé VI equation. This talk is based on joint work with Henrik Aratyn from the University of Illinois at Chicago.

## Orthogonal decomposition of $E_8$ and the Dempwolff group

Arne Meurman

## Vertex algebras and logarithmic conformal field theory I: Introduction

Antun Milas

Rational conformal field theories and rational vertex algebras have been studied extensively over the last two decades. These vertex algebras have semi-simple module categories and share many nice properties. Well-known examples include lattice vertex algebras, Virasoro minimal models, etc. More recently, new examples of irrational vertex algebras have emerged in connection with *logarithmic conformal field theory*, with new features such as *logarithmic modules* and correlation functions with logarithmic branch cuts.

In this talk, we shall begin with some basic notions in vertex algebra theory. Then I will discuss an algebraic approach to logarithmic conformal field theory based on the theory of vertex algebras and logarithmic intertwining operators. Considerable attention will be devoted to irrational vertex algebras of finite-representation type, for which many techniques from representation theory can be successfully applied.

## Vertex algebras and logarithmic conformal field theory II: Triplet vertex algebra $\mathcal{W}(p)$

Antun Milas

In the second part, I will focus on the triplet vertex operator algebra $\mathcal{W}(p)$, a mediator between the Virasoro vertex algebra $L(c_{p,1}, 0)$ and the rank one lattice vertex algebra $V_L$. Several results on the structure $\mathcal{W}(p)$ will be presented, including the $C_2$-cofiniteness and classification of irreducible $\mathcal{W}(p)$-modules. I will also discuss closely related logarithmic $\mathcal{W}(p)$-modules, irreducible characters and *pseudocharacters*. (This is a joint work with D. Adamović).

## Singular localization and a result of Carayol and Knapp

Dragan Miličić

For singular infinitesimal characters, the category of Harish-Chandra modules is the quotient category of the category of Harish-Chandra sheaves by the subcategory of objects with no global sections. This allows us to use geometry to do representation theory in such situations. We shall illustrate this by explaining a recent result of Carayol and Knapp on limits of large discrete series. This is a joint work with Peter Trapa.

## Topological Methods in Representation Theory

Ivan Mirković

Recently representation theory has imported two topological methods, Homotopical Algebraic Geometry and Topological Quantum Field Theory. The first one (Toen, Lurie, Ben Zvi, Nadler, Gaitsgory) is a very strong new tool. The second (Witten, Ben Zvi, Nadler) is a new organizational principle for representation theory which relates various developments in parallel with low dimensional topology. Another topoogical observation, that the cohomology classes affect geometry of a manifold (made popular in the realm of quantum cohomology) plays role in modular representation theory and conjecturally also theory of loop groups and quantum groups.

## Asymptotic cone of semisimple orbit for symmetric pairs.

Kyo Nishiyama

By a result of Borho and Kraft, it is known that the asymptotic cone of a semisimple element $h$ is the closure of a Richardson nilpotent orbit corresponding to a parabolic subgroup whose Levi component is the centralizer $Z_G(h)$ in $G$. In the talk, we give an analogue on a semisimple orbit for a symmetric pair, which is associated to an even nilpotent orbit. Also we give some examples associated to Siegel maximal parabolics.

## Vertex operator algebra analogue of embedding of $B_4$ into $F_4$

Ozren Perše

Let $L_B(-\frac{5}{2},0)$ (resp. $L_F(-\frac{5}{2},0)$) be the simple vertex operator algebra associated to affine Lie algebra of type $B_4^{(1)}$ (resp. $F_4^{(1)}$) with the lowest admissible half-integer level $-\frac{5}{2}$. We show that $L_B(-\frac{5}{2},0)$ is a vertex subalgebra of $L_F(-\frac{5}{2},0)$ with the same conformal vector, and that $L_F(-\frac{5}{2},0)$ is isomorphic to the extension of $L_B(-\frac{5}{2},0)$ by its only irreducible module other than itself. We also study the representation theory of $L_F(-\frac{5}{2},0)$, and determine the decompositions of irreducible weak $L_F(-\frac{5}{2},0)$-modules from the category $\mathcal{O}$ into direct sums of irreducible weak $L_B(-\frac{5}{2},0)$-modules.

## Combinatorial bases of representations of affine Lie algebras $A_1^{(1)}$ and $B_2^{(1)}$

Mirko Primc

In this talk I'll discuss different constructions of combinatorial bases of representations for some low rank affine Lie algebras and different approaches to proving linear independence of these bases. A particular connection of constructions for $A_1^{(1)}$ and $B_2^{(1)}$ illustrates advantages and disadvantages of Groebner basis type method compared to Capparelli-Lepowsky-Milas approach by using simple currents and coefficients of intertwining operators.

