Asymptotic stability in network with feedback and conflict resolution

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Preference Graph

A standard procedure in Decision Making (DM) is comparison (in pairs) of alternatives with respect to each criterion.

Preference graph $G = (V, \mathcal{A})$:

- the set of vertices V is the set of alternatives.
- $\alpha = (i, j) \in \mathcal{A}$ is an arc if and only if i and j are compared and i is more preferable than j.
- $\forall \alpha \in \mathcal{A}, \mathcal{F}_\alpha \geq 0$ is the intensity of the preference α.

$$(i) \xleftarrow{\alpha, \mathcal{F}_\alpha \geq 0} (j)$$

We call $\mathcal{F} : \mathcal{A} \to \mathbb{R}$ the `preference flow`.

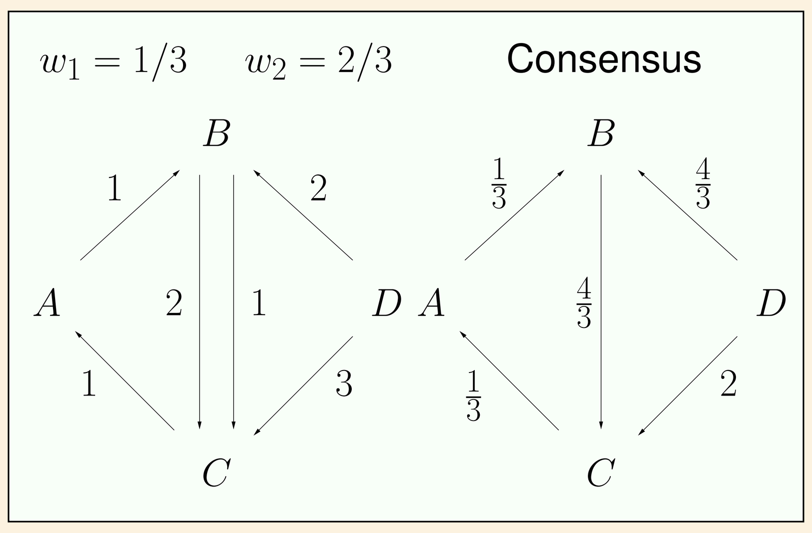
Aggregation of the flows.
`Consensus graph` $(V, \mathcal{A})$ and `consensus flow` $\mathcal{F}$ are defined as follows:

$$F_\alpha := \sum_{\substack{i=1 \\ \pm\alpha \in \mathcal{A}_i}}^{k} w_i \mathcal{F}_i(\alpha), \quad \alpha = (u, v), \qquad (1)$$

- $\mathcal{F}_i$ — the preference flow for i-th criterion C_i,
- w_i — the weight of C_i.

If $F_\alpha \geq 0$ then $\alpha \in \mathcal{A}$ and $\mathcal{F}(\alpha) := F_\alpha$, otherwise: $-\alpha \in \mathcal{A}$ and $\mathcal{F}(-\alpha) := -F_\alpha$.

An example (two criteria).



Demonstration: [use refresh button, F5]
`http://decision.math.hr/examples/graph/`

Potential Method (PM)

Let us denote:

- $n = \text{card}(V), m = \text{card}(\mathcal{A})$
- $A \in \mathbb{R}^{m \times n}$ incidence matrix of the preference graph.

Consistent flow. A preference flow $\mathcal{F}$ is `consistent` if the sum of its components along each oriented cycle is equal to zero. Because of the decomposition

$$N(A^\tau) \oplus R(A) = \mathbb{R}^m$$

the following statements are equivalent:

- $\mathcal{F}$ is consistent.
- $\mathcal{F}$ is an element of the column-space of the incidence matrix A of the preference graph.
- There exists $X \in \mathbb{R}^n$ such that $AX = \mathcal{F}$.
- Scalar product $y^\tau \mathcal{F} = 0$ for each cycle $y \in N(A^\tau)$.

