

Structure Equation Model of Heptathlon

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Run 100m hurdless



External viewer

About heptathlon (women)

Event	Day
run100 (hurdless)	First
hjump	First
shot put	First
run200	First
ljump	Second
javelin	Second
rundccc	Second

About heptathlon (women)

Event	Day	Results (Benchmark)
run100 (hurdless)	First	13.85 s
hjump	First	1.82 m
shot put	First	17.07 m
run200	First	23.8 s
ljump	Second	6.48 m
javelin	Second	57.18 m
rundccc	Second	127.63 s

About heptathlon (women)

Event	Day	Results (Benchmark)	Scores
run100 (hurdless)	First	13.85 s	1000
hjump	First	1.82 m	1000
shot put	First	17.07 m	1000
run200	First	23.8 s	1000
ljump	Second	6.48 m	1000
javelin	Second	57.18 m	1000
rundccc	Second	127.63 s	1000
Total Score			7000

Scoring function

International Association of Athletics Federations score:

$$IAAFscore(x) := a \cdot (\varepsilon \cdot (x - b))^c \quad (1)$$

Approximation: (Loglike)

$$f(x) := \lambda - \alpha * \log \left(\frac{x - a}{\beta} \right), \quad (2)$$

$$u : x \mapsto \frac{x - a}{\beta} \quad (\text{standardization}) \quad (3)$$

$x - a$ = the distance from the asymptote, β = unit length,
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Loglike approximation f enables the comparison between disciplines.

Scoring function

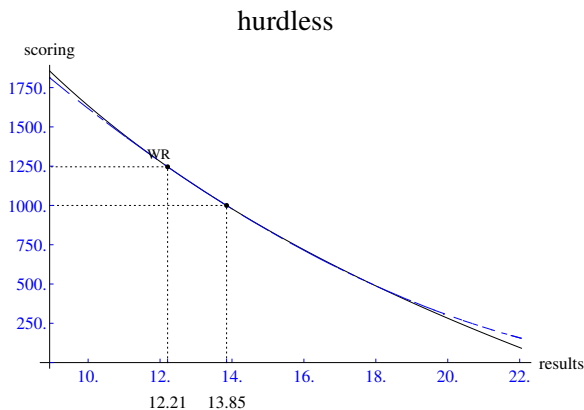


Figure 1 : Loglike-scoring (solid line) and IAAF-scoring (dashed line).

Problems (related)

There are two problems:

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All calculation is done on the results of the OI London 2012.

The first problem: Scoring

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Average score		Result \geq <i>bnchmrk</i>		Stand. Benchmarks	
Event	%	Event	%	Name	Val
run100	16.7	run800	96	run200	1.137
hjump	15.4	run200	87	run100	1.134
run200	15.1	run100	37	run800	1.127
run800	14.2	hjump	29	ljump	1.120
ljump	13.7	ljump	10	hjump	1.118
javelin	12.6	shot	1	shot	1.019
shot	12.4	javelin	0	javelin	1.011

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- Is there another evidence?

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- Is there another evidence? [▶ Later](#)

The second problem: SEM

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Event	AE	GS	Skill	RS	Speed	Mob	ES	SpE	StrE
100m	-	Med	High	High	High	High	High	Med	-
HJump	-	Low	High	High	High	High	High	-	-
Shot	-	High	High	Med	Low	Med	High	-	-
200m	Low	Med	Med	High	High	High	High	High	High
LJump	-	Low	High	High	High	High	High	-	-
Javelin	-	Med	High	High	Low	High	High	-	-
800m	High	-	Low	Low	Med	Low	-	-	High

AE=Aerobic Endurance

GS=Gross Strength

RS=Relative Strength

Speed=Running Speed

Mob=Mobility

ES=Explosive Strength

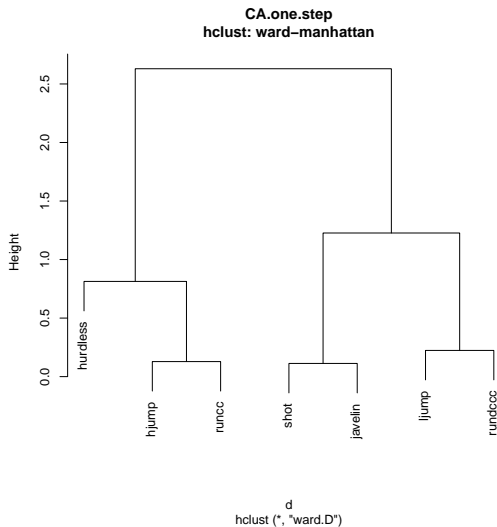
SpE=Speed Endurance

StrE=Strength Endurance

Table 1 : Motor skills impact on the event (score).