## Combinatorial bases of Feigin-Stoyanovsky's type subspaces of standard modules for affine Lie algebras of type $A_\ell$

Goran Trupčević

Let $\tilde{\mathfrak{g}}$ be affine Lie algebra of type $A_\ell^{(1)}$. Suppose we're given $\mathbb{Z}$-gradation of associated simple finite-dimensional Lie algebra $\mathfrak{g} = \mathfrak{g}_{-1} \oplus \mathfrak{g}_0 \oplus \mathfrak{g}_1$; then we also have the induced $\mathbb{Z}$-gradation of affine Lie algebra

$$\tilde{\mathfrak{g}} = \tilde{\mathfrak{g}}_{-1} \oplus \tilde{\mathfrak{g}}_0 \oplus \tilde{\mathfrak{g}}_1.$$

Let $L(\Lambda)$ be a standard module, ie. integrable highest-weight module for $\tilde{\mathfrak{g}}$. Standard modules of level 1 can be realized via vertex-operators associated to root lattice of $\mathfrak{g}$. Vertex-operators associated to the elements of weight lattice will then give intertwining-operators between standard modules.

Feigin-Stoyanovsky type subspace $W(\Lambda)$ is $\tilde{\mathfrak{g}}_1$-submodule of $L(\Lambda)$ generated by the highest-weight vector $v_\Lambda$,

$$W(\Lambda) = U(\tilde{\mathfrak{g}}_1) \cdot v_\Lambda \subset L(\Lambda).$$

In this paper we find combinatorial base of $W(\Lambda)$, given in terms of certain difference and initial conditions. Linear independence of the generating set is proved inductively by using coefficients of certain intertwining operators.

## A survey of vertex algebra

Markus Rosellen

In the first talk we do two things. First, we show that Lie, associative, and Poisson vertex algebras form full subcategories of certain categories of Lie, associative, and Poisson algebras, respectively. Second, we derive a new formulation of vertex algebras in terms of algebras with just one multiplication. Roughly speaking, a vertex algebra is an associative algebra such that its underlying Lie algebra is a vertex Lie algebra.

In the second talk we illustrate the usefulness of this new approach by discussing three topics: the notion of almost commutativity; enveloping and tensor vertex algebras; and representation theory.

## OPE-algebras: A vertex algebra formulation of conformal field theory

Markus Rosellen

It has been known for a long time that vertex algebras are the chiral algebras of conformal field theories (CFT). This means that the subspace of holomorphic fields of the "algebra" of fields of a CFT has the structure of a vertex algebra. The state space of the full CFT is of the form $V = \bigoplus_{i,j} M_i \otimes N_j$, where $M_i$ and $N_j$ are simple modules over the chiral and anti-chiral algebra. The "algebra" structure of the CFT V is constructed from intertwining operators for the modules $M_i$ and $N_j$ and is thus a generalization of a vertex algebra.

Two proposals have been made for a definition of this "algebra" structure on V. In 2003, the physicist Kapustin and the algebraic geometer Orlov defined OPE-algebras, constructed lattice OPE-algebras that generalize lattice vertex algebras, and explained their relation to sigma models of tori and to mirror symmetry. In 2007, Huang and Kong defined full field algebras and constructed examples from a class of "rational" vertex algebras. A basic difference between the two approaches is that the lattice OPE-algebras are parametrized by a moduli space and are rational only at special points. We present the results that have been obtained about OPE-algebras.

## Mocking the theta. Logarithmically

Alexei Semikhatov

Symmetries of logarithmic conformal field theories yield modular group transformations that nontrivially combine Mordell-type integrals and certain generalizations of theta functions.

## Quantum-group counterparts of logarithmic conformal field theory

Alexei Semikhatov

Part of the contents of logarithmic conformal field theories is reflected in certain structures defined on quantum groups. I review the existing evidence, which suggests that the similarities may extend quite far.

# Categorified actions and 2-representations

Zoran Škoda

Many objects in geometry and mathematical physics are of higher categorical nature, for example stacks and higher gauge theories. Symmetries in such circumstances are described by categorified group(oid)s, higher algebroids and so on. After basic examples and definitions, I will present several new constructions regarding categorified actions and 2-equivariant objects.