Potential X of a given flow $\mathcal{F}$ is the unique solution of the normal equation associated to $AX = \mathcal{F}$,

$$A^\tau AX = A^\tau \mathcal{F}, \quad \sum_{i=1}^{m} X_i = 0 \qquad (X)$$

and ranks are calculated by formula

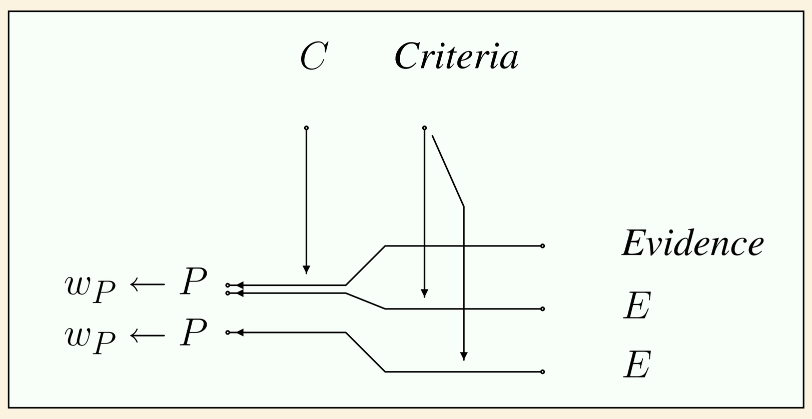
$$w = \frac{a^X}{\|a^X\|}, \quad a > 0 \qquad (w)$$

where a^X is calculated componentwise.

Measure of inconsistency $\mu(\mathcal{F})$ of the flow $\mathcal{F}$ is defined as an angle between $\mathcal{F}$ and the column space of A.

Theorem 1. *$\mathcal{F}$ is consistent iff $\mu(\mathcal{F}) = 0$.*

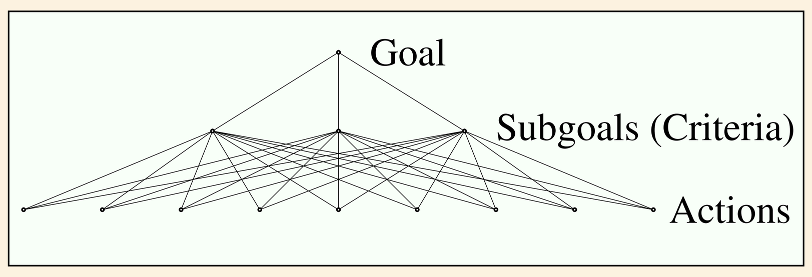
Goal oriented thinking – GOT



- ***Possibilities*** (P) lead towards the realization of a Goal. Human mind needs the ***Evidence*** (E) supported by ***Criteria*** (C) to conclude which Possibility is the best one, Baron [1].

[In medicine] Doctor is searching for evidence (laboratory result, state of the organs, ...) to select one of the several possible diagnosis.

- We represent GOT as a hierarchy with possibilities (***alternatives***, ***actions***) at the bottom of the hierarchy. The Goal is on the top.



Ranking procedure starts from the Goal. For each element of already ranked level, elements of the lower level may be ranked. Repeating this process we reach the bottom level. Methods:

- Analytical Hierarchy Process (AHP) (Saaty)
- Potential Method (PM) (Čaklović)

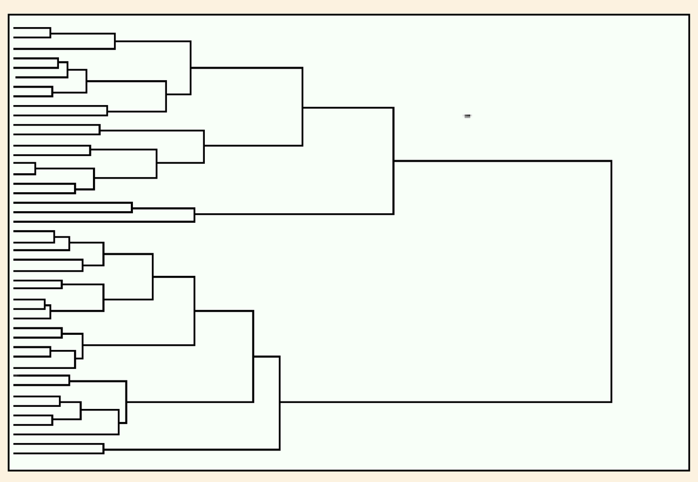
Conflict

Origin of the conflict.

