Cumulative Prospect Evaluation with Moving Reference Point

Doris Božović and Lavoslav Čaklović

Allianz Zagreb d.d. and Faculty of Natural Sciences
Zagreb, Croatia

ISCCRO'16, Zagreb, Croatia.

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Abstract

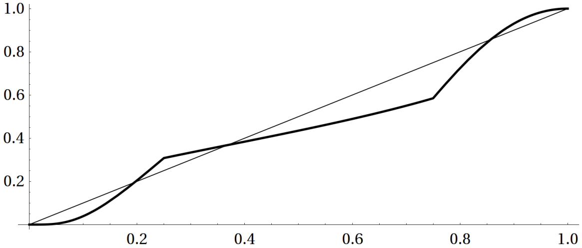
In this communication we shall explain *dynamic reference evaluation* of a prospect obtained from a time series (shares). The motivation comes from the Cumulative Prospect Theory using the ideas of Quiggin (1982) and Yaari (1987).

As an illustration of this evaluation we shall compare the shares of the companies: IBM, Western Digital and Apple in the period from 2010/07/01 to 2011/01/03 using the probability deformation according to Prelec (1998) and the original utility given by Tversky and Kahneman (1992).

Human perception of probability.

S1. For probabilities in the interval $[0, 1]$, that are bounded away from the end-points, decision makers overweight small probabilities and underweight large probabilities.

S2. For events close to the boundary of the probability interval $[0, 1]$, decision makers: (i) ignore events of extremely low probability and, (ii) treat extremely high probability events as certain.



Composite Prelec function[1] which satisfies S1 and S2 is given by:

$$w(p) = \begin{cases} 0 & \text{if } p = 0 \\ e^{-0.61266(-\ln p)^2} & \text{if } 0 < p \leqslant 0.25 \\ e^{-(-\ln p)^{0.5}} & \text{if } 0.25 \leqslant p \leqslant 0.75 \\ e^{-6.4808(-\ln p)^2} & \text{if } 0.75 \leqslant p \leqslant 1. \end{cases} \quad (1)$$

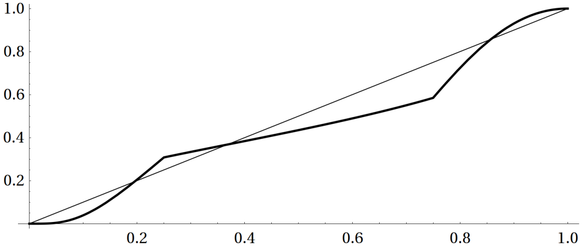


Figure 1 : Composite Prelec function.

[1] Prelec (1998)

Utility in Prospect Theory

Definition (Kahneman and Tversky (1979))

A utility function, $v(x)$, is a continuous, strictly increasing, mapping $v : \mathbb{R} \to \mathbb{R}$ that satisfies:

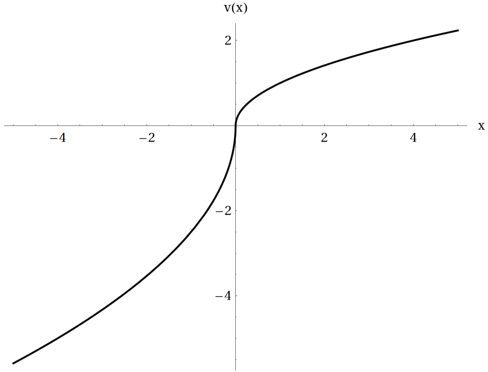
1. $v(0) = 0$ (reference dependence).
2. $v(x)$ is concave for $x \geqslant 0$ (declining sensitivity for gains).
3. $v(x)$ is convex for $x \leqslant 0$ (declining sensitivity for losses).
4. $-v(-x) > v(x)$ for $x > 0$ (loss aversion).

Tversky and Kahneman (1992) proposed the function

$$v(x) = \begin{cases} x^{\gamma} & \text{if } x \geqslant 0 \\ -\lambda(-x)^{\theta} & \text{if } x < 0 \end{cases} \tag{2}$$

$0 < \gamma, \theta < 1$ and $\lambda > 1$ (coefficient of loss aversion).

Asimetry of utility in Prospect Theory



$$v(x) = \begin{cases} x^\gamma & \text{if } x \geqslant 0 \\ -\lambda(-x)^\theta & \text{if } x < 0 \end{cases}$$

$\gamma = \theta = 0.5$ and $\lambda = 2.5$

Tversky and Kahneman (1992) estimated that $\gamma \simeq \theta \simeq 0.88$ and $\lambda \simeq 2.25$, but al Nowaihi and Dhami (2010) proved that $\gamma = \theta$.