Analysis

Correspondence Analysis



One step Correspondent Analysis

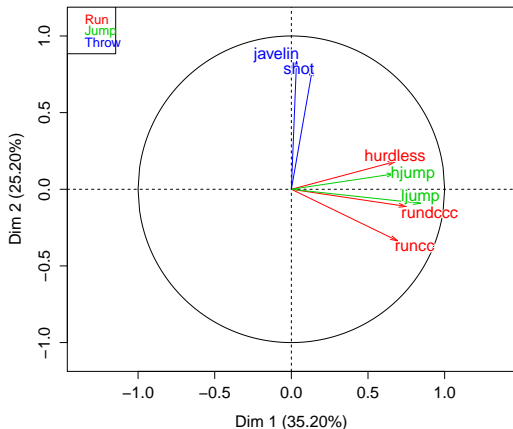
Event	w
hurdlless	14.1
hjump	13.5
runcc	13.4
rundccc	13.0
ljump	12.8
javelin	12.7
shot	12.1

R-code for one step in CA

```
residuals<-function(M){  
  M.P    <- M/sum(M)  
  M.r    <- apply(M.P,1,sum)  
  M.c    <- apply(M.P,2,sum)  
  M.Drmh <- diag(1/sqrt(M.r))  
  M.Dcmh <- diag(1/sqrt(M.c))  
  #M.res  <- M.Drmh %*% (M.P-M.r%o%M.c) %*% M.Dcmh  
  M.res  <- M.Drmh %*% (M.P) %*% M.Dcmh  
  colnames(M.res)<-colnames(M)  
  rownames(M.res)<-rownames(M)  
  return(M.res)  
}
```

Factor Analysis

Correlation circle



Loadings:

	Run	Jump	Throw
hurdless	0.886		0.305
hjump		1.008	
shot			0.406
runcc	0.863		
ljump	0.413	0.501	
javelin			0.834
rundccc	0.433	0.364	

	Run	Jump	Throw
SS loadings	1.889	1.399	0.953
Proportion Var	0.270	0.200	0.136
Cumulative Var	0.270	0.470	0.606

Marginal Scores

$$U(x_1, \dots, x_7) = \sum_{i=1}^t f_i(x_i), \quad x_i \text{ — } i\text{-th discipline.}$$

$$\text{toffM}_{ij} = \frac{i\text{-th marginal score}}{j\text{-th marginal score}} = \frac{\partial U / \partial u_i(u_{1000})}{\partial U / \partial u_j(u_{1000})}.$$

Trade-off matrix between marginal scores at *benchmark1000*

	run100	hjump	shot	run200	ljump	javelin	run800
hurdlless	1	1.66	11.2	0.99	1.6	15.57	0.96
hjump	0.6	1	6.74	0.6	0.96	9.37	0.58
shot	0.09	0.15	1	0.09	0.14	1.39	0.09
runcc	1.01	1.67	11.27	1	1.61	15.66	0.96
ljump	0.62	1.04	6.99	0.62	1	9.71	0.6
javelin	0.06	0.11	0.72	0.06	0.1	1	0.06
rundccc	1.04	1.73	11.69	1.04	1.67	16.25	1

Marginal Weights

Ranking from trade-off matrix	
Event	w
hjump	0.598
run100	0.198
run200	0.096
ljump	0.071
run800	0.023
shot	0.013
javelin	0.002

deg \rightarrow 21°

Motor Skill Matrix (msM)

Given: $semM$ — SEM matrix and $scoreM$ — score matrix

Table	hurdless	hjump	shot	runcc	ljump	javelin	rundccc
Ennis	1193.96	1050.72	813.017	1095.16	1000.	812.364	985.055
Schwarzkopf	1084.99	1012.61	845.729	909.259	943.087	894.308	958.25
Chernova	1052.87	974.936	805.681	1012.49	1019.21	789.217	971.822
Skujyte	978.972	1128.34	1016.17	849.839	927.46	882.697	818.46

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2° $scoreM * semP =: msM$ — motor skills matrix.