- We are not sure how to trade off one attribute relative to another.
- Shafir, Simonson & Tversky [5] propose to seek and construct reasons in order to resolve the conflict.
- Different frames, contexts, and elicitation procedures highlight different aspects of the options and bring forth different reasons that influence decision, cf. Payne, Bettman, & Johnson [6].
- Seeking an additional options in criteria–alternative context means adding (or removing) a new object in the hierarchy structure.
- ***We suppose*** that DM does not change the structure of the hierrarchy and does not change the preferences of the objects inside the hierarchy.
- ***The source of the conflict*** is the ***unknown importance*** w of his goals.

Measuring conflict.

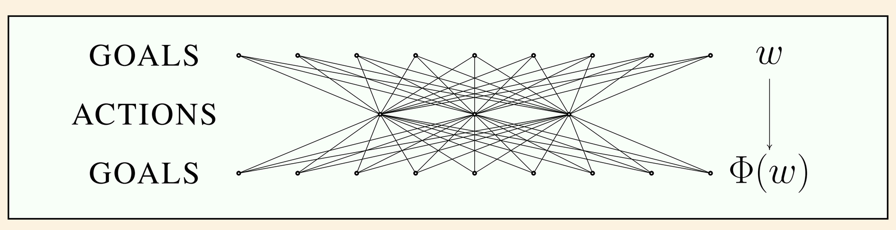
- Conflict exists whenever incompatible activities occur. An activity that is incompatible with another is one that prevents, blocks, or interferes with the occurrence or effectiveness of the second activity, Deutsch [4].
- The *size of the conflict* between two or more people can be measured by measuring dissimilarities between the preferences (Čaklović [3]).



Dendogram of a group decision. Possible conflict.

A model for conflict resolution.

- In the standard hierarchy the goals are criteria for actions...
- ...but, for each action there are some goals which support this actions more than the others.
- Each action has a tendency to rank the goals, maybe indirectly using some extra criteria.
- Hierarchy with ***feedback***:



Feedback. Self-duality

Self-duality

Duality in Decision Making.

- Dual objects are ***criteria***.
- They measure differences of the primal objects, ***alternatives***, with respect to some quality.
- In hierarchical structure, dual objects are placed in the level above the level under consideration.

Self-duality in decision process arises when some objects are also criteria for themselves.

[Self-ranking of the group] A group of Decision Makers attempt to rank themselves. Each one of them creates his own preference graph over the set of group members, including himself. Those preference graphs may be conjoined in one (consensus) graph and the group members' ranks may be calculated.

Conflict resolution using PM.

- In a self-dual hierarchical structure ***the first*** and ***the last*** level are ***equal***.
- Each element in the first level create a preference graph over the elements of the last level, i.e. over the same set, taking into account the preferences inside the hierarchy.
- Aggregation is done by formula (1).
- The problem is that we ***do not know*** the weights of the goals in the first level.
- We may start with some a priory given ranks w (1), (X) and (w) calculate new ranks $\Phi(w)$. Repeating the process

$$w \mapsto \Phi(w) \mapsto \Phi(\Phi(w)) \mapsto \cdots \Phi^n(w) \mapsto \cdots \qquad (2)$$

we expect to obtain asymptotic stability.

Theorem 2. **(Čaklović [2])** *Let X denotes a potential matrix of the group and let us assume*

$$2 \ln a \|X\|_\infty < 1$$

Then, the iterative process (2) *converge to the unique fixed point $w = \Phi(w)$ of*

$$\Phi(\xi) := \frac{a^{X\xi}}{\|a^{X\xi}\|_1}$$

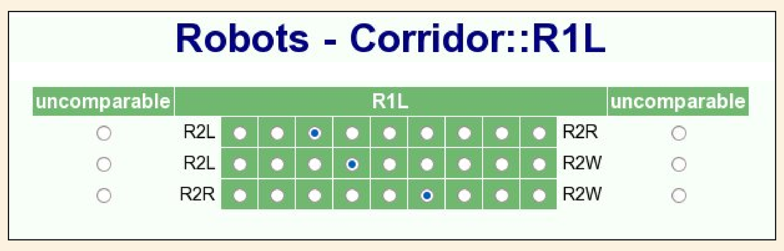
which is independent of the initial value of w.