Cummulative Prospect Theory in brief

We shall obtain prospect from the lottery

$$\ell = (y_{-m}, p_{-m}; \ldots; y_{-1}, p_{-1}; y_0, p_0; y_1, p_1; \ldots; y_n, p_n)$$

with probabilities $p_i \geqslant 0, \sum p_i = 1$ and welth outcomes

$$y_{-m} < \ldots < y_{-1} < y_0 < y_1 < \ldots < y_n$$

with y_0 as reference value.

Let us denote $x_i = y_i - y_0$ and

$$\ell = (x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0 = 0, p_0; x_1, p_1; \ldots; x_n, p_n) \qquad (3)$$

Rank dependent utility[2] for **positive part of the prospect** inspired by Choquet integral is given by

$$V^+(\ell) = \sum_{i=1}^{n} \underbrace{\left[w\left(\sum_{j=i}^{n} p_j\right) - w\left(\sum_{j=i}^{n} p_{j+1}\right)\right]}_{q_i^+} v(x_i) =: \sum_{i=1}^{n} q_i^+ v(x_i)$$

where $p_{n+1} = 0$.

Rank dependent utility for **negative part of the prospect**

$$V^-(\ell) = \sum_{i=1}^{m} \underbrace{\left[w\left(\sum_{j=i}^{m} p_{-j}\right) - w\left(\sum_{j=i}^{m} p_{-j-1}\right)\right]}_{q_i^-} v(x_{-i}) =: \sum_{i=1}^{m} q_i^- v(x_{-i})$$

where $p_{-m-1} = 0$.

[2] Quiggin (1982), Yaari (1987)

Cumulative prospect value[3] is defined as the sum of positive and negative Rank dependent utilities

$$\begin{aligned} V(\ell) &= V^{-}(\ell) + V^{+}(\ell) \\ &= \sum_{i=1}^{m} q_i^{-} v(x_{-i}) + \sum_{i=1}^{n} q_i^{+} v(x_i). \end{aligned} \tag{4}$$

Generally,

$$\sum_{i=1}^{m} q_i^{-} + \sum_{i=1}^{n} q_i^{+} \neq 1.$$

[3] with respect to the specified reference value

Dynamic referencing

Let us consider a time series[4] $(z_t)_{t=1,\ldots,k}$. We shall make a prospect from time series in the following way:

1. round the values of z,
2. calculate the relative frequences p of the rounded values y,
3. make the prospect according to (3).

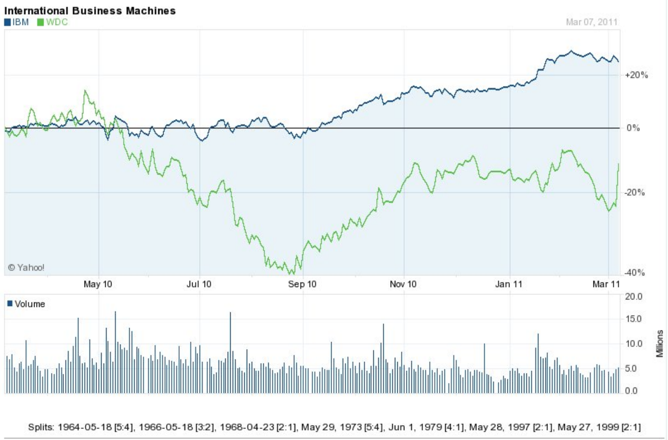
The idea of dynamic referencing is to take each y_i as the reference point and calculate the value of the corresponding prospect according to formula (4).

The sum of all those values we call the **D**ynamic **P**rospect **V**alue (DPV).

[4] share of some company in a time window

Ax example

We shall give the DPV for the shares: IBM, WDC, AAPL[5] in the period 2010/07/01 – 2011/01/03.



[5] Western Digital, Apple

Results:
The reference point for static evaluation is at the begining of the time period.

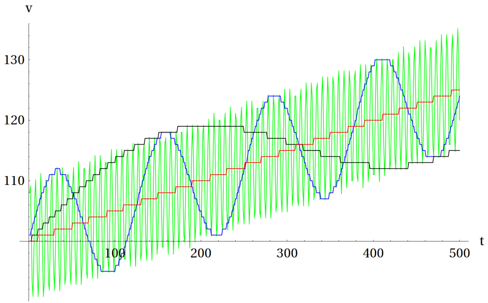
	IBM	WDC	AAPL
static eval.	3.495	-3.406	9.013
dynamic eval.	-0.43056	-1.46711	0.25247

Table 1 : Results of static and dynamic referencing: IBM, WDC i AAPL.

In both methods:

$$\text{AAPL} \succ \text{IBM} \succ \text{WCD}.$$

Simulation



Red ≻ Black ≻ Blue ≻ Green (intuition)

colour	Red	Black	Blue	Green
oscilation	No	Weak	High	Extrem
dyn. ref.	0.0041	0.0025	−0.0457	−1.703
static ref. (ref.v.=100)	3.5720	3.8416	3.5645	3.279

Table 2 : Moving reference point value of simulated shares

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