Table	AerEnd	GrStr	Skill	RelStr	Speed	Mob	ExpStr	SpEnd	StrEnd
Ennis	301	712	1097	999	1056	1037	1064	365	319
Schwarzkopf	281	694	1066	960	1005	1000	1027	324	292
Chernova	292	679	1049	955	1006	990	1013	336	307
Skujyte	246	721	1081	954	985	1005	1060	293	258

May be considered as a **redistribution of scores** over the skills.

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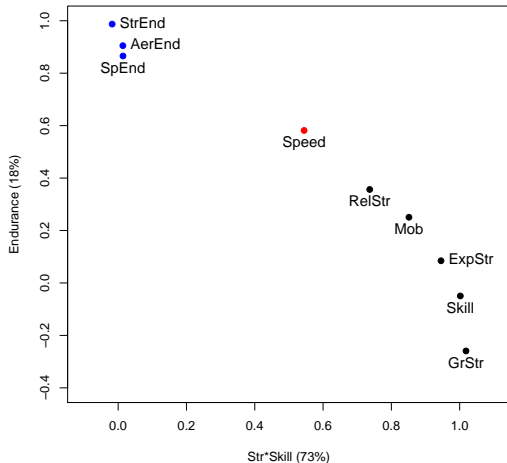
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Skills Table — Factor Analysis

Factor Analysis – Motoric Skills

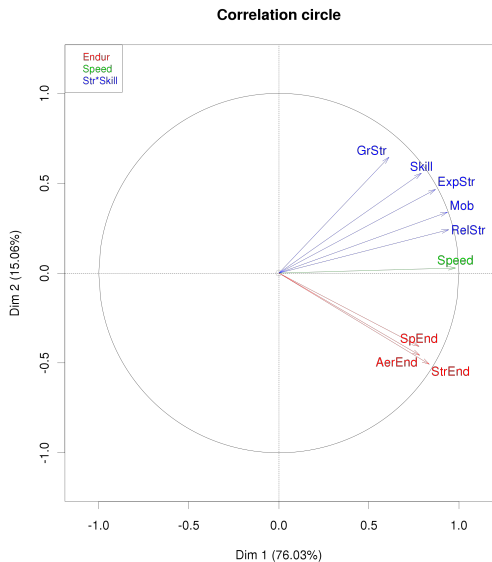


Loadings:

	Str+Skill	Endur	Eigval
AerEnd		0.905	6.544
GrStr	1.019		1.649
Skill	1.002		0.451
RelStr	0.737		0.289
Speed	0.544	0.582	0.037
Mob	0.851		0.028
ExpStr	0.945		0.002
SpEnd		0.866	0.000
StrEnd		0.987	0.000

	Str+Skill	Endur
SS loadings	4.499	2.881
Proportion Var	0.500	0.320
Cumulative Var	0.500	0.820

Skills table — Factor Analysis



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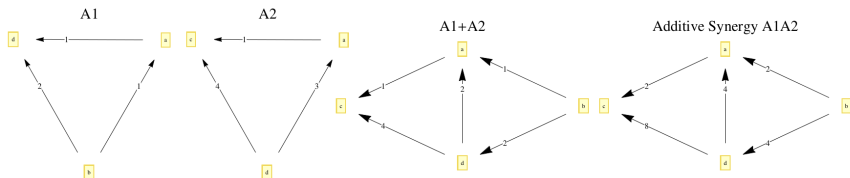
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- ▶ ?

Aggregation of synergy

Dealing with tables

	Decision table		Extended part	
	A1	A2	$A1 \oplus A1$	$A1 \circ A1$
<i>a</i>	2	4	$2 + 4$	$2 * 4$
<i>b</i>	1		??	??
<i>c</i>		5	??	??
<i>d</i>	3	1	$3 + 1$	$3 * 1$

Dealing with graphs



→ `motoric.skills.synergy.nb`

References

Heazlewood, I. T. (2011). Factor Structure of the Women's Heptathlon: Applications of Traditional Factor Analysis and Structural Equation Modelling . *Theories & Applications the International Edition*, 1(1):114–125.

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