The assumption of the theorem is mild because X is proportional to $1/n$ where n is the size of the group. Let Σ denotes the standard simplex. The following generalization is also true.

Theorem 3. *Assume that $1 \notin \text{spectrum}(\Phi'(\xi))$ $\forall \xi \in \text{Aff}(\Sigma)$. Then, Φ has a unique fixed point.*

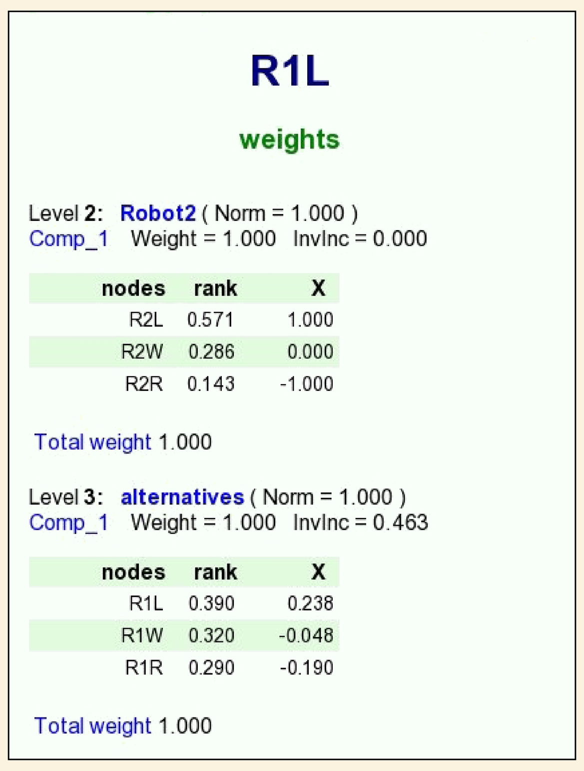
Robots – web interface

Pairwise comparison interface.



Web interface for setting preference flow.

Priority of actions with respect to R1L.



R1L

weights

Level 2: Robot2 (Norm = 1.000)
Comp_1 Weight = 1.000 InvInc = 0.000

nodes	rank	X
R2L	0.571	1.000
R2W	0.286	0.000
R2R	0.143	-1.000

Total weight 1.000

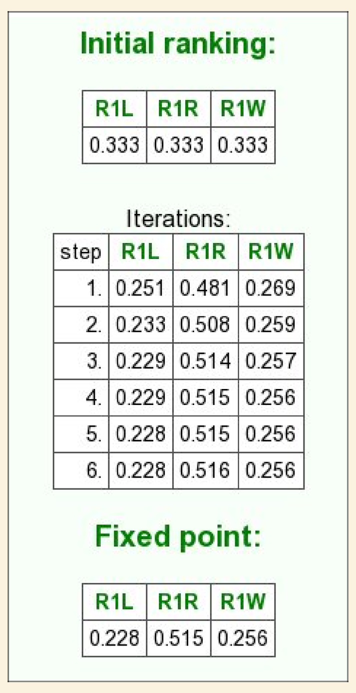
Level 3: alternatives (Norm = 1.000)
Comp_1 Weight = 1.000 InvInc = 0.463

nodes	rank	X
R1L	0.390	0.238
R1W	0.320	-0.048
R1R	0.290	-0.190

Total weight 1.000

Ranks of the (both) robots movements if the first robot turns left.

Selfranking $(\epsilon = 0.0001)$.



Initial ranking:

R1L	R1R	R1W
0.333	0.333	0.333

Iterations:

step	R1L	R1R	R1W
1.	0.251	0.481	0.269
2.	0.233	0.508	0.259
3.	0.229	0.514	0.257
4.	0.229	0.515	0.256
5.	0.228	0.515	0.256
6.	0.228	0.516	0.256

Fixed point:

R1L	R1R	R1W
0.228	0.515	0.256

Fixed point for the first robot.

Intelligent mobile robots

The problem. Two robots are passing through the corridor facing each other. To avoid the crash they can turn:

right (R), left (L), wait (no action) (W).