# Some new examples of the Extension of the Classical Gauss Series-product Identity

Tomislav Šikić

The central object of this talk will be a character $ch_{L(\Lambda_k)}$ of an irreducible highest weight module $L(\Lambda)$ of the affine Lie algebra $\widehat{\mathfrak{sl}}_n$, where $\Lambda_k$ can be any fundamental weight. We looked at the mentioned object from two different points of view. One point of view is based on the *character formula* for any dominant integral weight $\Lambda$ in the special case of the affine Lie algebras of type $A_l^{(1)}$ (see [1]). Another point of view is based on a bosonic and fermionic constructions of fundamental representations $L(\Lambda_k)$ of affine Lie algebra $\widehat{\mathfrak{gl}}_n$ parameterized by partitions $\underline{n}$ (see [2]). The "trace formula" of $L(\Lambda_k)$ for the special case $\widehat{\mathfrak{sl}}_n$ (also [2]), is an expression for a particular specialization $\mathcal{F}_{\mathbf{s}}$ of the character $ch_{L(\Lambda_k)}$. By using Gauss identity

$$\frac{\varphi(q^2)^2}{\varphi(q)} = \sum_{n\in\mathbb{Z}} q^{2n^2+n} ,$$

for two special choices of partitions $\underline{n}$, and the corresponding fundamental highest weights $\Lambda_k$, we can transform the "trace formula" of $L(\Lambda_k)$ into infinite products and obtain two infinite families of series-product identities ([3]). In this talk will be presented some new examples of series-product identities (out of this two families) based on the mentioned observations.

[1] Kac, V.G.: *Infinite dimensional Lie algebras* ($3^{rd}$ edition), Cambridge University Press, (1990).

[2] ten Kroode, F., van de Leur J.: Bosonic and fermionic realization of the affine algebra $\widehat{\mathfrak{gl}}_n$, Comm. Math. Phys. 137 (1991), 67-107.

[3] Šikić T.: An Extension of the Classical Gauss Series-product Identity by Boson-fermionic Realization of the Affine Algebra $\widehat{\mathfrak{gl}}_n$, (submitted to the "Journal of Algebra and Its Applications" 2008) .

## Self-normalizing reductive Lie subalgebras and subgroups

Boris Širola

We consider a class of pairs $(\mathfrak{g}, \mathfrak{g}_1)$, where $\mathfrak{g}$ is a semisimple Lie algebra and $\mathfrak{g}_1$ is reductive and self-normalizing in $\mathfrak{g}$, satisfying certain additional "rigidity conditions". We also consider pairs of groups $(G, G_1)$ which correspond to $(\mathfrak{g}, \mathfrak{g}_1)$ in an obvious way. We will have some evidence that such pairs, both of Lie algebras and groups, are worth studying (e.g., in some branching problems, and for the purposes of the geometry of orbits). In particular, for some interesting pairs of groups $(G, G_1)$, we will describe the structure of the normalizer $\mathcal{N}_G(G_1)$.

## Unipotent representations of $Sp(p, q)$ and $SO^*(2n)$.

Peter Trapa

To each (suitably integral) nilpotent coadjoint orbit for $Sp(p, q)$ and $SO^*(2n)$, we attach an irreducible unitary representation. In particular, this construction establishes the unitarity of Arthur's special unipotent representations for these groups.

## Lusztig's conjecture and unitary representations

David Vogan

Suppose $G$ is a complex reductive algebraic group. Write $N^*$ for the cone of nilpotent elements in the dual of the Lie algebra of $G$. The group $G$ acts on $N^*$ with a finite number of orbits $O_i^*$, each of which has an isotropy group $H_i$. The collection $(H_i)\hat{}$ of irreducible algebraic representations of $H_i$ is (almost) a cone in a lattice, with (almost) caused by the fact that $H_i$ may be disconnected. The rank of the lattice is equal to the rank of $H_i$, which is at most the rank of $G$.

Lusztig conjectured that there is a natural finite decomposition of the cone-in-lattice $G\hat{}$ into pieces in bijection with the various cones-in-lattices $\hat{H_i}$. This conjecture was proved by Bezrukavnikov, but there is in general no explicitly computable description of the bijection. (Pramod Achar has given such a description for $GL(n)$.)

I will explain how one can try to compute Lusztig's decomposition; how a knowledge of it can tell you about unitary representations of $G$; and how to extend all of these ideas to real reductive groups.

## Geometric methods in representation theory

Kari Vilonen

I will discuss a geometric point of view to representation theory of reductive groups. In particular, I will explain the theory of (mixed) Hodge modules of Saito and the role they, at least potentially, play in representation theory

## Kazhdan-Lusztig Polyonomials and Signature Computations

Wai Ling Yee

The Unitary Dual Problem is a difficult open problem in mathematics which has been open for more than 70 years. Today, this problem is most commonly studied in the setting of representations admitting invariant Hermitian forms (classified by Knapp and Zuckerman). The Unitary Dual Problem may then be reformulated as the problem of identifying which forms are positive definite by computing the signatures of the forms. In this talk, I will discuss the connection between this problem and module structure encoded by Kazhdan-Lusztig polynomials