Mathematical model. If the first robot is ***waiting (W)***, the second robot prefers moving (R) or (L) when compared with (W), and moving (R) is preferred to (L). More precisely:

$$\mathcal{F}^W_{(W,L)} = 0, \mathcal{F}^W_{(R,W)} = 2, \mathcal{F}^W_{(R,L)} = 2. \quad \text{(W-flow)}$$

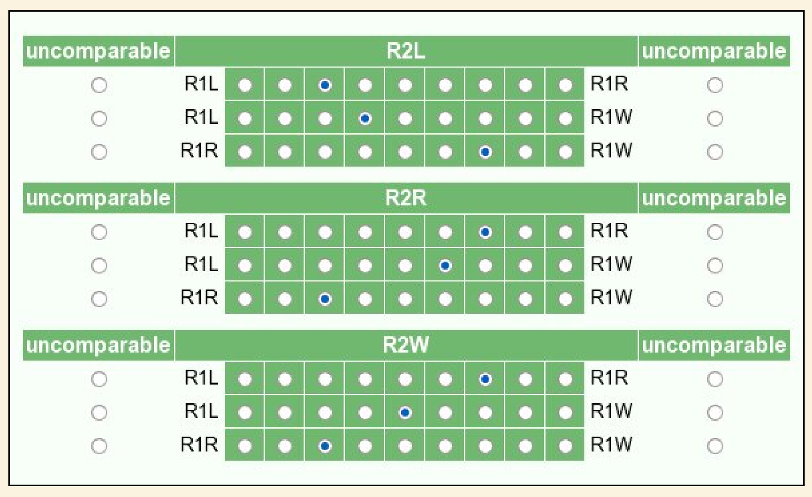
If the first robot ***turns left (L)*** the response of the second robot is structured by the preference flow $\mathcal{F}^L$:

$$\mathcal{F}^L_{(L,W)} = 1, \mathcal{F}^L_{(W,R)} = 1, \mathcal{F}^L_{(L,R)} = 2. \quad \text{(L-flow)}$$

If the first one turns ***right (R)***, the response of the second is structured by the preference flow $\mathcal{F}^R$:

$$\mathcal{F}^R_{(W,L)} = 1, \mathcal{F}^R_{(R,W)} = 2, \mathcal{F}^R_{(R,L)} = 2. \quad \text{(R-flow)}$$

These preferences are written in the memory of both robots.



Preferences of the robot's responses.

Hierarchy.

- In the first level are options of the first robot, in the second level are options of the second robot.
- Ech element in one level is a criterion for the elements in the other level (***feedback***).
- The third level is equal to the first one.

	actions		
1st robot	R1L	R1W	R1R
2nd robot	R2L	R2W	R2R
1st robot	R1L	R1W	R1R

Levels of the self-hierrarchy.

Calculation.

- The flows for each root (L, R, W) are defined by (L-flow), (R-flow) and (W-flow).
- Iteration process (2) gives the folowing values for fixed point (with precision $\epsilon = 0.0001$).

step	R1L	R1R	R1W
1.	0.251	0.481	0.269
2.	0.233	0.508	0.259
⋮	⋮	⋮	⋮
6.	0.228	0.516	0.256

Fixed point for the first robot.
(R) option has the highest rank.

Conclusion.

- Moving right (R) has the highest rank, than wait (W) and moving left (L) has the lowest rank.
- This is so because we encourage moving right and discourage moving left in the preference flows.
- The reader can perform ranking procedure starting the `robotscorridor.php` script on URL `http://decision.math.hr/examples/`.

References

[1] Jonathan Baron, *Thinking and deciding*, Cambridge Univ. Press, 1994.

[2] Lavoslav Čaklović, *Self–duality in Group Decision*, Preprint, `http://decision.math.hr/papers/selfduality.pdf`.

[3] ______, *Graph Distance in Multicriteria Decision Making context*, Metodološki zvezki (Advances in Methodology and Statistics) **1** (2003), no. 19, 25–34.

[4] Morton Deutsch, *The resolution of conflict: Constructive and destructive processes*, CT: Yale University Press, 1973.

[5] Shafir Eldar, Itamar Simonson, and Amos Tversky, *Reason–based choice*, Cognition **49** (1993), 11–36.

[6] J.W. Payne, J.R. Bettman, and E.J. Johnson, *Behavioral decision research: a constructive process perspective*, Annual Review of Psychology **43** (1992), 87–131.